REVIEW

of the thesis

Branching Processes – Optimization and Applications

for awarding the educational and scientific degree "Doctor",
Area 4. Natural sciences, mathematics and informatics,
Professional direction 4.5. Mathematics, Science major "Theory of Probability and Mathematical Statistics" (01-01-13)

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1. Basic information.

The submitted dissertation is in English and contains 196 pages of text, of which 18 pages are a bibliography (213 titles), 4 pages - Conclusion and 2 pages - Appendix, which presents some main results of the Perron-Frobenious theory on non-negative matrices.

In addition, an Abstract (in Bulgarian) with 45 pages of text plus 4 pages of bibliography is presented. This abstract is also given in English as ABSTRACT with 43 pages of text and bibliography.

I received these documents in hard copy, but in addition, all the documentation on the dissertation is well presented on the website https://intranet.fmi.uni-sofia.bg/index.php/s/zYAoUtQLvGuU2sU. There are the publications and all the documents necessary for the procedure, which are properly formatted.

The author Kaloyan Nikolayev Vitanov is a full-time doctoral student at the Department of Probability, Operational Research and Statistics, Faculty of Mathematics and Informatics, SU "St. Kliment Ohridski". Born in 1992, graduated from NPMG in 2011, followed bachelor's (2011-2015) and master's (2015-2017) programs of FMI - SU, was enrolled as a doctoral student in 2018, worked as a part-time teacher at SU (2014 -2015), and also in BISAM (Sofia) as Quantitative Researcher (2016-2020) and Senior Quantitative Researcher (2020-2022).

The jury was appointed by order RD 38-308/01.07.2022 of the Rector of SU "St. Kliment Ohridski". The pre-defense of the dissertation was held at the FMI of SU on 06/09/2022. Documentation and all protection procedures are in accordance with the requirements of the law and relevant regulations.

2. Structure, analysis, scientific and scientific-applied contributions of the dissertation.

Broadly speaking, the thesis deals with branching stochastic processes and their applications related to mutation and optimization. Multidimensional (multitype) Sevastyanov, Bellman-Harris and Bienayme-Galton-Watson models are mainly used. The dissertation has an Introduction (21 pages), two main chapters, a concluding part, an appendix and a bibliography.

In fact, the dissertation begins with a detailed literature review (Chapter 1. Introduction), where a significant part of the development of branching processes and some of their main applications (213 titles) is well presented, with special attention paid to Sevastyanov's multi-type branching processes. The basic system of nonlinear integral equations for the generating functions and the equations for the degeneracy probabilities is presented. Special attention is paid to problems related to corresponding stochastic optimization (Stochastic sequential decision problems). Overall, this part leaves a very good impression and shows that the PhD student has entered well into the thesis problem and presents the conceptual organization of the thesis well.

Chapter 2 (95 pages) finds an interesting connection between a natural model of mutation and multitype (multidimensional) branching Sevastyanov processes. Theorem 2.1 shows that, in fact, the mutation process can be represented as a Sevastyanov multitype process, where the multivariate individual distribution of the number of offspring of a particle is obtained as a randomized polynomial distribution generated by the two corresponding univariate distributions in the mutation model, i.e. for the individual pgf in the Sevastyanov process we have $h_i(y,s)=f_i(y,u_i(s))$, where $f_i(y,x)$ is the pgf of the number of offspring in the mutation model, and $u_i(s)=(u_{i1})s_1+\dots+(u_{in})s_n$, $s=(s_1, \dots, s_n)$, the parameter y is the age of the particle at the time of reproduction, $i=1,\dots,n$. In fact, in a polynomial scheme, the parameter number of independent trials is randomized, in each of which we have n number of independent outputs with probabilities u_{ik}, $k=1,\dots,n$; $u_{i1}+\dots+u_{in}=1$. In this way, many results of its corresponding branching Sevastyanov process can be transferred into the mutation model. It would be interesting to consider some particular cases. For example, *h_i(y,s)=exp{-m_i(y)[1- u_i(s)]}* when the randomizing distribution of the *i-th* type is Poisson with parameter *m_i(y).*

Further on in §2.2.3, the probabilities of extinction in the two cases of zero and non-zero age of the initial particle are considered, and two interesting illustrations (numerical and graphical) are presented at the end of the paragraph.

In §2.2.4 - §2.2.6, problems related to the emergence of particles (in particular, mutants) generated by a fixed subset *W_e* of the set of all types *W* are studied. Theorem 2.4 presents the corresponding nonlinear integral equations for the pgf. Some interesting implications and illustrative studies with graphs are also indicated. In Theorems 2.6 and 2.7, the time distribution for the appearance of a «successful» particle is found, i.e. which gives rise to a non-degenerate process. In §2.2.7, schemes for numerical calculation of the obtained systems of integral equations are presented. In §2.2.8, some examples with three types of lifetimes (exponential, log-normal, and gamma) and 6 types of mutations are considered.

In §2.3 a class of decomposable processes is studied, where the set of types *W* is divided into two subsets *W_e* and *W_0*, the types of *W_e* can give rise to all types of *W*, and the types of *W*_0 can only reproduce. The most popular particular case of this scheme are the so-called processes with immigration, where *W_e* has only one type, which is always self-producing and gives rise to a random number of types of *W_0*, which are interpreted as immigrants. In the general case, the equations for the pgf and the probabilities of extinction are given. Proposition 2.1 gives the probabilities of extinction in the case when the types in *W_e* are not supercritical. In Proposition 2.2 and 2.3, the times for the first successful mutant in *W_0* generated by *W_e* are investigated. In Theorem 2.8, a nonlinear integral equation is found for the probability that the time of the first successful mutant (from *W_e* to *W_0*) exceeds a given fixed time t at which there is degeneracy at *W_e*. Finally, §2.3.2 systematizes all analogous results, but in the case of multitype decomposable Bellman-Harris processes.

