

Review

On a procedure for defense of a dissertation

"A Class Töplitz C^ -Algebras"*

For acquisition of educational and scientific degree "doctor" (PhD)

from

Candidate: **Nikolay Petrov Buyukliev,**

Scientific Field: **4. Natural Sciences, Mathematics, and Informatics**

Field of Study: **4.5. Mathematics,**

Doctoral program: **„Mathematical Analysis“,**

Faculty of Mathematics and Informatics (FMI),

Sofia University "St. Kliment Ohridski"

Review by: **Assoc. Prof., PhD, Nikolay Antonov Ivanov, FMI-SU,**

As a member of the jury, according to order № ПД-38-223/28.04.2023 г. from the Rector of the Sofia University.

1. General characteristics of the dissertation and the presented materials

The candidate has presented a complete set of documents for the procedure: Application, dissertation, abstract (in Bulgarian and in English), declaration of originality, reference of the minimal national requirements, report from the head of the MA group, similarity report, protocol of originality, opinion on (non)plagiarism, request for discussion of the dissertation (pre-defense), order of enrollment in doctoral studies for self-study, certificate of completed study, order of unenrollment, order of suspension of study, internal review of the dissertation, transcript of the meeting of the MA group in connection of the pre-defense, diploma for Master's degree in Mathematics, curriculum vitae, list of publications and teaching materials, and directory of the publications for the procedure.

The presented dissertation “A Class of Töplitz C^* -Algebras” has volume of 56 pages and is written in Bulgarian. It consists of a total of six chapters, with the first being introductory, the second chapter containing some technical information, and the remaining four chapters containing the author’s original results. The bibliography contains 52 titles.

2. Personal data and personal impressions about the candidate

Nikolay Petrov Buyukliev was born on 08.05.1959 in Gabrovo.

In 1977, he graduated from the Mathematical High School “Acad. Ivan Gyuzelev” in Gabrovo, with distinctions.

In 1977, Nikolay Buyukliev participated in the International Mathematics Olympiad, where he won a silver medal.

In the period 1977-1979, he served in the military.

In 1984, he completed Master’s degree at FMM of University of Sofia with honors. He wrote a master’s thesis titled “Wiener-Hopf Operators”, under the supervision of Roni Levi.

In the period 1985-1993, Nikolay Buyukliev was an assistant, in the period 1993-1997, he was a senior assistant, in the period 1997-2014, he was a principal assistant, in the period 2015-2022, he was an assistant, and from 2023 to the present day, he has the position ‘mathematician’. The candidate held all these positions at the Faculty of Mathematics and Informatics of the Sofia University “St. Kliment Ohridski”.

I have known Nikolay since 2016. He has a subtle sense of humor and a friendly attitude towards his colleagues. He is always ready to discuss mathematics – both posing problems and thinking about them. Nikolay has great teaching experience and a wonderful attitude towards students. Above all, he has a rich knowledge of mathematics - both general mathematics and one related to his scientific research.

During the writing of the dissertation, I have helped Nikolay several times in organizing and writing up in English the publications involved in the dissertation. I am very familiar with the topic.

3. In-depth analysis of the candidate's scientific and applied-scientific achievements contained in the submitted dissertation and the publications included in the procedure

The first chapter of the dissertation is introductory. In this chapter, the structure of the dissertation is discussed.

The second chapter contains general technical results that are used in the work. There are given: general definition of C^* -algebra; definition and index of Fredholm operator; definition of groupoid; definitions of groupoid C^* -algebras; examples of groupoid C^* -algebras; definition of K -theory of C^* -algebra, including exact hexagon, Meier-Vietoris formula, Künneth formula; definition of cyclic cocycles and cyclic cohomologies.

In the third chapter, C^* -algebras of Wiener-Hopf groupoids $T = C^*(\Gamma)$ are considered, where the Wiener-Hopf groupoid $\Gamma = (Y \times G)|X$ is a reduction of a group of transformations, where $X \subset Y$ are topological spaces. If $F \subset X$ is an invariant and closed subset and if $C_{\text{red}}^*(\Gamma|U)$ is the ideal of the compact operators in $C_{\text{red}}^*(\Gamma)$, where $U = X \setminus F$, linear sections $\beta: C_{\text{red}}^*(\Gamma|F) \rightarrow C_{\text{red}}^*(\Gamma)$ are constructed in the exact sequence

$$0 \rightarrow C_{\text{red}}^*(\Gamma|U) \rightarrow C_{\text{red}}^*(\Gamma) \rightarrow C_{\text{red}}^*(\Gamma|F) \rightarrow 0,$$

in the special case when there is a contraction $\lambda: X \rightarrow F$.

