STATEMENT REPORT

on the procedure for defense of a dissertation thesis
entitled: “Branching processes – optimization and applications”
for obtaining the scientific degree “Doctor” (educational and scientific)

by Kaloyan Nikolaev Vitanov
In the Scientific field: 4. Natural Sciences, Mathematics and Informatics,
Professional field: 4.5. Mathematics,
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The statement was written by: assoc. prof. Vessela Kirilova Stoimenova, PhD (SU, FMI) member of the scientific jury according to Order No. RD-38-308 / 01.07.2022 issued by the Rector of Sofia University.

1. General characteristics of the PhD thesis and the presented materials

The text of the PhD thesis has 196 pages, which include an introduction, two chapters, a conclusion, an appendix and a bibliography. It is written in English.

The Introduction presents the historical development of branching processes and an overview of the literature in the field. In Sections 1.3 - 1.4, a definition of the classical multitype Sevastyanov branching process and its probability generating functions, integral equations and probabilities for extinction are given. Paragraph 1.4. points to some of the basic concepts in the studied processes - "particles", "successors", "ancestors", "mutation" and their interpretation in the presented work. Section 1.5 gives a brief introduction to stochastic optimization problems with sequential decision making. Paragraph 1.6 gives the organizational structure of the work.

The thesis is composed of two separate topics, addressed in the consecutive Chapters 2 and 3. Although they deal with different issues in their nature, they are logically connected to each other and Chapter 3 is a kind of continuation of Chapter 2.

Chapter 2 introduces the multitype Sevastianov processes through probabilities of mutation between types (MSBPM process) (Definition 2.1), comments on their relationship to the classical Sevastyanov process, and derives results for particle populations avoiding extinction. In Definition 2.2, the probability generating functions of the MSBPM process starting with 1 particle of type i at age 0 and of the process starting with one particle of type i at age a are derived, and in Theorem 2.1 and Corollary 2.1 - the corresponding systems of integral equations. In Subsection 2.2.3, the probabilities of extinction are defined and investigated. Section 2.2.4 considers the reproduction of the particles of a given subclass of particle types towards all particles types within the process (in particular, the number of particles of type j derived from an ancestor of the given subclass at time t) and derives the associated integral equations. Section 2.2.5 defines a “successful” particle as a particle that gives rise to a nonextinct MSBPM process, and the random variable “time until occurance of the first successful particle” (Definition 2.9). Theorem 2.6. investigates the properties of its distribution in the case when the process starts with particles of age 0, and in Theorem 2.7 the ages of the initial particles are allowed to be nonzero. Subsection 2.2.6 aims to investigate immediate the risk of the occurrence of a "successful" particle through risk functions. Subparagraph 2.2.7 presents two numerical schemes for calculating the integral equations obtained in the previous paragraphs in the case of a process starting with a particle of age 0 and with a particle of age a. The results so far are new and unpublished.

In Section 2.3, particular cases of a decomposable MSBPM process are studied – a decomposable multitype Sevastyanov branching process through probabilities of mutation between types (DMSBPM) and a decomposable multitype Bellman-Harris branching process through probabilities of mutation between types (DMBHBPM). While the results on the first process were published by Vitanov and Slavtchova-Bojkova in 2022 in [7], the results on the second is a summary of Slavtchova-Bojkova and Vitanov's work from 2019 ([5]). DMBHBPM is a special case of DMSBPM in which the dependence of the reproduction on the age of the particles does not hold.

Chapter 3 aims to study stochastic optimization problems with sequential decision making (SDP) concerning systems with dynamics specified by a branching stochastic process. At the beginning, concepts such as exogenous information variables, control variables, transition function, state variables, objective function are explained, finite and infinite horizon optimization problem (Definition 3.2) and Bellman optimality equation (Definition 3.3) are defined. . It is noted that standard algorithms using the latter equation require traversal of all elements of the state space and solution space, making it difficult to include branching processes in the model. Sections 3.4, 3.5 and 3.6 consider SDPs with dynamics specified respectively by a Bieneme-Galton-Watson process, by a multi-type branching Bellman-Harris process with exponential lifetimes of all particle types through probabilities of mutation between types (MBHBPM) and from MSBPN process. The latter case allows for future research, the idea being to define a new state space by which the MSBP process and the Sevastianov process can be reduced to the other cases. Based on the Bellman optimality equation, a new proof of the S. Pliska theorem is presented in the case of the Galton-Walson process. A similar result is considered for the other two cases. In Section 3.7, an algorithm of the type of dynamic programming by approximations is discussed. The results of Chapter 3 have not been published yet.

The Approbation and the Scientific Contributions are included in the Conclusion.

Basic theorems of matrix algebra are presented in the Appendix.

The bibliography includes 213 titles.

2. Information and personal impressions about the candidate.

I have known the PhD student since he was a student in the Master's program in Probability, Actuarial Science and Statistics, highly motivated and with many interests. I had the opportunity to participate in the defense of his master's thesis, which was also in the area of branching stochastic processes, and to see his huge progress in the scientific field during the PhD process.

3. Analysis of the contributions.

The PhD thesis is an original study in the field of continuous-time multitype branching processes in the context of populations escaping extinction. The considered types of process are integrated into optimization problems with sequential decision making, which is an interesting and novel approach in the field with a potential for multiple applications and for future development. I believe that the presented results in the dissertation work and scientific publications represent an original contribution to the field.

4. Approbation of the results.

The results were reported at seven scientific forums and published in 4 papers, three of which have an IF.

The publications meet the minimum national requirements (under Article 2b, Paragraphs 2 and 3 of the RSARB) and the additional requirements of SU "St. Kliment Ohridski" for the acquisition of an educational and scientific degree "doctor" in the scientific field of the procedure.

5. Assesment of the thesis synopsis

The abstract is written in Bulgarian and presents the results and content of the dissertation work correctly.

6. Critical remarks and comments

I have no significant remarks on the candidate's work. I believe that the work would have benefited if more explanations related to the use of the software, algorithms and procedures, programmatic implementation were included.

7. Conclusion

After getting acquainted with the thesis and the corresponding materials, with respect to the work done by the candidate, I consider that the thesis fulfills the requirements for a PhD degree of the Bulgarian law and the corresponding state and university regulations. I recommend that Kaloyan Nikolaev Vitanov should be given a PhD degree in field of 4.5. Mathematics (Probability Theory and Mathematical Statistics).

24.09. 2022 г. Reviewer: ....

(assoc. prof. Vessela Stoimenova)