

REVIEW

under the procedure for acquisition of the educational and scientific degree “Doctor”

by candidate **Matey Boyanov Konstantinov**

of the PhD Thesis entitled:

“Subdifferential analysis of convex-like functions”

In the Scientific field: **4. Natural Sciences, Mathematics and Informatics**

Professional field: **4.5. Mathematics,**

Doctoral program: **“Operations research”,**

Department: **“Probability, operations research and statistics”,**

Faculty of Mathematics and Informatics (FMI),

Sofia University “St. Kliment Ohridski” (SU),

The review is prepared by: Prof. Boyan Georgiev Zlatanov, Dr.Sci - University of Plovdiv “Paisiy Hillendarski”, Faculty of Mathematics and Informatics, Department of Mathematical Analysis, in my capacity as a member of the scientific jury, according to Order No. RD-38-113/ 06.03. 2023 of the Rector of Sofia University and the decision of the first meeting of the jury, I was chosen to present a review.

1. 1. General characteristics of the dissertation thesis and the presented materials

According to the Rules for the Development of the Academic Staff of the SU, the dissertation candidate has submitted the complete set of documents, which includes:

- 1. Application Form*
- 2. PhD Thesis*
- 3. Abstract in Bulgarian*
- 4. Abstract in English*
- 5. Declaration*
- 6. Certificate of compliance with the minimum national requirements*
- 7. Report from the research supervisor on readiness for defense*
- 8. List of publications or those accepted for printing*
- 9. Enrollment order for PhD study*
- 10. Curriculum Vitae*
- 11. Index of Similarity Report*
- 12. Protocol for verification of originality*

13. *Opinion of the scientific supervisor*
14. *Report, order and record of the internal pre-discussion of the thesis*
15. *Certificate of exams taken during training for the PhD degree*
16. *An Order for finishing of the PhD study*
17. *Internal pre-review*

The PhD thesis is based on 3 publications in journals indexed in SCOPUS and WoS. The PhD student has successfully passed the required examinations for the training to acquire the educational and scientific degree “doctor”.

This quick review shows that Matey Konstantinov satisfies the formal minimum national requirements for obtaining a PhD degree in the Scientific field of higher education: 4. Natural sciences, mathematics and informatics; professional field: 4.5. Mathematics and has successfully completed his studies at FMI at SU.

The PhD thesis, submitted for defense, is 79 pages long, consists of a Preface, three chapters, a conclusion, an appendix and a bibliography of 55 items.

After reading the thesis, I can say that the cited literature is the one that is needed for obtaining the described results and there are no artificially inserted citations in the list.

The methods used by the author can generally be systematized as classical techniques from the geometry of Banach spaces, non-smooth analysis, subdifferentials and variational principles applied creatively to solve the set problems.

2. Short CV and personal impressions of the candidate

The PhD student was born in 1994 in Bulgaria. He graduated consecutive secondary education at Sofia Mathematical High School "Paisiy Hilendarski" in 2013, acquired bachelor's and master's degrees in mathematics and applied mathematics respectively at Sofia University "St. Kliment Ohridski" - Faculty of Mathematics and Informatics in 2017 and 2019, respectively.

I do not know the PhD student personally and have not had the opportunity to hear a presentation of him at a conference or seminar.

3. Content analysis of the scientific and applied achievements of the candidate, contained in the presented PhD thesis and the publications to it, included in the procedure

In the preface, the author has justified the importance of research in the field of non-smooth analysis. The fundamental concepts and their properties of non-smooth analysis are presented.

The PhD student has preferred to include the necessary concepts and results of other authors where the need for them arises, instead of presenting them in an introductory chapter. The author has preferred the main technical statements, which are ten in number, to take them out in the last chapter “Appendix”, which gives the reader the opportunity to concentrate on the essential ideas of the main theorems and their proof technique.

A good impression is made by the presence of eight figures that contribute to the easier perception of geometric elements in proofs and concepts.

In the first chapter, a new proof is given of a result which gives the characterization of a closed set in a Hilbert space to be r -prox-regular. This result, as specified by the author, is known, but a different proof is given in the dissertation, which is based on the modulus of convexity in Hilbert space.

