Review

for the competition for the academic position
"Professor"
in the Professional Field 4.5 Mathematics
(Ordinary differential equations, Hamiltonian systems)
for the needs of the Sofia University "St. Kliment Ohridski", Faculty of Mathematics and Informatics (FMI of SU),
announced in Newspaper of State, No 56 from 30.06.2023 and on the Internet pages of FMI and SU "St. Kliment Ohridski"

The review is written by Prof. DSc Mikhail Ivanov Krastanov from Михаил Иванов Кръстанов from FMI of SU as a member of the Scientific Jury for the competition by Order № РД 38-519/29.08.2023 of the Rector of the Sofia University.

The only applicant who has applied for the position is:

Assoc. Prof. DSc Ognyan Borisov Ghristov

from the Department of Differential equations at the Faculty of Mathematics and Informatics of the Sofia University "St. Kliment Ohridski".

I. Description of the presented documents

1 Information concerning the documentation

The documentation presented by the applicant is in accordance with the requirements of the Law on the Development of the Academic Staff in the Republic of Bulgaria (LDASRB), the Regulations for the application of LDASRB and the Regulations for the conditions and procedures for holding academic positions in SU and the accompanying rules of the Sofia University.

To participate in the competition, the candidate, Associate Professor DSc Ognyan Borisov Hristov, presented a list of 6 titles of publications in foreign scientific journals. A total of 26 other documents were also presented. The submitted documents include: the announcement in the State newspaper; application for participation in the competition; resume; diplomas for Master's, Doctor's degrees, doctor of science and associate professor; office memo and certificate for work experience in the specialty, issued by the SU; documents showing meeting the minimum requirements; list of all publications; list of publications as well as files with themselves publications and their summaries with which he participates in the competition; reference from the systems "The Authors"; author reference of the contributions in the articles, presented for the competition; reference for the execution of the additional indicators from art. 122 of LDASRB; list of citations; reference for his participation in various research projects; lectures on ordinary differential equations, dynamical systems and Hamiltonian systems.

All documents are well designed and presented in a work-friendly format. The author reference of the presented results is comprehensive and correctly reflects the scientific contributions in the articles presented in this competition.

2 Information for the applicant

Associate Professor DSc Ognyan Borisov Hristov completes his higher education in the Faculty of Mathematics and Informatics of SU "St. Clement Ohridski" in 1984 with a specialization in Stability and control of mechanical systems. In 1994 he received educational and scientific degree "candidate of mathematics sciences", and in 2017 he defended his dissertation work "Algebraic, analytical and geometric studies on some finite-dimensional and infinite-dimensional Hamiltonian systems" and was awarded the scientific degree "Doctor of Science" in the scientific specialty 4.5 Mathematics (Differential Equations). In the period 1986 - 1989, he worked as assistant professor at "Angel Kanchev" Technical University, Ruse. From 1991 until now, he has been working at the University of St. Kliment Ohridski". At first as an assistant (from 1991 to 2001), from 2001 until now - as associate professor. I should note here that Associate Professor Hristov was Head of the Differential Equations Department since April 2018. until March 2019. I was impressed that Associate Professor Hristov was visiting professor at one American university, as well as at many prestigious European universities: University of Karlsruhe, Germany 1993-94, Warsaw University, Poland 1997, Humboldt University, Germany 2000, University of L'Aquila, Italy 2004, University of Kansas, USA 2012, University of Groningen Holland, 2012, University of Utrecht Holland, 2012, Warsaw University of Technology, Poland 2014.

