REVIEW

by competition for the academic position of associate professor

in a professional direction
4.5. Mathematics (Differential Equations),

for the needs of Sofia University, St. Kliment Ohridski,

Faculty of Mathematics and Informatics (FMI),

announced in SG No. 24 of 2023

and on the FMI and SU websites

The review was prepared by: Associate Professor, Doctor Angel Ivanov Zhivkov – Sofia University, Faculty of Mathematics and Informatics, "Differential Equations"department,
in my capacity as a member of the scientific jury for the competition 4.5. Mathematics (Differential Equations) according to Order No. RD-38-245/12.05.2023 of the Rector of Sofia University.

They submitted documents for participation in the announced competition the following candidates:

1. **Georgi Ivanov Georgiev**, chief assistant doctor FMI, SU, "Differential Equations"department
2. **Svetlin Georgiev Georgiev**, chief assistant, Ph.D FMI, SU, "Differential Equations"department

General description of the materials presented

1. About Georgi Ivanov Georgiev

The documents presented in the competition from the candidate corresponds to the requirements of ARASRB, PPZRASRB and the Regulations for the terms and conditions for the acquisition of scientific degrees and occupying academic positions at the University of St. Clement Ohridski (PURPNSZADSU).

Scientific papers

To participate in the competition, the candidate submitted 8 articles, which I will number (1) to (8):

(1) published in Fractal Frac., 2021 (IF 3.57), joint with T. Boev,
(2) in AIP Conference Proceedings, 2022 (SJR 0.18),
(3) in AIP Conference Proceedings, 2021 (SJR 0.18),
(4) in Chaos, Solitons & Fractals, 2020 (IF 5.94),
(5) in AIP Conference Proceedings, 2018 (SJR 0.17),
(6) in BAS Reports, 2018, (IF 0.21),
(7) in SIGMA, joint with O. Hristov, 2015 (IF 0.45),
(8) in Chaos, Solitons & Fractals, joint with O. Hristov, 2015 (IF 1.61).

The studies reviewed in the above list of publications are in the field of Hamiltonian systems and their study integrability. Even the publication (1) about the existence of a global solution of the classical Derichlet problem with non-zero boundary values conditions for the fractional Laplace equation is also related to the study of Bessel's generalized equation and the project to find his differential Galois group.

This is an unexplored variant of the generalized hypergeometric equation, investigated in publication (7). Two types of fourth-order equations with a property are considered there of Penleve - polynomial type and without moving singular points. These equations can be written as a Hamiltonian system. In this publication it is proved that the equations, as follows denoted by F-XVII of the Cousgrove classification and generalized variants of $P_{II}^{(2)}$ and $P_{II}^{(3)}$ of the hierarchy P_{II} are non-integrable in rational first integrals except for some parameters. It is shown that these equations with Painleve property have normal variational equations that are generalized hypergeometric equations and their differential group is found of Galois. Because normal variations have an essential feature, c these cases in the generators of the Galois Group there are Stokes Matrices, which are calculated explicitly.

In publication (8), non-integrability of the system describing the stationary solutions of the Bose–Einstein model or as in the particular Bose–Fermi case. It has been proven that the only integrable cases are those for which the variables separate. Here again, the variational equations around a suitable particular are considered solution and three approaches are used in Morales–Ramis Theory. The first is Kovacic's Algorithm; the second – the study of variations from line 3; the third – Poincare theory–Arnold–Melnikov–Ziglin the study of the Melnikov integral.

In publication (6), the Hamiltonian system with the potential of Dyson and her non-integrability. A different proof is shown for meromorphic non-integrability than what is already known, which is a bit more total result of

previously achieved.

In publication (5) the Shazi-Carzon cosmological model was investigated for non-integrability. Here the approach is a little different – directly studying the geometry of solutions and proving them conditional non-periodicity. The problem here is that the equations of motion are not in the proper form, and no amount of changing the variables brings them in an easy-to-research version.

In publications (2), (3) and (4) the closed ion model was investigated and despite some inconsistencies and inaccuracies such as degenerate cases in (4), the cases that are non-integrable are shown. Here there is proved a direct connection between differential Galois theory and classical Galois theory.

In publication (1), the classic Dirichlet Problem was solved in the three-dimensional and one-dimensional cases for Laplace's fractional equation with nonzero boundary conditions. A variant of the classic is used Hoermander's approach viewed through the prism of fractionals Laplacians.

