

**SCIENTIFIC OPINION**  
**on procedure for PhD attainment**

**Title**

***“Variational Analysis without Variational Principles”***

**with candidate: Stoyan Raychev Apostolov**

Area of high education: **4. Natural Sciences, Mathematics and Informatics,**  
Professional direction: **4.5. Mathematics,**  
Doctoral program: **“Mathematical Analysis“**, chair: **“Mathematical Analysis“**,  
**Faculty of Mathematics and Informatics at Sofia University “St Kliment Ohridski”**,

by **Prof. Dr. Nikolay Vassilev Zhivkov, Institute Mathematics and Informatics, BAS** as a member of **Scientific Jury**, by order № ПД 38-309 / 01.07.2022. г. by the Rector of Sofia University..

**1. General description of the dissertation and the documents presented**

The dissertation has 72 pages in 6 chapters: Introduction, Preliminaries, three chapters containing main results, and Conclusion. There is also a bibliography with 61 scientific sources.

**2. Personal and professional information about the candidate**

My acquaintance with the candidate began with his active participation in a scientific contract “Nonlinear Analysis, Variational Methods and Optimization” between a group of mathematicians and the National Scientific Fund at the Ministry of Education and Science, in Bulgaria, and also with his participation in two workshops: on “Mathematical Analysis” in the Faculty of Mathematics and Informatics at Sofia University, and “Operations Research, Probability and Statistics” at the Institute of Mathematics and

Informatics at the Bulgarian Academy of Sciences. I have excellent impressions of his work.

**3. Analysis of the scientific achievements contained in the dissertation and the publications included in this procedure.**

The theme, of the dissertation presented, is from the area of the non-smooth variational analysis. In the last decades, enormous interests sustain methods with non smooth objects under investigation. From one point of view, these interests serve theoretical consolidation of knowledge in various mathematical areas, and on the other, the new methods allow new and more general mathematical models to be solved for various application problems, also due to the advances of computer technologies.

Main notion in dissertation is transversality, as well as, different variations of it, like strong transversality, subtransversality, tangential transversality, intrinsic transversality and other. The concept arise in topology and is about local separation (or non separation) of two sets in a point, but also plays role in other mathematical problems from Calculus of Variations, Conditional Optimization, Parametric Optimization, Optimal Control, Approximation theory and other.

In the dissertation, the author explicitly states what his scientific contributions are. Some of these remarkable results are:

- Characterizations of subtransversality, transversality, and intrinsic transversality allowing implications about how different notions of transversality are related one to another.
- A general sufficient condition for tangential transversality and its application to abstract Lagrange rule in different optimization settings.
- Generalization of the notion of intrinsic transversality to infinite-dimensional spaces.
- Demonstration of the fact that transversality and subtransversality can be employed to give characterizations for the notions of metric regularity and metric subregularity.
- Sufficient condition for continuity in metric space setting of the optimal value function depending on parameter in parametric optimization problems.

#### 4. **Approbation of results**

The dissertation is based on three papers, published in J.Convex Analysis, Доклади БАН, and Set-Valued and Variational Analysis, and another accepted paper, four all. These are joint works with two coauthors. The coauthors declare equal participation of the candidate. Results from this dissertation have been given as lecture talks on five scientific meetings with international participants.

The scientific work in the dissertation corresponds to the national criteria (according to the Bulgarian Law), and to the additional requirements of Sofia University for attainment of PhD qualification in the area of Mathematics. The results in this dissertation do not appear in previous procedure of the candidate for attaining scientific degree or title. There is no evidence for plagiarism.

#### 5. **Quality of abstract of dissertation**

The abstract of dissertation has 39 pages and 61 sources of scientific information, and precisely and thoroughly describes the research results, and also complies with the necessary requirements.

#### 6. **Critical notes and recommendations.**

No critical errors found. No remarks.

#### 7. **Conclusion**

After I have acquainted with the dissertation and the related scientific papers and all other documents, and after analyzing their scientific importance, **I confirm**, that the presented dissertation, and the scientific publications therein, correspond to the requirements necessary, with respect to the Bulgarian Law, and Additional Requirements of Sofia University Rulebook, **for attainment a PhD title of Stoyan Raychev Apostolov in the area of Mathematics, professional direction Mathematical Analysis**

According to the afore stated, I **recommend** to the scientific jury and to the competent organ of Faculty of Mathematics and Informatics at Sofia University to give PhD title in the area of **Mathematics**, professional direction **Mathematical Analysis** to **Stoyan Raychev Apostolov**.

27. 09.2022 r

Scientific opinion by

Prof. Dr.Nikolay Zhivkov