3 General characteristic of the scientific work and achievements of the applicant

All 6 articles submitted for participation in the competition are published in well known mathematical journals. The paper [37] is published in Nonlinear Dynamics (with impact factor 5.022/2020 and falls into quartile Q1); three articles have been published in journals, falling into groups with quartile Q2 (the article [36] was published in Symmetry (with an impact factor of 2.645/2019), the article [38] has been published in European Physical Journal Plus (with impact factor 3.4/2021), the article [39] was published in Discrete and Continuous Dynamical Systems - Series B (with impact factor 1.2/2021)); the article [35] is published in Advances in Mathematical Physics (with impact factor 0.936/2018 and falls into a group with quartile Q3) and the article [23] is in Lecture notes in computer sciences (with impact rank 0.302/2018). All articles are standalone. In addition to summaries of the articles of Bulgarian and English, associate professor Hristov presented for us facilitation and reference for their scientific contributions.

Here I must note that the scientific publications have not been used in previous applications and no plagiarism has been established in them.

In the table below, it can be seen that the scientific works correspond to the minimum national requirements (according to Art. 2b, paras. 2 and 3 of the LDASRB) and respectively of the additional requirements of SU “St. Kliment Ohridski” to occupy the academic position "Professor"position in the scientific field and professional direction of the competition.

Group	**А**	**Б**	**В**	**Г**	**Д**	**Е**
Minimum points required	50	–	100	200	100	100
Points achieved	50	–	105	225	200	170

4 Characteristics and evaluation of the teaching activity of the applicant

Associate Professor Hristov’s teaching activity began in 1986. as an assistant at "Angel Kanchev" Technical University, Ruse. From 1991 and until now he works as a lecturer at the University of St. Clement Ohridski”. From his appointment until now, Assoc. Prof. Hristov has full auditorium employment. He teached seminars in linear algebra, geometry, analysis, ordinary and partial differentials equations, numerical methods and analytical mechanics. Next he teaches lectures of the differential and integral calculus courses, and ordinary and partial differential equations. I should note that he also has many specialized courses, such as Hamiltonian systems and chaos: analytical methods - summer semester 1997; non-integrable dynamic systems. motion of a rigid body around a fixed point - summer semester 1998; dynamical systems - summer semester 2000, 2009, 2013, 2022 Winter Semester 2002; Hamiltonian systems - winter semester 2001, summer 2021, 2022; differentiable approach in theory of general economic equilibrium - summer semester 2003, 2004;. analytical theory of differential equations - winter semester 2005; microeconomics - winter semester 2006, 2007, 2008; calculus of variations and applications in economics - summer semester 2006, 2007, 2008, 2009; algebraic groups and differential theory of Galois - winter semester 2015, 2022; matrix groups (together with A. Bozhilov) - winter semester 2021, 2022. Here I should note that he has lectured on dynamical systems for PhD students at the University of Tuzla, Bosnia and Herzegovina, in May 2011.

Associate Professor Hristov has written lectures on ordinary differential equations, by Dynamical systems and Hamiltonian systems. As far as I know, he is respected colleague in the Faculty of Mathematics and Informatics. He has many master students: Hristo Iliev (he works at Bulgarian Academy of Sciences and the American University), Nikolay Dimitrov (he works in Canada), Petya Hristova, Petya Brainova, Hermina Garabedova, Ioana Yordanova, Ventsi-slava Boteva, Katya Zafirova, Penka Zafirova, Stanislav Andreev, Svilen Popov, Anton Bykov, as well as the successfully defended PhD student: Georgi Georgiev 2015 (he works at FMI, SU).

5 Analysis of the scientific achievements of the applicant contained in the documents and publications presented for the competition

I will present briefly the submitted publications for participation in the competition:

In the article [23], the stationary solutions of a partial differential equation (known as the equation of Fischer-Kolmogorov-Petrovsky-Piskunov) are studied, assuming that the diffuse parameter $\varepsilon > 0$ is small. It is formulated a singularly perturbed boundary value problem on the interval [0, 1]. Such problems admit solutions that contain internal boundary layers known in the literature as "spikes". The first assertion concerns the estimation of the maximal number of internal boundary layers. Then there are studied the possible solutions for which the boundary layers appears only at the endpoints of the interval [0, 1]. The following result holds: For small values of the parameter $\varepsilon > 0$ there exist exactly four solutions that do not have internal boundary layers and they are approximated uniformly by explicitly given functions with exponentially small error. Also, there are studied the possible solutions with n internal boundary layers. It is proven similar result, where the positions of the internal boundary layers are calculated approximately

