REVIEW

under the procedure for acquisition
of the educational and scientific degree "Doctor"

in Professional Field 4.5. Mathematics,
doctoral program "Operations research",
At Sofia University "St.Kliment Ohridski",Faculty of
Mathematics and Informatics
Candidate: Margarita Nikolaeva Nikolova
Reviewer: Prof.Nadezhda Kostadinova Ribarska, DSci
October 8, 2023

I am writing this review in my capacity as a member of the scientific jury, according to Order No. RD-38-383/12.07.2023 of the Rector of Sofia University. The presented dissertation "Local properties of dynamical systems" (Локални свойства на динамични системи) consists of 60 pages and is written in English. It contains an introduction, four chapters, a conclusion and a bibliography including 38 titles. An abstract in Bulgarian and an abstract in English (the first one 39 pages long and the second one 37 pages long), as well as all other documents required by the procedure (including a report from the anti-plagiarism system) are submitted. I accept for review all submitted materials.

Margarita Nikolaeva Nikolova was born on November 15, 1990. He graduated from the National Science and Mathematics High School "Academic Lyubomir Chakalov" in 2009 and immediately began her studies at the Faculty of mathematics and informatics of the Sofia University "St. Kliment Ohridski", majoring in Applied Mathematics. She graduated as a Bachelor of Applied Mathematics in July 2013. She completed her master's degree in the master's program "Optimization" and graduated with honors in 2018 with a thesis under the supervision of Prof. DSci Mikhail Krastanov. From July 2019 Margarita Nikolova is a full-time doctoral student in the doctoral program "Operations Research" of FMI, SU with scientific supervisor Prof. DSci Mikhail Krastanov. Since the fall of 2022 she occupies a position of a researcher (first level) at FMI, SU. For many years she taught classes in Differential and Integral Calculus - 1, Differential and Integral Calculus - 2, Mathematical Analysis, Introduction to Statistics, Random Processes.

I have known Margarita Nikolova since 2010, when she was a sophomore and took the compulsory exam in Mathematical Analysis I was teaching. She performed brilliantly (in an extremely strong environment) and even then I

was impressed both by her mathematical thinking and by the fact that she is incredibly quiet and humble. After that, she took all the elective courses I taught with flying colors. For at least ten years, he has been my assistant in Mathematical Analysis, then also in other disciplines that I teach. As a teacher, Margarita is extremely responsible and organized (both personal observations and feedback from students). Gradually, I began to have full confidence in the overall delivery of the training – including the precise grading of both assignments and theory questions. I am happy to hear her opinion on any matter of interest to the community. I believe that Margarita Nikolova is a mature colleague as a teacher and as a member of the academic community. The presented dissertation convinces me of her qualities as a scientist as well.

The dissertation is devoted to the problem of small time local controllability of a non-linear system at a point. A geometric approach proposed and developed by Bianchini, Hermann, Hermes, Hirshorn, Krener, Kunita, Lobry, Stefani, Sussmannn and others is followed. Here it is appropriate to note that the scientific supervisor of the doctoral student is a world-renowned specialist in this field and one of the scientists developing the theory in question. The basic idea of the geometric approach is that the local properties of the reachable set of an analytic control system are defined by the properties of the Lie algebra, generated by the vector fields defined by the control system, which are calculated at the starting point. No necessary and sufficient condition for small-time local controllability is known in terms of the properties of this algebra, except in some special cases. Two new sufficient conditions and one necessary condition in these terms are obtained in the dissertation. They go beyond the known theory and their proofs contain new ideas with potential for further development.

The first chapter of the dissertation is an introduction, and the second chapter contains some necessary preliminary information - definitions, Campbell-Baker-Hausdorff formula, basic statements about tangent vectors to the reachable set. I would like to point out that the technique used is extremely difficult and was mastered beautifully by the PhD student.

In the fifth chapter of the dissertation, the following necessary condition for controllability is formulated and proved in differential-geometric terms natural to the problem: Let us be given a nonlinear control system Σ in $\mathbb{R}^n$, whose right-hand side is continuous in both phase and control variables and Lipschitz with respect to the phase variable in a neighborhood of the origin uniformly with respect to the control. The control takes values in a compact subset U of $\mathbb{R}^m$. The basic assumption is that there exists an proper subspace L of $\mathbb{R}^n$ such that the largest linear space containing the convex closed cone generated by L and the possible values of the right-hand side when the phase variable varies in a neighborhood of zero, and the control in U, is contained in L. Then the system Σ is not small time locally controllable at zero. This chapter also presents an example of a parameter-dependent control system

for which a characterization of the small time local controllability property is obtained based on the values of the parameter. The motivation of the basic assumption formulated above is also given there. My question to the PhD student is how this assumption is verified in the general case. Several corollaries of the obtained main result are formulated in this chapter. It is shown that, for the considered class of controllable systems, these corollaries entail Sussman's necessary condition from 1978, Stefani's necessary condition from 1986, and Krastanov and Velov's necessary condition from 2005 for switching systems. Chronologically, this is the first result obtained by the PhD student.

