

REVIEW

under the procedure for acquisition of the educational and scientific degree “Doctor”
by candidate Margarita Nikolaeva Nikolova,
of the PhD Thesis entitled: “Local properties of dynamical systems”,
In the Scientific field: **4. Natural Sciences, Mathematics and Informatics**
Professional field: **4.5. Mathematics, Doctoral program Operations Research,**
Faculty of Mathematics and Informatics (FMI), Sofia University “St. Kliment Ohridski”

The review has been prepared by: **Prof. Dr. Vladimir Veliov, Vienna University of Technology**, as a member of the scientific jury for the defense of this PhD thesis according to Order № ПД-38-383/12.07.2023 of the Rector of the Sofia University.

This review follows the structure and the specific points suggested in compliance with the rules and regulation of the Sofia University “St. Kliment Ohridski”.

1. General characteristics of the dissertation thesis and the presented materials

The presented materials have been prepared in accordance with the Law on the development of the Academic Staff in the Republic of Bulgaria (LDASRB), the Regulations for the application of the LDASRB, as well as the Regulations for the Terms and Procedures for Acquiring Scientific Degrees and Holding Academic Positions (RT LDASRB) in SU" St. Kl. Ohridski". These include: PhD Thesis in English; Abstract in Bulgarian; Autobiography in Bulgarian; List of scientific publications on the topic of the dissertation; Reference on the fulfillment of the minimum requirements under Article 2b, para. 2 and 3 of the LDAS of the Republic of Bulgaria; Application by the PhD student; Doctoral student's declaration of originality and absence of plagiarism; Report from the research supervisor; Similarity report; StrikePlagiarism.com software product report for no plagiarism; Protocol for verification of originality; Statement regarding the anti-plagiarism procedure signed by the supervisor and others.

The dissertation contains 60 pages and consists of introduction, one chapter containing preliminary material, 3 more chapters presenting the author's contributions, and a short concluding chapter including a list of 8 seminar or conference presentations. The bibliography contains 38 items, including 3 publications of the author.

2. Personal impressions of the candidate

I know Margarita since about three years ago, and in all this time I have been kept informed about her work. I have also attended several of her seminar and conference talks and witnessed the progress she made in the quality of her presentations. My general impression is that Margarita is a knowledgeable and skillful mathematician and a hard and dedicated worker. Her work has often been a subject of discussion between her scientific advisor, Prof. M. Krastanov, and myself. I also mention that along with his numerous contributions in variational analysis and control, Prof. Krastanov is a worldwide renown specialist in controllability theory, which is the subject of the dissertation under review.

3. Content analysis of the scientific and applied achievements of the candidate, contained in the presented PhD thesis and the publications to it, included in the procedure

The new results are given in chapters 3-5. Chapter 3 presents a sufficient condition for Small Time Local Controllability (STLC) of a system in which the components of the drift

term are homogeneous polynomials of second order and the controlled vector fields are constant. The result extends a result by Agular (2012) in two directions: (i) the system is much more general than in Agular (2012); (ii) the control is constrained in a cone, which is a substantial difference with Agular (2012) where the control is unconstrained. The proof of this result involves an elegant, although complicated, construction of tangent vector fields to the reachable set. The concept of tangent vector fields to the reachable set and a general sufficient controllability condition are introduced by Hermes (1978) and further developed in works by M. Krastanov and others. The construction of appropriate elements of the tangent set involves the fundamental idea by Sussman (1987) "that local controllability follows whenever brackets with certain symmetries can be "neutralized," in a suitable way, by writing them as linear combinations of brackets of a lower degree" (citation from Sussman [1987]). The notion of "degree" is essentially refined, which allows to obtain more general controllability results.

Chapter 4 investigates the STLC of the same system as Chapter 3, however the obtained sufficient conditions are different. Here, a strictly increasing finite sequence of subspaces is constructed, such that if the last subspace coincides with the whole state space, then the system is STLC. Neither of the theorems in Chapters 3 and 4 is more general than the other. Therefore, the union of their claims (given in Corollary 4.1.2) extends each of them. We mention that a cone, denoted by K_2 appears in both theorems. However, this symbol has different meanings in the two chapters, which may lead to a confusion. In fact, longer Lie brackets may appear in the proof of the theorem in Chapter 3, while more operations of building recessive spaces and cone hulls are involved in the theorem in Chapter 5. Correspondingly, the proofs of the two theorems present different ideas for "neutralization" (in the sense of Sussman) of bad Lie brackets. Both proofs are masterpieces that may be useful in other cases of controllability analysis.

Chapter 5 presents a necessary condition for STLC of a general non-linear system with compact control constraints. The formulation of this theorem is very simple and elegant, substantially extending, in the same time, a well-known result by Sussman (1978). The value of this theorem is demonstrated by giving a complete characterization of the STLC property for a three dimensional system with two constant controlled vector fields and one general (homogeneous) quadratic uncontrolled vector field, which is a remarkable achievement. A part of the chapter is devoted to proving the controllability of this particular system for certain parameter configurations. In the same time, the previous two chapters provide sufficient controllability conditions for these kind of systems. I wonder if the results from chapters 3 and 4 are not applicable here for a more direct proof.

The three main results in Chapters 3-5 are new, high level mathematical achievements in the area. The proofs are correct and non-trivial. The results will certainly influence the research in the local controllability theory, which, in addition to its academic and methodological values, is an important tool in many applications of control theory.

4. Approbation of the results

The results presented in the dissertation are published in 3 papers: two of them in the reputable journals *Automatica* and *Systems & Control Letters*, and the third one in *Comptes rendus de l'Academie Bulgare des Sciences*. The results are recently published and have a small visible impact so far. However, they substantially improve and extend recent results by leading authors in the field of local controllability and bring new productive ideas, therefore an impact of the research in this area is to be expected.