The rest of the papers presented in the competition study Hamiltonian systems. Hamiltonian systems are used as models in almost all physics. They provide a convenient way to describe the motion of systems according to the laws of the classical mechanics. Convenience is that the Hamiltonian scalar function encodes all the information of the $2n$ first-order dynamical equations. Hamiltonian approach, however, gives much more than this simplification. Indeed, if we allow more general functions $H(q, p, t)$ and a more general relation between canonical moments and velocities, then almost all models of classical physics can be formulated by Hamiltonian system (including the electromagnetic forces which are cannot be obtained from scalar potential). But most impressive to me is the fact that the quantum mechanics can be derived formally also from the classical mechanic model by replacing the canonical momentum in the Hamiltonian by a differential operator.

The Hamiltonian structure of a system imposes strong constraints on its solutions: When the Hamiltonian H does not depend on the time (i.e. the system is autonomous), the energy of the system $E := H(q,p)$ is constant. Similarly, if the Hamiltonian is independent of one of the configuration variables, then this one variable is constant. This provides a simple explanation for the relationship between the symmetries of the system and its invariants. And that's it the meaning of Emma Noether's theorem.

An important geometric corollary is that knowing the n invariants, it is sufficient to characterize fully the $2n$ solution equations for a Hamiltonian system with n degrees of freedom. This follows from the Liouville's integrability theorem. Furthermore, if the orbits of such a system are bounded, then almost all orbits must lie on an n-dimensional torus. This strong structural persistence of Hamiltonian dynamics is unexpected even in the middle in the 20th century, when physicists began the first computer simulations of dynamical systems.

A dynamic system is integrable when it can be solved analytically. This can almost never be done (in terms of the elementary functions). There is a class of Hamiltonian systems (described by the action and angle variables), whose solutions can be obtained analytically and there is well adopted definition of integrability for Hamiltonian dynamics, due to Liouville, in which each integrand is Hamiltonian system is locally equivalent to one of these systems. When a Hamiltonian system with two or more degrees of freedom is not equivalent to a system of this class, it usually has chaotic movement.

Let an analytic function $H_0(p,q)$ be given and let the corresponding Hamiltonian system is integrable, i.e. there exist n independent first integrals $F_1, F_2, \ldots, F_n$, which are in involution. According to the Liouville-Arnold Theorem, when the set

$$M^n := \{(p,q) \in R^{2n} : \ F_i(p,q) = c_1, i = 1, \ldots, n\}$$

is compact, then it is diffeomorphic to an n-dimensional torus T^n that is invariant with respect to the trajectories of the system. Locally, one can introduce the variable action - angle (I, φ) and the Hamiltonian system yields the form

$$\dot{I} = 0, \ \dot{\varphi} = \frac{\partial H_0}{\partial I},$$

For Hamiltonian systems, a vector $\omega = (\omega_1, \ldots, \omega_n)$ is defined in a natural way. Its components are called frequencies. The frequencies are said to be resonant if there exist integers k_i, $i = 1, 2, \ldots, n$ such that $k_1\omega_1 + k_2\omega_2 + \cdots + k_n\omega_n = 0$. If the frequencies are non-resonant, then the motion is conditionally periodic with frequencies $\omega_1, \ldots, \omega_n$, and the trajectory of the upper Hamiltonian system is a dense set of points in the M^n torus.