In the third and fourth chapters of the dissertation two sufficient conditions for small time local controllability are presented for a class of polynomial control systems whose right-hand side is a sum of constant vector fields and a polynomial vector field, which is homogeneous of the second degree. In both results new ideas appear which go beyond the established paradigm.

The sufficient condition for small-time local controllability discussed in chapter three is a generalization of a recent result by Aguilar in the sense that the class of control systems considered by the PhD student is substantially more general and, moreover, restrictions are imposed on the values of admissible controls. The proof rests on the formula of Campbell-Baker-Hausdorff. It should be noted that this controllability condition goes beyond the classical Sussmannn result. In Sussmannn's theory, to have small-time local controllability, it is necessary to neutralize the "bad Lie brackets" by suitable "good brackets". In the proof of the considered sufficient condition, some "bad Lie brackets" are allowed to neutralize each other in an appropriate way.

In the fourth chapter, a control system of the same class as defined in chapter 3 is studied. A certain structure of cones and linear spaces contained within each other is defined, and it is proved that the elements of these cones and subspaces are tangent to the reachable set. Thus defined structure refines and generalizes similar known structures for partially linear systems and for linear systems with switches (developed by Veliov and Krastanov). The resulting sufficient condition neither contains nor is contained in the sufficient condition of the previous chapter. The difference is that here a "larger" linear space is identified, generated by values at the origin of Lie brackets of length 3, while in chapter three "fewer" Lie brackets, but of greater length, are used. I should note that Corollary 4.1.2 unifies the two sufficient conditions. Two specific four-dimensional examples of control systems of the class under consideration are included in this chapter. These examples at first sight are similar but have completely different properties in terms of local controllability. It is important to emphasize that the proofs of the main results in the third and fourth chapters are not only an achievement from a technical point of view, but also contain ideas that will undoubtedly be used and further developed in the future.

The results of the dissertation have been published in three papers – one in Automatica, one in Systems & Control Letters, and one in Proceedings of the Bulgarian Academy of Sciences. The three articles are joint with the supervisor. All three journals have an impact factor. Automatica is in the first quartile of Web of Science Q1 for Automation & Control Systems and Systems & Control Letters in Q2 of Web of Science for Operations research & management science and in Q1 of SCOPUS for the categories of Computer Science and Control and Systems Engineering. I want to emphasize that two citations of Margarita Nikolova's publications have already been noticed – one in a preprint from arXiv and one in a doctoral thesis defended at a French university.

Part of the results of the dissertation were reported by the doctoral student at prestigious scientific forums: Large-Scale Scientific Computations, Sozopol, June 7 - 11, 2021, Large-Scale Scientific Computations, Sozopol, June 5 - 9, 2023, and 16-th International Workshop on Well-Posedness of Optimization Problems and Related Topics, Borovets, July 2 - 7, 2023. Margarita Nikolova took part in the Spring School on Variational Analysis 2015, Paseky (Czech Republic), as well as in conferences 15-th International Workshop on Well-Posedness of Optimization Problems and Related Topics, June 28 - July 6 2023, Borovets, Bulgaria, 15th Viennese Conference on Optimal Control and Dynamic Games, 12 - 15 July 2022, Vienna, Austria and in International Conference "Mathematics Days in Sofia", July 7 - 10, 2014, Sofia, Bulgaria.

The results obtained in Margarita Nikolova's dissertation and the publications based on these results exceed a lot the minimum national requirements (according to Art. 2b, Para. 2 and 3 of the ZRASRB) and, respectively, the additional requirements of SU "St. Kliment Ohridski" for the acquisition of the educational and scientific degree "doctor" in professional field 4.5 Mathematics (Operations Research).

The abstracts (in Bulgarian and in English) accurately and comprehensively reflect the results, described in the dissertation. As a remark, I could say that they are too long.

Some remarks. The introduction of the dissertation is not written carefully enough. It is easy to notice a lot of language mistakes. There are occasions when the non-specialist reader can easily get lost due to some inaccuracies. For example, at the end of paragraph 2.2, the definition of $Exp\ (\mathcal{S})_N$ is not quite correct, and in the evaluation after that, t appears only in the right-hand side of the inequality.

The dissertation contains original results. Referencing other people's results is comprehensive and correct. The presentation is good. The dissertation work contains scientific results that are an original contribution to the scientific

field and which can be developed in future.

In conclusion, the dissertation of Margarita Nikolaeva Nikolova is an original research in the field of local controllability of dynamic systems. The obtained results contain substantial contributions and are published in prestigious journals. I confirm that the presented dissertation and the scientific publications related to it meet the requirements of ZRASRB, the Regulations for its application and the relevant Regulations of SU "St. Kliment Ohridski" for the candidate's acquisition of the educational and scientific degree "doctor" in professional field 4.5 Mathematics. In particular, the candidate satisfies the minimum national requirements in the professional field and no plagiarism has been found in the scientific papers submitted.

Based on the above, I strongly recommend the scientific jury to award Margarita Nikolaeva Nikolova the educational and scientific degree "Doctor" in Scientific Area 4. Natural sciences, Mathematics and Informatics, Professional field 4.5.Mathematics, doctoral program "Operations Research".

08.10.2023

(Prof. N. Ribarska, DSci)