In the fourth chapter, the results of the third chapter are applied to the calculation of the indices of some Fredholm operators in a dense subalgebra of the C^* -algebra of a Wiener-Hopf groupoid of a group of transformations.

Here some conditions are imposed on the linear section to define the dense algebra. An index formula that uses A. Connes' cyclic cohomologies is applied to calculate the index.

In the fifth chapter, a comment is made on the K-theory of C^* -algebras of Wiener-Hopf operators $\mathcal{B}(\mathbb{R}^n, P)$ associated to \mathbb{R}^n and a polyhedral cone $P \subset \mathbb{R}^n$. Under certain conditions on P , a lattice of ideals in the C^* -algebra is found, the exact Meier-Vietoris sequence is applied to the resulting pullback, and a Fredholm operator of index one is constructed, which guarantees that the image of the index map on K_0 of the compact operators is surjective. This allows to find the K-theory of $\mathcal{B}(\mathbb{R}^n, P)$.

In the sixth chapter, the C^* -algebra of the Wiener-Hopf operators associated to the discrete Heisenberg group, $H_3(\mathbb{Z})$, and to the semigroup, P , of the 'positive' elements, is considered. After representing it as a groupoid C^* -algebra, a technically difficult description of the lattice of ideals of this C^* -algebra is given. This allows (in principle) the K-theory to be calculated.

The results of this dissertation are original. Whenever results and ideas from other publications are used, the references to those publications is complete and thorough.

4. Approbation of the results

The results in the third chapter have been published in the publication Buyukliev N., Linear Cross-sections and Fredholm Operators in a Class of Groupoid C^* -algebras,

to appear in Ann. Sofia Univ., Fac. Math. and Inf.

The results in the fourth chapter have been published in the publication Buyukliev N., An Index Formula in a Class of Groupoid C^* -algebras,

to appear in Ann. Sofia Univ., Fac. Math. and Inf.

The results in the fifth chapter have been published in the publication Buyukliev N., K-theory of the C*-algebra of Multivariable Wiener-Hopf Operators, Ann. Sofia Univ., Fac. Math. and Inf., **1**, (1997), 115-119.

The results in the sixth chapter have been published in the publication Buyukliev N., The C*-algebra of Toeplitz Operators of the Discrete Heisenberg Group H_3 , submitted for publication.

The dissertation contains scientific results that are an original contribution to science and contribute to the existing theory of C*-algebras. It shows that the candidate has a deep scientific knowledge of the theory of C*-algebras and an ability for independent research. The results of the dissertation can be continued in future research.

In the following table, it is demonstrated that the minimal scientometric requirements are met:

Indicator	Min. number of points	Points of the candidate
Dissertation	50	50
Publications	30	54

There is no plagiarism, proven by law, in the submitted dissertation under this procedure.

5. Quality of the abstract

The version of the abstract in Bulgarian contains 14 pages and cites 28 titles. The English version contains 12 pages and cites 28 titles. The abstract adequately presents the results of the dissertation.

6. Critical notes and recommendations

I recommend that the candidate continue his research in the field of C^* -algebras. For example, he can think of a class of examples satisfying the conditions imposed in chapter four.

7. Conclusion

Having read the dissertation under this procedure myself, having examined the accompanying publications, and based on analysis of their significance and scientific contributions they contain, **I confirm** that the presented dissertation and scientific publications with it, as well as the quality and the originality of the results and achievements presented in them, meet the requirements of LDASRB, the Regulations of SU “St. Kliment Ohridski” for the candidate’s acquisition of educational and scientific degree “**doctor**” (PhD) in scientific field 4. Natural sciences, Mathematics, and Informatics and field of study 4.5 Mathematics.

Based on the above, **I recommend** the scientific jury to award **Nikolay Petrov Buyukliev** an educational and scientific degree “**doctor**” (PhD) in scientific field 4. Natural sciences, Mathematics, and Informatics, field of study 4.5 Mathematics, doctoral program “Mathematical Analysis”.

29.05.2023 г.

Reviewer: Assoc. Prof., PhD, Nikolay Ivanov