Suppose now that the system is slightly different from an integrable system, i.e., consider the following perturbed Hamiltonian of the small parameter $\varepsilon > 0$:

$$H = H_0(I) + \varepsilon H_1(I, \varphi) + \cdots, \ H_1(I, \varphi + 2\pi) = H_1(I, \varphi).$$

The main question is how this perturbation affects the trajectories. Specifically, are the trajectories destroyed or preserved? Andrei Kolmogorov gave an answer to this question in 1954: If the Hamiltonian function is analytic and 2π - periodic, with respect to φ, and the so-called condition is satisfied of Kolmogorov

$$\det\left(\frac{\partial^2 H_0}{\partial I_j \partial I_k}\right) \neq 0,$$

then the torus M^n (which is obtained for $\varepsilon = 0$) only slightly is deformed for small $\varepsilon > 0$. This was rigorously proven by Vladimir Arnold in 1963 (for analytic Hamiltonian systems). Jurgen Moser proved in 1962 a similar result for one class of smooth maps, and the general the result is known now as the KAM theorem.

The paper [35] studies the integrability of a Hamiltonian system, given by the following Hamiltonian:

$$H = \sum_{j \in Z} \left[\frac{p_j^2}{2} + \frac{C}{2}(q_{j+1} - q_j) + V(q_j)\right], \dot{q}_j = p_j. \tag{1}$$

Here the constant $C > 0$ measures the interaction between two neighboring particles (with unit masses) and $V(x)$ is a nonlinear potential. This Hamiltonian is called a Klein-Gordon chain. The case where $C = 1$. This article assumes that

$$V(x) := \frac{a}{2}x^2 + \frac{b}{2}x^4, \text{ where } a > 0 \text{ is irrational number.}$$

The paper considers a periodic chain, assuming that the Hamiltonian system corresponding to (1) has n degrees of freedom. The cases that correspond to values of $n = 2, 3, 4, 5, 6$. The following two results were obtained:

Theorem 2. The Klein-Gordon periodic chain is integrable at $n = 2$ only for $b = 0$.

Theorem 3. The periodic normal form $\bar{H} = H_2 + \bar{H}_4$ of the Klein-Gordon chain is
(i) fully integrable and KAM nondegenerate when n =2,3,4;
(ii) fully integrable and KAM nondegenerate when $n = 5$ for all a except for the case $a = 5(3\sqrt{5} - 5)/16$;
(iii) fully integrable for $n = 6$.

The paper [36] is a generalization of the paper [35]. It again considers a Hamiltonian system set from a periodic Klein-Gordon chain for the case $C = 1$ and a function

$$V(x) := \frac{a}{2}x^2 + \frac{b}{2}x^4, \text{ where } a > 0.$$

Again, we denote by n the number of degrees of freedom. The following assertion is proven:

Theorem 1. The Klein-Gordon periodic chain is integrable only for $b = 0$.

The Galois differential group is used in the proof and the approach developed by Morales-Ramis-Simo.

The system under consideration possesses the important discrete R and S symmetries. Using these symmetries, an R, S-symmetric is constructed resonant fourth-order normal form $\bar{H}$. This normal one form turns out to be Liouville integrable. Therefore, the periodic A KG chain can be seen as a perturbation of the integrable normal form of Birkhoff. If n is odd, the integrals of normal form are quadratic. Then one can introduce the global variables action-angle. It turns out that this normal form is KAM non-degenerate. And this proves the existence of quasi-periodic low energy level solutions. When n is even, the resonance normal form also allows the existence of integrals that are not quadratic. The following is proved

Theorem 2. Let $a = 1$. Then the normal form of fourth order $\bar{H} = H_2 + \bar{H}_4$ of the periodic chain of Klein-Gordon is:

(i) fully integrable and KAM non-degenerate when the number of the degrees of freedom n is an odd number;

(ii) fully integrable when n is an even number.

Next, a Klein-Gordon chain with n particles is considered and fixed boundary conditions. The normal form of is calculated fourth row. This normal form is shown to be completely integrable, has n number of quadratic first integrals, and is KAM non-degenerate. And from here it turns out that almost all low-energy solutions are quasi-periodic.

