

STATEMENT REPORT

under the procedure for acquisition of the educational and scientific degree “Doctor”
by candidate Hristina Yordanova Belcheva,

of the PhD Thesis entitled: “Applications of the Variational Analysis Methods”,

In the Scientific field: **4. Natural Sciences, Mathematics and Informatics**

Professional field: **4.5. Mathematics**

Doctoral program “Operations research”, Department „Probabilities, operations research and statistics”, **Faculty of Mathematics and Informatics (FMI), Sofia University “St. Kl. Ohridski” (SU),**

The statement report has been prepared by: prof. DSc Mikhail Ivanov Krastanov, Faculty of Mathematics and Informatics, Su “St. Kliment Ohridski”, as a member of the scientific jury for the defense of this PhD thesis according to Order № RD-38-685/ 5.11.2025 of the Rector of the Sofia University.

1. General characteristics of the dissertation thesis and the presented materials

The presented materials have been prepared in accordance with the Law on the Development of the Academic Staff in the Republic of Bulgaria (ZRASRB), the Regulations for the Implementation of the ZRASRB, as well as the Regulations for the Conditions and Procedures for Obtaining Academic Degrees and Holding Academic Positions (PURPNSZAD) at Sofia University "St. Kliment Ohridski". They include: Dissertation in English, Abstract in Bulgarian. Abstract in English, Curriculum Vitae in Bulgarian, List of scientific publications related to the dissertation topic, Report on the fulfillment of the minimum requirements under Article 2b, paragraphs 2 and 3 of the ZRAS of the Republic of Bulgaria, Application from the doctoral candidate, Declaration of originality and absence of plagiarism by the doctoral candidate, Readiness report from the scientific supervisor, Similarity report from the software product StrikePlagiarism.com for plagiarism check, Protocol for originality verification, Statement regarding the procedure for preventing plagiarism, signed by the scientific supervisor and other relevant documents.

The dissertation of Hristina Belcheva consists of 59 pages and includes an introduction, three chapters presenting the obtained results, a summary, and a bibliography. It is written in English. In the introduction, a brief motivation for the research is provided, and the results achieved are summarized. The main notations, concepts, and statements used in the dissertation are then introduced. Each chapter begins with a brief history of the task under discussion, aimed at motivating the research within it. **Chapter 1** discusses the problem of finding a common perturbation function for a given sequence of eigenfunctions, semi-continuous from below, and bounded from below, such that each of the perturbed functions reaches a strict minimum. **Chapter 2** extends the approach presented in Chapter 1 to a parameterized family of epi-continuous functions defined in a complete metric space. **Chapter 3** introduces a perturbation method suitable for minimization problems with constraints. The conclusion outlines the main contributions of the dissertation, the author's publications, and a list of presentations at seminars, national, and international conferences. The bibliography, consisting of 40 titles, is comprehensive and demonstrates a deep understanding of the subject matter.

2. Short CV and personal impressions of the candidate

From the presented creative biography, it is evident that Hristina Belcheva was born on September 18, 1995. She completed her secondary education at the Sofia Mathematics High School "Paisii Hilendarski" and her higher education at the Faculty of Mathematics and Informatics of Sofia University. In 2018, she obtained her Bachelor's degree in "Applied Mathematics," and in 2020, she earned her Master's degree in "Computational Mathematics and Mathematical Modeling". From February 1, 2022, to February 1, 2025, she is a full-time doctoral student in the doctoral program "Operations Research" at the Faculty of Mathematics and Informatics, Sofia University, with her academic advisor being Prof. DSc Nadia Zlateva.

I met Hristina Belcheva during the candidate doctoral exam. She made a strong impression on me with her straightforwardness and openness. Later, I had the opportunity to attend her presentations at

the "Optimization" seminar, at the Spring Scientific Session of the Faculty of Mathematics and Informatics, as well as at international conferences such as the International Workshop on Well-Posedness of Optimization Problems and Related Topics in 2023 in Borovets, and the Week of Mathematics and Informatics in 2024 at Duni. This allowed me to closely observe her development as a mathematician.

3. Content analysis of the scientific and applied achievements of the candidate, contained in the presented PhD thesis and the publications to it, included in the procedure

In the first chapter, the concept of uniform epi-convergence for a sequence of functions is defined, which is closely related to the well-known concept of epi-convergence. A method is proposed for finding the same perturbation function for all elements of a uniformly epi-convergent sequence of eigenfunctions, semi-continuous from below, and bounded from below, defined in a complete metric space, such that the strict minima of the perturbed functions converge to the strict minimum of the perturbed boundary function. It should be noted that this approach does not require any assumption of convexity

Chapter 2 extends the approach presented in Chapter 1 to a parameterized family of uniformly epi-continuous functions defined in a complete metric space. As an application, a result of the Stechkin type is obtained regarding the uniqueness and continuity of the metric projection (with respect to a nearby equivalent norm) onto a fixed non-empty, closed, bounded, and convex set at each point of a pre-fixed dense set in a separable Banach space.

