STATEMENT REPORT

under the procedure for acquisition of the educational and scientific degree "Doctor"

by candidate Hristina Yordanova Belcheva**,**
of the PhD Thesis entitled: "Application of Variational Analysis Methods",

In the Scientific field: **4. Natural Sciences, Mathematics and Informatics**

Professional field: **4.5. Mathematics**

Doctoral program "Operations Research", Department „ Probability, Operations Research and Statistics", **Faculty of Mathematics and Informatics (FMI)**, **Sofia University "St. Kl. Ohridski" (SU).**

The statement report has been prepared by Prof. DrSc Julian Petrov Revalski, Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, as a member of the scientific jury for the defense of this PhD thesis according to Order № РД-38-685 / 05.11.2025 г. of the Rector of the Sofia University.

1. **General characteristics of the dissertation thesis and the presented materials**

   The dissertation, comprising 59 pages and written in English, consists of an Introduction, three chapters, a Conclusion, information on publications related to the dissertation and the approbation of the results, and a bibliography of 40 references.

2. **Short CV and personal impressions of the candidate**

   Hristina Belcheva was born in 1995. She graduated from the Sofia High School of Mathematics "Paisii Hilendarski" in 2014 and from the Faculty of Mathematics and Informatics at Sofia University "St. Kliment Ohridski" in 2020, obtaining a Master's degree in Computational Mathematics and Mathematical Modeling. During the period February 2022 – February 2025, she was a full-time PhD student in the doctoral programme "Operations Research" at the Faculty of Mathematics and Informatics, Sofia University. She has participated in six research projects, including three internal projects, one funded under the National Recovery and Resilience Plan, and one supported by the National Science Fund. Although I do not know the PhD candidate personally, the submitted

materials leave a very positive impression of her serious and professional approach to every aspect of her work as a doctoral student, including the dissertation defense procedure.

3. **Content analysis of the scientific and applied achievements of the candidate, contained in the presented PhD thesis and the publications to it, included in the procedure**

The presented dissertation is devoted to the derivation of parametric variational principles for optimization problems in complete metric spaces **X** and to their applications in the study of properties of optimization problems and norm-related properties in Banach spaces. The perturbation space **P** is a Banach space of uniformly continuous and bounded functions on **X**, endowed with a norm that dominates the usual supremum norm and possesses a property that appropriately separates the points in **X**.

In Chapter One, the parametrization is associated with a sequence of functions converging suitably to a limit function. This setting allows the generic result of Theorem 1.3.1—namely, the existence of a dense **G_δ** set of perturbations ensuring a strong minimum of the perturbed function—to yield, as an intersection, a nonempty generic set of perturbations that simultaneously guarantee the attainment of a strong minimum for both the perturbed functions and the limit function, as well as the convergence of these minima to the minimum of the limit function (Theorem 1.4.1). This result is applied in Theorem 1.5.1 to study, under appropriate assumptions, the convergence of solutions of perturbations of a sequence of constrained minimization problems, where the objective functions converge uniformly to a limit function and the feasible sets converge to the limiting admissible set in the sense of Painlevé–Kuratowski.

In Chapter Two, the parameter space is a complete metric space that is not necessarily countable, which necessitates a modification of the perturbation space. In this setting, the perturbations are functions depending also on the parameter and are subject to additional conditions beyond those imposed in Chapter One. Under suitable assumptions, a generic variational principle is established for each fixed parameter (Theorem 2.1.1). The author introduces conditions of uniform epi-continuity for a parametric family of functions and, assuming separability of the parameter space, proves a parametric variational principle in Theorem 2.4.3. This result is applied to investigate properties related to the generic existence of solutions and the well-posedness of best approximation problems in separable Banach spaces (Theorem 2.5.4), in the spirit of classical lines of research originating from results by Stechkin in the mid-1960s.