In [37], an analytical Hamiltonian with equilibrium at zero is considered. The second-order normal form is assumed to be positive definite, and the frequencies are in 1:2:2 resonance. is studied normal form integrability up to order four. This shape contains too many parameters, making a complete analysis difficult this integrability task. Ferhulst propose the hypothesis, that the fourth-order terms are an obstacle for integrability when they are in general position. Integrability analysis has done in the complex domain using only variational equations of first order by applying the approach of Ziglin-Morales-Ruiz and Ramis. Main tool are different techniques from differential Galois theory and the important observation of Lyapunov that if the general solution of the equation in variations is not unique, then the nonlinear system does not admit an analytic first integral. Using this Lyapunov observation, it is shown that the existence of an additional first integral reduces to the solution of an appropriate linear system with respect to the parameters. And the solution of this linear system leads to the existence of a non-trivial first integral. I have to note that when the analysis is too difficult, numerical experiments have been done. These experiments show chaotic behavior of the trajectories, which corroborates non-integrability.

It is shown in a paper by I. Fakkousy, J. Kharbach, W. Chater, M. Benkhali,

A. Rezzouk, M. Quazzani-Jamil from 2020 that the three-dimensional Henon-Hyles system is integrable in the sense of Liouville at certain parameter values. The numerical experiments, done by the above authors for parameter values close to to the calculated ones, show a chaotic behavior of the trajectories. And this precludes integrability. However, there is a theoretical possibility that other cases of integrability exist, for parameter values that are far from the founded ones. Using the Morales-Ramis approach to the variational equations up to the order three, it was proved in [38] that no other values exist of the parameters for which this system is integrable.

In [39], an analytical Hamiltonian with equilibrium at zero is considered. The second-order normal form is assumed to be positive definite, and the frequencies to be in resonance. It is studied the integrability of the normal form up to order three, aiming to find more integrable cases when the frequencies are in proportion $1 : 2 : 1 : 2$ resonance. For this purpose, the normal form is simplified: first, the quadratic terms are eliminated and then using of a unitary transformation, two matrices are diagonalized simultaneously. As a result, a normal form is obtained depending only on four parameters. For the normal form thus simplified there are found two new integrable cases obtained for particular values of the parameters. A non-integrability theorem is proved for the case, when parameter values are different from these found values. The proof uses the approach of Morales-Ruiz and Ramis, and the Lyapunov's observation.

The papers submitted for participation in the competition clearly show, that Associate Professor DSc Ognyan Hristov is a highly qualified specialist in the area of differential equations. His papers on integrability of Hamiltonian systems made me strong impression with their depth and the non-triviality of the obtained results.

6 Critical remarks and recommendations

I have no critical remarks and recommendations

7 Personal impressions for the applicant

I have known associate professor Ognyan Hristov for about 30 years. He stands out for precision, high scientific morality, criticality and self-criticism towards one's scientific and pedagogical activity. Possesses a high collegial spirit. He has a well-deserved authority not only among colleagues from the Faculty of Mathematics and Informatics, but also throughout all mathematical society in Bulgaria.

8 Conclusion for the application

Conclusion for the application After my careful and critical reading of the documentation and the publications presented for the competition and my analysis of their significance and the scientific and scientific-applied contributions **I confirm** that the scientific contributions are sufficient (as required by the law and the additional requirements of the Sofia University) for the position ”Professor“ in the scientific field of the competition. In particular, the applicant satisfies the minimal national requirements for the scientific field and there is not a plagiarism in the presented publications for the competition.
I give my **positive evaluation** for the application.

II. CONCLUSION

Based on the above, I **strongly recommend** the Scientific Jury to suggest that the Council of the Faculty of Mathematics and Informatics of the Sofia University ”St. Kliment Ohridski“ to elect Associate Professor DSc Ognyan Borisov Ghristov for the academic position ”Professor“ in the professional field 4.5 Mathematics (Ordinary differential equations, Hamiltonian systems).

27.10.2023 Signature:
Sofia

/Prof. DSc Mikhail Ivanov Krastanov/