In chapter two, a characterization of uniformly regularized functions from below, which are defined on a Hilbert space, is obtained. New concepts are introduced that are generalizations of already known concepts: epi uniformly lower regular function in Hilbert space, which is a generalization of the notion of uniformly primal lower nice for proper lower semicontinuous function; uniformly epi prox-regular of an epigraph set, which concept differs from the well-known notion uniform prox-regularity. Two results are proved which give a relation between the function f and its epi set, namely if f is epi r lower regular function, then its epi graph C is epi r prox-regular set; if the epigraph set C of the function f is epi r prox-regular, then the function f generating it is $\frac{r}{\sqrt{2}}$ lower regular function. In Theorem 2.3.2, a characterization was found for an epigraph set to be epi r prox-regular set, without including in the equivalent conditions the function f that generates the epigraph set C . A characterization was also obtained for a function to be epi r lower regular.

In Chapter Three, a new variant of the classical epsilon subdifferential method is presented, with the help of which a new proof of the Moreau-Rockafellar theorem is given, that a proper, lower semicontinuous and convex function defined on a Banach space is determined up to a constant by its subdifferential, i.e. if the subdifferential of a function g contains the subdifferential of a function f , then f and g differ by a constant. A construction with a different approximating sequence is proposed, which is a variant of the epsilon subdifferential method, and an estimate is made from above (Lemma 3.2.3) of the required number of iterations, which estimate improves the previously known estimate, in some sense. Four technical lemmas are also formulated for the proof.

In the conclusion section, the contributions in the presented PhD work are correctly described.

4. Approbation of the results

The PhD thesis is based on 3 publications in journals indexed in SCOPUS and WoS. Two of the journals have IF in WoS and Q4 and the third has SJR. At the start of the procedure, only one of the articles had been published. All three articles are currently published, with two of the articles already indexed in WoS and one of them in SCOPUS. The results were reported at three scientific forums and five seminars. Two of the scientific forums are international, held in Bulgaria. The other forum is the FMI spring scientific session at SU, which has become a tradition over the years and is a nice one, where specialists in mathematics, informatics and education in mathematics and informatics can share their new ideas or results.

a) The PhD student has submitted a thesis and thus satisfies the group of indicators “A” with 50 points. The total number of points with which the candidate participates in the group of indicators “Д” in the procedure for acquiring the PhD degree is 102 points. It follows that Matey Konstantinov satisfies the minimum national requirements. The PhD student has successfully passed the required examinations for the training to acquire the PhD degree and accordingly satisfies the requirements of SU "St. Kliment Ohridski" for the acquisition of an educational and scientific degree “Doctor” in the Professional field: 4.5. Mathematics.

The author's first publication is from 2021 and the other two were in print at the start of the procedure, so it is quite natural that there are still no citations to the PhD student's research papers.

For years, in the world and in Bulgaria for delight, the authors of articles, in general speaking, in the field of functional analysis, are arranged in alphabetical order. That is why I consider that the contributions of the authors, the PhD student and his research supervisor, in the three articles on the thesis are equal, as far as one can speak of equal contribution of a student and a teacher.

b) The results presented by the candidate in the dissertation work and scientific works to it do not repeat such from previous procedures for acquiring a scientific title and academic position as it is a first procedure for academic growth of the candidate.

c) there is no plagiarism proven in the legally established order in the submitted dissertation work and scientific papers under this procedure in the sense of the “Law on the Development of the Academic Staff in the Republic of Bulgaria” in the Republic of Bulgaria.

5. Qualities of the abstract

The presented abstracts in Bulgarian and in English meet the requirements of the SU and correctly systematize the results of the presented thesis.

6. Critical notes and recommendations

I have no any critical notes or recommendations of the thesis and the documents presented except those related to my personal layout preferences, which I consider inappropriate to comment on here.

7. Conclusion

Having become acquainted with the PhD thesis presented in the procedure and the accompanying scientific papers and on the basis of the analysis of their importance and the scientific and applied contributions contained therein, **I confirm** that the presented PhD thesis and the scientific publications to it, as well as the quality and originality of the results and achievements presented in them, meet the requirements of the “Act on Development of the Academic Staff in the Republic of Bulgaria”, the Rules for its Implementation and the corresponding Rules at the Sofia University “St. Kliment Ohridski” (FMI-SU) for acquisition

by the candidate of educational and scientific degree “Doctor” in the Scientific field 4. Natural Sciences, Mathematics and Informatics, Professional field 4.5. Mathematics. In particular, the candidate meets the minimal national requirements in the professional field and no plagiarism has been detected in the scientific papers submitted for the competition.

Based on the above, **I strongly recommend** the scientific jury to award Matey Boyanov Konstantinov, the educational and scientific degree “Doctor” in the Scientific field 4. Natural Sciences, Mathematics and Informatics, Professional field 4.5. Mathematics, Doctoral program: “Operations research”.

April 27, 2023

Reviewer:

/Boyan Zlatanov, Professor, Dr.Sci. /