Chapter 3 presents a perturbation method suitable for minimization problems with constraints. It is shown that, under certain conditions, correct placement in a compact modulus can be achieved for the perturbed problems. The approach is applied in Orlicz sequence spaces, for which a suitable perturbation space is constructed. A non-trivial result is proven, establishing, in a uniform manner, the non-existence of bell-shaped functions with certain properties in these spaces.

4. Approbation of the results

The results of the dissertation have been published in three articles, all in reputable journals with an impact factor and impact rank:

- 1. Hristina Topalova**, Perturbation Method in Orlicz Sequence Spaces, 16-th International Workshop on Well-Posedness of Optimization Problems and Related Topics, 3 - 7 July, 2023, Borovets, Bulgaria <http://www.math.bas.bg/~bio/krast/WP23/>
- 2. Hristina Topalova**, Perturbation Method in Orlicz Sequence Spaces, FMI Spring Science Session, 25.03.2023, Sofia, Bulgaria <https://fmi.uni-sofia.bg/bg/proletna-nauchna-sesiya-na-fmi-2023>
- 3. Hristina Topalova**, Simultaneous perturbed minimization of a convergent sequence of functions, FMI Spring Science Session, 25.03.2024, Sofia, Bulgaria, <https://www.fmi.uni-sofia.bg/bg/proletna-nauchna-sesiya-na-fmi-2024>
- 4. Hristina Topalova**, Simultaneous perturbed minimization of a convergent sequence of functions, International Conference on Optimization: Challenges and Applications, 27-29 May, 2024, Alicante, Spain, <https://sites.google.com/gcloud.ua.es/icoca75boris/home>
- 5. Hristina Topalova**, Simultaneous perturbed minimization of a convergent sequence of functions, Week of Mathematics and Informatics, September 23-27, 2024, Duni Royal Resort, Bulgaria <https://www.fmi.uni-sofia.bg/bg/week-mathematics-and-informatics>
- 6. Hristina Topalova**, Generic continuity of the perturbed minima of certain parametric optimization problems, 4th International Conference on Variational Analysis and Optimization, January 14-17, 2025, Santiago, Chile, <https://eventos.cmm.uchile.cl/lopezcerda2025/>
- 7. Hristina Topalova**, Generic continuity of the perturbed minima of certain parametric optimization problems, FMI Spring Science Session, 22.03.2025, Sofia, Bulgaria, <https://www.fmi.uni-sofia.bg/bg/node/10153>

8. **Hristina Topalova**, Continuity of minima of a sequence of functions, Joint Doctoral School SU-UoA part 1, August 24-30, 2025, Sofia, Bulgaria, <https://doctoral-school.fmi.uni-sofia.bg/en>
9. **Hristina Topalova**, Continuity of minima of a parameterized family of functions, Joint Doctoral School SU-UoA part 1, August 24-30, 2025, Sofia, Bulgaria, <https://doctoral-school.fmi.uni-sofia.bg/en>
10. **Hristina Topalova**, Continuity of minima of a sequence of functions. Applications, Joint Doctoral School SU-UoA part 2, September 21-27, 2025, Aizuwakamatsu, Japan, <https://doctoral-school.fmi.uni-sofia.bg/en>

5. Qualities of the abstract

The abstract has been written in two versions: in Bulgarian, consisting of 31 pages, and in English, consisting of 30 pages. It accurately reflects the content of the dissertation and complies with the requirements of the Law on the Development of the Academic Staff in the Republic of Bulgaria (ZRASRB) and the Regulations for the Conditions and Procedures for Obtaining Academic Degrees and Holding Academic Positions at Sofia University "St. Kliment Ohridski." The numbering of the statements, definitions, literary sources, and others in both the abstract and the dissertation is consistent, which facilitates reading.

6. Critical notes and recommendations

I have no critical remarks regarding the substance. I would recommend avoiding the use of quantifiers. The accumulation of several quantifiers such as "for every" and "there exists" makes it difficult to understand the corresponding mathematical text (see, for example, pages 13, 21, 22, 23, and 29).

7. Conclusion

After reviewing the dissertation and the accompanying scientific works presented in the procedure, and based on the analysis of their significance and the scientific and applied contributions contained within them, I confirm that the presented dissertation and the related scientific publications, as well as the quality and originality of the results and achievements presented, meet the requirements of the Law on the Development of the Academic Staff in the Republic of Bulgaria (ZRASRB), its implementing regulations, and the corresponding Regulations of Sofia University "St. Kliment Ohridski" for the awarding of the educational and scientific degree "Doctor" in the scientific field **4. Natural Sciences, Mathematics, and Informatics**, and the professional field **4.5. Mathematics**. In particular, the candidate satisfies the minimum national requirements in the professional field, and no plagiarism has been detected in the scientific works presented for the competition.

Based on the above, I **recommend** that the scientific jury award **Hristina Yordanova Belcheva** the educational and scientific degree of "Doctor" in the scientific field **4. Natural Sciences, Mathematics, and Informatics, professional field 4.5. Mathematics** (Operations Research).

Date: 04.01.2026

Signature:

/Prof. DSc Mikhail Ivanov Krastanov/