

REPORT

according to the procedure for the defence of a PhD Thesis entitled

"Variational analysis without variational principles"

for the acquisition of PhD degree of the Sofia University

by the PhD student: **Stoyan Raychev Apostolov**

in the field of higher education: **4. Natural sciences, mathematics and informatics**

professional direction: **4.5. Mathematics**

doctoral program: **Mathematical Analysis of the Faculty of Mathematics and Informatics (FMI) of the SU "St. Kliment Ohridski"**

The referee report is prepared by Stanimir Troyanski, associated member of the Institute of Mathematics and Informatics, Bulgarian Academy of Science (e-mail troyanski@math.bas.bg).

The PhD thesis, entitled "Variational analysis without variational principles", is written in English, and is a text of 72 pages, A4 format, 61 titles were cited. It consists of an Introduction, Preliminaries, and three chapters, divided into paragraphs, in which the results of the dissertation are presented, and Conclusion. The PhD thesis is in the field of Variational Analysis (Optimization). In contrast to the Classical Analysis in Optimization, roughly speaking, problems are considered in which the functions are not smooth and are set-valued. This necessitates the creation of new approach for solving the set tasks. However, the developed tools of Analysis (Mathematical and Functional, Differential Geometry), Topology (General and Differential) is the basis of the methods for solving the tasks of Optimization.

To work in this field, the author masters a large volume, Analysis and Topology knowledges and applies it in different directions of Variational Analysis.

The dissertation explores the concept of transversality and related concepts. Transversality is a classical concept of Mathematical Analysis

and Differential Topology is also a natural and convenient concept in some parts of Variational

Analysis. Recently intensively studied by a number of leading specialists in Optimization. The extension of transversality to subtransversality is useful for instance for deriving necessary optimality conditions of the Pontryagin maximum principle.

The author studies and applies two intermediate concepts of transversality. In fact, we have the following chain of implications.

transversality \implies tangential transversality \implies

intrinsic transversality \implies subtransversality

and neither implication is invertible.

A general sufficient conditions of tangential transversality introduced by M. Bivas, M.Krastanov and N. Ribarska are investigated and applied to abstract Optimal Control in Banach spaces. Tangential transversality is sufficient for subtransversality The well-known sufficient condition for tangential transversality using compactly epi-Lipschitz sets is weakened to a symmetric condition with respect to the sets. The result yields an abstract version of the well-known

Aubin condition from which is then applied to an abstract optimal control problem in Banach spaces.

In addition of tangential transversality in the thesis

examines intrinsic transversality. The metric nature of intrinsic transversality is established. The relation of intrinsic transversality and tangential transversality is clarified. The equivalence of author characterization of intrinsic transversality and the primal characterization of intrinsic transversality of N.H. Thao, H.T. Bui and N.D.Cuong. In the setting of Hilbert spaces is proved, while in general Banach spaces author characterization is less restrictive.

The thesis is based on four articles, three of which have been published, respectively, in the Journal of Convex Analysis, Set-Valued and Variational Analysis, Comptes rendus de l'Academie bulgare des Sciences, the latter being a preprint. We will note that the first two journals are specialized

journals in Optimization. This gives reason to believe that the dissertation's results are subject to serious international expertise.

Parts of the thesis have been reported in various scientific forums. From the direct analysis, it can be seen that the author has mastered a significant mathematical knowledges and has skilfully applied it in solving specific optimization tasks. In my opinion, the results of the thesis, both in terms of quantity and quality, more than meet the standard requirements for a PhD thesis.

Based on the above, I strongly recommend that the scientific jury award Stoyan Raichev Apostolov the educational and scientific degree "doctor" in a scientific field: 4.Natural sciences, mathematics and informatics, professional direction: 4.5. Mathematics (Mathematical Analysis).

Sofia, 30 September 2022

S. Troyanski