Teaching and educational-pedagogical activities. Georgi Georgiev leads or has led:

a) lecture courses at FMI or BF of SU:
– "Mathematics Faculty of Biology,
– "Differential equations FMI, special "Mathematics
– "Differential equations FMI, special, "Mathematics and Informatics" – optional course,
b) exercises on differential equations in FMI
– majors "Mathematics", "Applied Mathematics", "Mathematics and informatics".

My assessment of the candidate's teaching-pedagogical activity is very good.

I have no critical notes or recommendations on the science and the teaching activity of the candidate.

Personal impressions of the candidate. I have known Georgi since 1989, when he was visiting the workshop of Vasil Tsanov and Emil Horozov. After graduating from FMI, he worked for many years at P?I. After defending his doctoral thesis with a factual academic supervisor associate professor Ognyan Hristov and admission is at FMI as a senior assistant, he scores fast progress in his research.

He has a sense of humor.

Conclusion of the application. After getting acquainted with the materials presented in the competition and scientific works and based on the analysis of their significance and scientific-applied contributions contained in them, I confirm, that they meet the requirements of ZRASRB, The regulations for its application and the relevant Regulations of the University of St. Kliment Ohridski to borrow from the candidate for the academic position of associate professor in the scientific field and professional direction of the competition.

In particular, the applicant satisfies the minimum national requirements requirements in the professional direction and has not been established plagiarism in the scientific works submitted for the competition.

I give my positive assessment to the candidature of Georgi Georgiev.

2. About Svetlin Georgiev Georgiev

The documents presented in the competition from the candidate corresponds to the requirements of ?ARASRB, PPZRASRB and the Regulations for the terms and conditions for the acquisition of scientific degrees and occupying academic positions at the University of St. Clement Ohridski" (PURPNSZADSU).

Scientific papers

To participate in the competition, the candidate submitted two works.

The joint article with T. Xiang

T. Xiang and S. Georgiev. Noncompact-type Krasnoselskii fixed point theorems and their applications. Mathematical Methods in the Applied Sciences, Vol. 39, Issue 4, 2016, pp. 833-863

was published in a journal with a high Impact Factor in which Svetlin Georgiev is a member of the collegium.

The article offers methods for solving different types perturbation equations arising in applied sciences. These methods are based on generalizations of abstract theorems about existence of fixed points x of operator equations $Tx + Sx = x$ where x belongs to a convex closed subset of a Banach space, and the operators S and T are of different type. 8 such variants of theorems are indicated.

Applications of the above results follow next. The equation

$$\Big[v_3\frac{\partial}{\partial x}+\sigma(x,v)+\lambda\Big]\psi(x,v)=\int_{\mathbb{S}^2} r\big(x,r,r',\psi(x,v')\big)\,dv'$$

defines the asymptotics of the energy distribution $\psi(x,v)$, depending on the variables $x\in[0,1]$ and $v=(v_1,v_2,v_3)\in\mathbb{S}^2$, the functions $\sigma,\lambda\in\mathbb{C}$ and r are known. It characterizes possible energy leakage along channel boundaries ($\psi(0,v)_{|v\in\mathbb{S}^2}$ is the input boundary, and $\psi(1,v)_{|v\in\mathbb{S}^2}$ is the outgoing boundary).

A theorem was formulated and proved that if 4 conditions hold, then the above equation has a solution and it is unique.

Формулирана и доказана е теорема, че ако са в сила 4 условия, то горното уравнение има решение и то е единствено.

In the next paragraph, Darboux's first quadrant task is discussed:

$$\begin{aligned} &u_{xy}(x,y) \;=\; \lambda u(x,y)+\mu\, g(x,y,u(x,y))\,, \qquad x\geq 0,\ y\geq 0,\\ &u(x,0)=\phi(x)\,,\quad u(0,y)=\psi(y)\,, \end{aligned}$$

where λ and μ are non-negative constants, ϕ and ψ are C^1–functions and g is continuous.

Conditions for λ, μ, and g are found, where the above Darboux problem has a global C^1– solution u, the derivative u_{xy} exists and is continuous. The proof of this theorem is broken down into 12 lemmas and two propositions.