In conclusion to this Second Chapter, it can be noted that the dissertation has well mastered some techniques related to multi-type branching processes (and especially those of Sevastyanov), which are well aptly applied in the mutation model defined in this way. Moreover, these theoretical results are very well illustrated with numerical examples and interesting graphs (25 in number).

Chapter 3 is devoted to the so-called Sequential Decision Problems (SDPs), i.e. optimization problems with sequential decision making, the main ideas and overview of the problem are presented in §3.1 and also in §1.5. In §3.2, 5 main components of the so-called "Universal Modeling Framework" are presented. The basic formulation of SDPs is given in Definition 3.2. It is specified that in the following only tasks with a finite horizon will be considered. In §3.3, the Bellman optimality equations (see (3.2) and (3.3)) are introduced and investigated, and Theorem 3.1 presents the solution of these equations. Note that in Proposition 3.3 it is seen that (3.4) is a direct iteration of (3.5). In §3.3.3 some algorithmic problems related to dynamic Bellman programming are discussed. Notice that §3.4 introduces a multitype Bienayme-Galton-Watson process as determining the dynamics in SDPs, as well specified in 8 points (Model 1, pp. 142-146). As noted in point 3, the set of possible solutions is finite, and as seen in point 5, each solution means choosing a corresponding individual distribution, i.e. thus, the process becomes inhomogeneous in time. Definition 3.4 introduces the so-called maximum return operator *R*, with the help of which the optimal solution (3.9) and the corresponding optimal policy (3.10) are presented in Theorem 3.2. Note that the operator *R* depends of the matrix of individual mathematical expectations *M(x)* and the discount factor. In §3.5, SDPs are considered, where the dynamics is now determined by multitype Bellman-Harris processes with mutation when the particle lifetimes are exponentially distributed. In this case, the process is Markovian, so the optimal solution found in Theorem 3.3 is analogous to that in the discrete case. In §3.6, SDPs are considered in the case of Sevastyanov-type dynamics. In the definition of the so-called Model 3 assumes that, along with the number of offspring, their ages are also given, which leads to a Markovian character of the system (see Proposition 5.6). In §3.7, basic principles of Approximate Dynamic Programming are discussed and Algorithm 3.2 is proposed. In the last §3.8 of Chapter 2, two examples of discrete-time and continuous-time Markov branching processes are proposed. Processes have three types of particles (individuals) and are decomposable. The types are conventionally named Bachelor, Master and Ph.D. For each type, a table with 3 possible solutions and a final horizon 4 is presented. An application of Theorems 3.2 and 3.3 is shown.

Chapter 3 shows that the author student entered a relatively little researched problem by mastering and further developing the relevant technique.

The dissertation ends with a "Conclusion", where the main scientific contributions are noted, which can be fully accepted, as they also correspond to what was noted above in the respective chapters.

The abstract correctly reflects the structure, main ideas and results of the dissertation and meets the main academic requirements.

The results of the dissertation have been published in 4 articles, of which 2 in the prestigious international journal Stochastic Models (with Impact Factor), one - in Reports of the BAS and one - in the Proceedings of 21 European Young Statistician Meeting. All results have been reported at international conferences abroad, as well as in our country.

2. Notes and recommendations.

The thesis is written clearly and comprehensibly, but some things could have been presented better.

For example, the title of the dissertation does not fully reflect the essence of the problem. "Branching Processes - Mutation, Optimization, Applications" would be much more adequate. There are some redundant repetitions, for example, (3.21)=(3.24). On page 162, third line from top, the given formula is not well explained. In addition to the above-mentioned remarks, some more obviously correctable errors and shortcomings could be pointed out, which, however, do not seriously hinder the reader.

It would be good for the dissertation to try to formulate some problems for future research. For example, it would be interesting to transfer some limiting results from branching process theory to mutation models. Since the general theory of decomposable processes is not well developed, it would be interesting to study the asymptotic behavior of some particular cases.

3. CONCLUSION

The presented dissertation has all the qualities of a serious scientific study in the field of Stochastics. The obtained interesting results define a new direction in the study of mutational phenomena using branching stochastic processes. One part of the contributions has a definite theoretical character, and another part provides new methods and algorithms for approximate calculation. A good impression is left by the numerous examples and graphs that well illustrate the theory. The obtained results have been reported at international conferences and seminars and published in prestigious publications. All of this gives us full reason to conclude that all the conditions of the law and its regulations, as well as that of FMI - SU, which apply to obtaining the scientific degree "Doctor" in the relevant specialty, are satisfied. That is why I definitely suggest that the honorable jury should evaluate the highly presented dissertation work, and that its author, Kaloyan Nikolayev Vitanov, should be awarded the educational and scientific degree "DOCTOR" in District 4. Natural Sciences, Mathematics and Informatics, Professional Direction 4.5. Mathematics, Scientific specialty "Probability Theory and Mathematical Statistics" (01-01-13).

19.09.2022

Reviewer:

/Professor Nikolay M. Yanev, PhD, Dr.Sc./