

OPINION

In connection with the procedure for the defense of a PhD Thesis on the topic:

"Local Properties of Dynamical Systems"

Presented for the acquisition of the educational and scientific degree "doctor"

candidate: **Margarita Nikolaeva Nikolova**

Field of higher education: **4. Natural sciences, mathematics and informatics** Professional

direction: **4.5. Mathematics**, PhD program: **"Operations Research"**, Department:

"Probability, Operations Research and Statistics",

Faculty of Mathematics and Informatics (FMI),

Sofia University "St. Kliment Ohridski" (SU)

The opinion is prepared by Professor DSc Petar Stoyanov Kenderov, retired, associate member of the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences, member of the scientific jury, according to Order No. RD 38-383/12.07.2023 of the Rector of Sofia University.

1. General characteristics of the PhD Thesis and the presented materials

The PhD thesis is presented in English. It contains an Introduction, four chapters, a Conclusion and a Bibliography. The thesis consists of 60 pages. The bibliography covers 38 papers, three of which are joint articles of the candidate and her supervisor (published in the period 2020-2023). The earliest cited references are from Brunovsky (1976), Krener (1977), Agrachev and Gamkrelidze (1978), and Sussmann (1978). In addition to those mentioned, many other names of recognized authorities in the field of the PhD thesis are presented in the cited literature.

An Abstract in Bulgarian (38 pages) and a translation of the Abstract in English (37 pages) are also presented. In addition to the PhD thesis, the Abstract and the Abstract in English, I was provided with all the documents required by the defense procedure (21 in total), equipped with the necessary signatures and certifications. The candidate's academic supervisor is Prof. Mihail Ivanov Krastanov, who is a well-known specialist in the field of the PhD thesis.

2. Data and personal impressions about the candidate

Margarita Nikolaeva Nikolova graduated her secondary education at the famous National Science and Mathematics High School "Academic Lyubomir Chakalov" in 2009. She received her bachelor's degree in mathematics from the Faculty of Mathematics and Informatics of Sofia University (in 2013). In 2015, she completed a three-semester master's education in the same faculty, majoring in Applied Mathematics (Optimization). According to the Master's degree, the report from the State Examination is dated 31.11.2018, and the diploma itself was issued on 19.12.2019. Almost all exam grades passed during higher education, with the exception of only 2-3, are "excellent 6". Margarita Nikolova was enrolled in doctoral studies by order of the Rector of Sofia University from 16.07.2019. The doctoral studies were terminated on 15.07.2022 with the "right to defense the degree".

Already during her master's study from 2013, she began working as a part-time assistant at the Faculty of Mathematics and Informatics of Sofia University "St. Kliment Ohridski". She has delivered seminars on Differential and Integral Calculus (Part 1 and Part 2), Mathematical Analysis, Introduction to Statistics, Random Processes.

She also participated in the scientific research developed at the Faculty of Mathematics and Informatics. She was a member of the research teams in a total of 10 research projects. Eight of them were financed by the Scientific Research Fund of Sofia University, one was under the Scientific Research Fund of the Ministry of Education and Science, and she currently participates in project No. BG-RRP-2.004-0008-C01 from the SUMMIT program, SU "St. Kliment Ohridski", in line with the "National Plan for Recovery and Sustainability of the Republic of Bulgaria".

I have not had direct contact with the candidate and I cannot judge about her personal qualities, but I have witnessed goodwill and respect to her on the side of her colleagues. This, in itself, is a good certificate of her personal qualities.

3. Content analysis of the candidate's scientific and scientific-applied achievements, contained in the presented PhD thesis and the publications to it, included in the procedure.

The problems considered in the PhD thesis are from the field of Controllability of dynamic (with respect to the time t) systems Σ , where the dynamics of a point $x(t) \in \mathbf{R}^n$ in the time interval $[0, T]$ is determined by the systems of ordinary differential equations (ODE) $\dot{x}(t) = f(x(t), u(t))$, where $f: \mathbf{R}^n \times \mathbf{R}^m \rightarrow \mathbf{R}^n$ is a continuous map. *Admissible*

trajectories of the system are all absolutely continuous curves $\mathbf{x}(t): [0, T] \rightarrow \mathbf{R}^n$ for which there exists a measurable map $\mathbf{u}(t): [0, T] \rightarrow \mathbf{R}^m$, such that the ODE system is satisfied for almost every t from $[0, T]$. The map \mathbf{u} is called a *control* and satisfies the additional requirement that its values belong to some given in advance compact and convex subset U of \mathbf{R}^m .