Next, the author considers a class of difference equations

$$\Delta u(n)=a(n)u(n)+\lambda b(n)f(u(n-\tau(n)))+g(n)\,,\quad n\in\mathbb{Z}\,,$$

where $\Delta u(n)=u(n+1)-u(n)$, a,b,τ and g are ω–periodic functions, and λ is a constant.

Different types of conditions are indicated for a,b,τ,g and λ, where we can guarantee the existence of a solution $u=u(n)$, as well as to estimate the growth of these solutions.

Finally, a theorem on the existence and uniqueness of is proved the solution of the perturbed Volterra equation

$$u(t)=\int_a^t k(t,s)u(s)ds+f(t,u(t))\,,\quad t\in[a,b]$$

for special values of the kernel k and the perturbation f .

The second scientific work presented by S. Georgiev is the standalone book (402 pages)

S. Georgiev. *Integral Equations on Time Scales*, Atlantis Press, 2016.

According to WikipediaA, "In mathematics: Time-scale calculus, the unification of the theory of difference equations with differential equations."
S. Georgiev's book is a supplement to the foundational work

M. Bocher, A.Peterson, *Dynamic Equations on Time Scales: an Introduction with Applications* (Birkhauser, Boston, 2003).

Over 50 new theorems necessary for the practical calculations of various integrals equations on time scales and the reduction of dynamic to integral equations. Of course, the proof of most of these theorems is relatively easy.
Hundreds of concrete examples of
– Volterra's integral equations,
– integro-differential equations,
– equations of the Fredholm type,
– Hilbert-Schmidt integral equations with symmetric kernels,
– Laplace transform,
– solutions in the form of lines ("series solution"),
– nonlinear integral equations on time scales.

The theoretical part of the book, plus the detailed calculations in it, in my opinion, make it a good textbook on "Time-scale calculus".

Teaching and educational-pedagogical activities. Svetlin Georgiev has an excellent list of courses led by him.

Mandatory – in FMI or BF of SU:

– "Differential equations and applications" special, "Informatics"
– "Equations of mathematical physics special„Applied math",
– "Partial differential equations", special, "Mathematics",
– "Mathematics and informatics special "Biology".
– "Mathematical analysis of functions of many variables spec. "Engineering Physics", "Medical Physics".

Elective courses – at FMI SU:

– "Wave images",

– "Integral equations",
– "Tensor calculus",
– "Clifford's Analysis of Differential Equations",
– "Theory of semigroups and applications",
– "Introduction to the theory of discrete dynamical systems and chaos",
– "Dynamic computation on time scales".

Most of the elective courses have been written and issued (in foreign publishing houses) relevant textbooks, monographs or books.

My assessment of the candidate's teaching-pedagogical activity is very good.

I have no critical notes or recommendations on the science and the teaching activity of the candidate.

Personal impressions of the candidate. I have known Svetlin Georgiev since 2001, when I was a reviewer of his doctoral dissertation. Since then he has made an unexpectedly successful for me leap in his science development.

Conclusion of the application. After getting acquainted with the materials presented in the competition and scientific works and based on the analysis of their significance and scientific-applied contributions contained in them, I confirm, that they meet the requirements of ZRASRB, The regulations for its application and the relevant Regulations of the University of St. Kliment Ohridski" to borrow from the candidate for the academic position of associate professor in the scientific field and professional direction of the competition. In particular, the applicant satisfies the minimum national requirements requirements in the professional direction and has not been established plagiarism in the scientific works submitted for the competition.

I give my positive assessment to the candidature of Svetlin Georgiev.

OVERALL CONCLUSION

Both candidates have the qualities to hire in an academic position associate professor in a professional field 4.5. Mathematics (Differential Equations) at Sofia University, St. Kliment Ohridski, Faculty of Mathematics and Informatics.

I would make the following comparison between them.

The volume of Svetlin Georgiev's scientific output is extraordinary – a total of 49 articles, 40 participations in conferences abroad, 16 books, incl. four of them published by Springer or Birkhauser. I rate the post qualities of both as approx equal.

The teaching activity of Svetlin Georgiev is more diverse, there are actually textbooks written on most of what he read elective courses.

Based on the above, I recommend the scientific jury to rank the two candidates as follows:

1. **Svetlin Georgiev Georgiev**
2. Georgi Ivanov Georgiev

Sofia, July 10, 2023 Prepared the review:

(Assoc. Dr. Angel Zhivkov)