The final Chapter Three is devoted to the derivation of appropriate variational principles in Orlicz sequence spaces. One of the objectives (Theorem 3.2.2) is to obtain, in an arbitrary Banach space, a generic set of perturbations ensuring generalized well-posedness—referred to by the author as “well-posedness modulus compact”—of the corresponding perturbed minimization problem, in particular guaranteeing the attainment of a minimum. Specifically, in Orlicz spaces, under suitable conditions on the Orlicz function generating the norm and with an appropriately chosen perturbation space, variational principles are established (Theorems 3.3.3 and 3.3.4).

These principles are then used to study the nonexistence of bump functions in such spaces possessing certain differentiability properties.

The results obtained by the PhD candidate in this dissertation constitute an important and original contribution to the field of variational analysis and its applications. The candidate has mastered advanced techniques related to the proof of variational principles in optimization, previously developed by authors such as Deville, Godefroy and Zizler, Ivanov, and Ivanov and Zlateva, and has further developed and applied them in a nontrivial and appropriate manner within the framework of the dissertation. I consider the contribution to be fully sufficient for the award of the PhD degree.

4. Approbation of the results

The dissertation is based on three publications by the candidate, co-authored with the academic supervisor, published respectively in the journals *Set-Valued and Variational Analysis* (IF: 1.1; Q2 according to Web of Science), *Optimization* (IF: 1.8; Q1 according to Web of Science), and *Positivity* (IF: 0.9; Q2 according to Web of Science). In my assessment, the candidate's contribution to the joint publications is equal to that of the co-authors. The results of the dissertation have been presented at ten scientific events, three of which were prestigious international conferences. No documentation of citations has been provided, which I consider reasonable given the recent publication of the papers. As evidenced by the submitted report, the candidate exceeds the required national quantitative criteria by more than a factor of three.

It is clearly stated that:

a) the scientific works meet the minimum national requirements (under Art. 2b, para. 2 and 3 of ADASRB*) and respectively to the additional requirements of Sofia University "St. Kliment Ohridski" for acquiring the educational and scientific degree "Doctor" in the scientific field and professional field of the procedure;
b) the results presented by the candidate in the dissertation work and scientific works to it do not repeat such from previous procedures for acquiring a scientific title and academic position;
c) there is no plagiarism proven in the legally established order in the submitted dissertation work and scientific papers under this procedure.

5. Qualities of the abstract

The abstract, in both Bulgarian and English, meets all formal requirements and provides an accurate presentation of the results and content of the dissertation.

6. Critical notes and recommendations

Apart from a few typographical errors and some omitted additional assumptions (for example, concerning the set S on page 43 of the dissertation), a certain lack of awareness

of earlier results related to the topic of the dissertation can be observed. For instance, the idea of using (generic sets of) perturbations from a Banach space of continuous functions predates the works [7,8] cited by the author as the first of this kind; it already appears in papers by Lucchetti and Patrone and by Stegall from the late 1970s, as well as in works by Choban and Kenderov from the mid-1980s. It would also have been desirable for the substantial contributions of the Bulgarian mathematical community—such as those by Kenderov and Zhivkov—to the study of points of uniqueness of best approximation problems in Banach spaces to be reflected in the dissertation. These remarks, however, do not diminish the merits of the presented dissertation.

7. Conclusion

Having become acquainted with the PhD thesis presented in the procedure and the accompanying scientific papers and on the basis of the analysis of their importance and the scientific and applied contributions contained therein, **I confirm** that the presented PhD thesis and the scientific publications to it, as well as the quality and originality of the results and achievements presented in them, meet the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Rules for its Implementation and the corresponding Rules at the Sofia University "St. Kliment Ohridski" (FMI-SU) for acquisition by the candidate of educational and scientific degree "Doctor" in the Scientific field 4. Natural Sciences, Mathematics and Infromatics, Professional field 4.5 Mathematics. In particular, the candidate meets the minimal national requirements in the professional field and no plagiarism has been detected in the scientific papers submitted for the competition.

Based on the above, **I strongly recommend** the scientific jury to award Hristina Yordanova Belcheva, the educational and scientific degree "Doctor" in the Scientific field 4.Natural Sciences, Mathematics and Informatics, Professional field 4.5 Mathematics.

Date: 04.01.2026

Signature: .....
Prof. DrSc Julian Revalski/