The set-theoretical union of the endpoints at the time T of all admissible trajectories starting from a given point \mathbf{x}_0 at the zero time is called *reachable set in time T* . By definition, the system Σ is *small-time locally controllable (STLC)* at the point \mathbf{x}_0 if and only if the reachable set in time T contains \mathbf{x}_0 in its interior for all $T > 0$. The STLC property is an important characteristic of the system not only from a theoretical but also from a practical point of view. As with many other problems related to applications in practice, finding conditions that are both necessary and sufficient for the small-time local controllability of a sufficiently large (general) class of controllable systems remains an open challenge. A number of conditions have been found for different classes of systems that are only sufficient for the STLC property (cf., for example, Sussmann (1978, 1983, 1987), Bianchini and Stefani (1993), Hirschorn and Lewis (2004), Krastanov (2009)). There are also several necessary conditions for the STLC property (cf., for example, Stefani (1986), Kawski (1987), Krastanov (1998)). The dissertation contains new results in this direction, by building on the existing knowledge in this area. Two sufficient conditions for STLC are found (Theorem 3.4.1 and Corollary 4.1.2 of Theorem 4.1.1), as well as one necessary one (Theorem 5.1.1 and its corollaries). A curious detail here is that the proven necessary condition for the STLC property is formulated as a sufficient condition for the absence of the STLC property.

A significant contribution of the PhD thesis is the presentation of interesting examples of controllable systems that demonstrate the applicability of the obtained results and highlight their advantages in comparison with the results obtained by other authors. The basis of the considerations in the PhD thesis is the fact that the values at the starting point of the elements of the Lie algebra generated by the vector fields of the system contain significant information about the local properties of the reachable set. The skillful handling with the differential-geometric nature of the problem, with the topological essence of the conclusions (the reachable set must contain the initial point in its interior) and with the use of a contemporary mathematical techniques (Lie brackets and Lie algebras) are important characteristic features of the PhD thesis.

4. Approbation of the results

In the dissertation, three publications are indicated in connection with the obtained results. All publications are in journals with impact factor. Two of the papers are in elite journals (falling into quartiles Q1 and Q2).

Results of the PhD thesis have been reported at three international conferences and at numerous optimization seminars at the Faculty of Mathematics and Informatics of Sofia University "Kliment Ohridski".

From the documents provided to me in connection with this procedure, it can be seen that the minimum national requirements (according to Art. 2b, paras. 2 and 3 of the LDASRB) and the additional requirements of SU "St. Kliment Ohridski" for the acquisition of the educational and scientific degree "doctor" in the scientific field and professional direction of the procedure are filled with a huge surplus.

The plagiarism check was negative.

Until now, the candidate has not participated in procedures for obtaining a scientific degree and/or a scientific position, which excludes the possibility that these results and works of her have already been used in such a sense.

5. Qualities of the abstract

The abstract correctly and fully reflects the content of the PhD thesis. The handling of the literature (both in the abstract and in the dissertation) makes a particularly good impression. The contributions of other authors are noted with the necessary detail and expertise.

6. Critical notes and recommendations

I have no major criticisms. In the expressions in the English language, there are sometimes roughnesses, but this does not lead to inaccuracies in the statements.

The paper of

Cesar O. Aguilar и Andrew D. Lewis, *Small-time local controllability for a class of homogeneous systems* (SIAM Journal on Control and Optimization, vol. 50, No. 3, pp. 1502–1517)

contains a necessary and sufficient condition for the STLTC for a class of controllable systems of interest. The approach is analytical (without Lie brackets and Lie algebras). Is it possible to use the methods of the PhD thesis to obtain more general and stronger results?

7. Conclusion

Having become acquainted with the PhD thesis presented in the procedure and the accompanying scientific papers and on the basis of the analysis of their importance and the scientific and applied contributions contained therein, **I confirm** that the presented PhD thesis and the scientific publications to it, as well as the quality and originality of the results and achievements presented in them, meet the requirements of the law LDASRB, the Rules for its Implementation and the corresponding Rules at the Sofia University “St. Kliment Ohridski” (FMI-SU) for acquisition of educational and scientific degree “Doctor” in the Scientific field **4. Natural Sciences, Mathematics and Informatics**, Professional field **4.5. Mathematics**, doctoral program *Operations Research*. In particular, the candidate meets the national requirements in the professional field and no plagiarism has been detected in the published scientific papers.

Basing on what was written above, **I recommend** to the scientific jury to award Margarita Nikolaeva Nikolova the educational and scientific degree “Doctor” in the Scientific field 4. Natural Sciences, Mathematics and Informatics, Professional field 4.5 Mathematics.

07.10. 2023

Reviewer:

/Prof. DSc Petar St. Kenderov/