In my opinion, the dissertation meets the national and the additional requirements of Sofia University “St. Kliment Ohridski” for acquiring the educational and scientific degree “Doctor” in the scientific field and professional field of the procedure. The presented results do not repeat such from previous procedures for acquiring a scientific title and academic position. I do not see any plagiarism. What concerns the contribution of the author of the dissertation (given that the results are published with the co-authorship of Prof. Krastanov), I can say the following: Prof. Krastanov is a world leading specialist in the controllability theory and his experience and ideas have certainly helped a lot in the work of Margarita. However, as far as I know, not only the huge technical work (a lot of which is, unfortunately, not explicitly included in the dissertation), but also additional substantial creative work was done by Margarita.

The scientific metrics of these articles, compared with the minimum requirements for the educational and scientific degree "doctor", according to Resolution No. 26 of February 13, 2019 on the amendment and addition of the Regulations for the implementation of the Law on the development of the academic staff in the Republic of Bulgaria, adopted with Decree No. 202 of the Council of Ministers of 2010 (promulgated, SG No. 75 of 2010; amended and supplemented, No. 19 of 2011, No. 9 of 2012, No. 62 of 2013, No. 60 of 2014, No. 57 of 2015 and No. 56 of 2018) are as follows: All publications fall into **Group G7** and collect a **total of 171 points**, with a minimum requirement of **30 points** for the acquisition of the educational and scientific degree "doctor" in the scientific field **4. Natural sciences, mathematics and informatics**, professional direction **4.5 Mathematics**. The first publication is in Q1 and is valued at 75 points, the second is in Q4 and is valued at 36 points, and the third is in Q2 and is valued at 60 points. The obtained **111 points** significantly exceed the minimum requirements for obtaining the educational and scientific degree "doctor" in the scientific field and professional direction of the procedure.

5. Qualities of the abstract

The abstract is, in fact, a shortened version of the dissertation in which the proofs are removed. Therefore, it correctly and fully presents its content. The abstract is too long (38 pages), but this seems to be consistent with the rules. At this place, I mention that the title of the dissertation is too general and does not give accurate information about the subject and the content.

6. Critical notes and recommendations

The thesis is shorter than usual. This is not a principle drawback, but the author could have given a more comprehensive and broader introduction to the area (including the historical development), much more explanations of the assumptions and the results, more technical details in the proofs, and more comparison with known results. Substantial technical parts of some of the proofs are not included, which makes the verification difficult. Instead of writing "There are examples ..." the author could have given such examples. Motivations and potential applications are not indicated. Ability of writing mathematical texts is not convincingly demonstrated. With some additional effort, the manuscript could have been made self-contained and much more readable and valuable.

At many points it is not clear which assumptions do apply, some notations are not defined (or multiple notations are used for the same object), formulations of some results are not accurate or complete. A few examples:

- P. 9, in Definition 2.3.1. $A^0(x_0)$ is not defined. For which t the inclusion has to be fulfilled?
- P. 10: What is $\alpha(t_1, \dots, t_k)$? What are θ and w (the latter is defined on a later page)?
- P. 12: S and S^+ seem to denote the same object.
- P. 16: $=$ after (3.1).
- P. 17: In some parts m is assumed to be greater than or equal to 1, in other parts $m=1$.

This leads to confusion.

- P. 19: What is on line 9?
- P. 19: Theorem 3.1.2 is very vaguely formulated. The claim should start with "There exists

An natural number r such that for every $\epsilon > 0$

- P. 20: The assumption that U is symmetric with respect to the origin is missing in Theorem 3.2.1.

Moreover, the paper contains too many misprints. Several, out of many, examples follow:

- P. 8: N and N denote the same thing; the estimation before Section 2.3 contains two missing symbols t_1 and t_2 and an undefined symbol $\$t\$$.
- P. 10: $\|x\|$ should be $\|x - x_0\|$.
- P. 12: after the long formula, typo in a_1 and O_1 .
- P. 24: "Theorem 1" should be "Theorem 3.2.1".
- P. 44: "Theorem 3" should be "Theorem 5.1.1".

The large number of grammatical mistakes can be tolerated, but the large number of missing (or wrongly placed) dots and other punctuation at the end of exposed formulas is a just a sign of carelessness.

7. Conclusion

Having become acquainted with the PhD thesis presented in the procedure and the accompanying scientific papers and on the basis of the analysis of their importance and the scientific and applied contributions contained therein, **I confirm** that the presented PhD thesis and the scientific publications to it, as well as the quality and originality of the results and achievements presented in them, meet the requirements of the LDASRB in the Republic of Bulgaria, the Rules for its Implementation and the corresponding Rules at the Sofia University "St. Kliment Ohridski" (FMI-SU) for acquisition by the candidate of educational and scientific degree "Doctor" in the Scientific field **4. Natural Sciences, Mathematics and Informatics**, Professional field **4.5. Mathematics**, doctoral program **Operations Research**. In particular, the candidate meets the national requirements in the professional field and no plagiarism has been detected in the published scientific papers.

Based on the quality and the importance of the obtained scientific results, and although I have made several critical remarks (hopefully to be taken into account in the future work of Margarita), **I recommend** the scientific jury to award Margarita Nikolaeva Nikolova the educational and scientific degree "Doctor" in the Scientific field 4. Natural Sciences, Mathematics and Informatics, Professional field 4.5 Mathematics.

08.09.2023

Reviewer:

Prof. Dr. Vladimir Veliov