STATEMENT REPORT

under the procedure for acquisition of the educational and scientific degree "Doctor"

by candidate **Nikolay Petrov Buyukliev**

of the PhD Thesis entitled:

"A Class of Teo plitz C*-algebras"

In the Scientific field: **4. Natural Sciences, Mathematics and Informatics**

Professional field: **4.5. Mathematics,**

Doctoral program: **"Mathematical analysis",**

Department: **"Mathematical analysis",**

Faculty of Mathematics and Informatics (FMI),

Sofia University "St. Kliment Ohridski" (SU),

The review is prepared by: Prof. Boyan Georgiev Zlatanov, Dr.Sci - University of Plovdiv "Paisiy Hillendarski", Faculty of Mathematics and Informatics, Department of Mathematical Analysis, in my capacity as a member of the scientific jury, according to Order No. RD-38-223/28.04.2023 of the Rector of Sofia University and the decision of the first meeting of the jury, I was chosen to present a review.

1. General characteristics of the dissertation thesis and the presented materials

According to the Rules for the Development of the Academic Staff of the SU, the dissertation candidate has submitted the complete set of documents, which includes:

1. *Application Form*
2. *PhD Thesis*
3. *Abstract in Bulgarian*
4. *Abstract in English*
5. *Declaration*
6. *Certificate of compliance with the minimum national requirements*
7. *Report from the head of the department on readiness for defense*
8. *Index of Similarity Report*
9. *Protocol for verification of originality*
10. *Opinion of no plagiarism*
11. *Report of the internal pre-discussion of the thesis*
12. *Order of the internal pre-discussion of the thesis*
13. *Enrollment order for PhD study*

14. Certificate of exams taken during training for the PhD degree

15. An Order for finishing of the PhD study

16. An Order for interruption of the PhD study

17. Opinion of the internal pre-discussion of the thesis

18. Record of the internal pre-discussion of the thesis

19. 1985 Higher Education Diploma with a five-year course of study recognized as a Master Degree

20. Curriculum Vitae

The PhD thesis is based on 4 publications in journals. One of the publications is indexed in zbMATH Open (Zentralblatt MATH) u MathSciNet. Another two of them are accepted for a publication in God. Sofij. Univ., Fak. Mat. Inform., which is indexed iin the mentrioned above two data bases. The fourth publication is a manuscript, submitted and under review in C. R. Acad. Bulg. Sci. The PhD student has successfully passed the required examinations for the training to acquire the educational and scientific degree "doctor".

This quick review shows that Nikolay Buyukliev satisfies the formal minimum national requirements for obtaining a PhD degree in the Scientific field of higher education: 4. Natural sciences, mathematics and informatics; professional field: 4.5. Mathematics and has successfully completed his studies at FMI at SU.

The PhD thesis, submitted for defense, is 56 pages long, consists of a six chapters and a bibliography of 52 items.

After reading the thesis, I can say that the cited literature is the one that is needed for obtaining the described results and there are no artificially inserted citations in the list.

The methods used by the author can generally be systematized as a theory of C-algebra and more precisely Toeplitz and Wiener-Hopf operators and grouoid algebra generated by Wiener-Hopf operators, applied creatively to solve the set problems. Wiener-Hopf operators are widely used in physics to predict what might happen in the future if we have information about what happened in the past. The idea of considering not a single operator, but an algebra of operators and based on the found properties of the algebra to derive properties for individual representatives of it determines the interest to study C*-algebras. The main topic in the presented dissertation work is to investigate some properties of groupoid algebras generated by the Wiener-Hopf operator.*

The volume of 56 pages is related to the specifics of the subject. Results in the field of C-algebras are difficult to obtain, and in many cases the proofs are short.*

2. Short CV and personal impressions of the candidate

The PhD student was born in 1859 in Bulgaria. He graduated consecutively obtained a secondary education at the Mathematical School in the city of Gabrovo and a higher education at

Sofia University "St. Kliment Ohridski" - Faculty of Mathematics and Informatics with a specialization in real and functional analysis, respectively in 1977 and 1984.

The PhD student trained me in mathematical analysis when I was a student at Sofia University "St. Kliment Ohridski". My memories of the problem solving classes he led are of exceptional knowledge of the topic, ability to present the solutions and the techniques used in a comprehensible manner. Nikolay Buyukliev was able to engage the attention of the audience both with standard tasks and with significantly more difficult ones, which of course I attribute to the fact that he was a participant in the Bulgarian team for the 19th International Mathematical Olympiad, where he won a silver medal.

3. Content analysis of the scientific and applied achievements of the candidate, contained in the presented PhD thesis and the publications to it, included in the procedure

In the introduction, the author has justified the importance of research in the field of the C-algebra of Toeplitz and Wiener-Hopf multidimensional operators and a program for their study. The fundamental concepts, their properties, the problems arising from them and the possible techniques for solving them are presented. Chapter 2 is dedicated to a more detailed introduction to the topic of the investigated problemss in the PhD thesis. The concepts of groupoids and related algebras, K-theory and cyclic homologies of C*-algebras, Fredholm operator index are introduced.*

A good impression is made by the presence of numerous diagrams in the text, which facilitate the perception of the formulated results and their proofs. There are also illustrative examples.

Chapters 3, 4, 5 and 6 are devoted to new results obtained by the author.

In the third chapter, continious linear cross-sections of groupoid Wiener-Hopf algebras are considered. The results in this chapter are based on the article accepted for publication [Buyukliev N., Linear cross-sections and Fredholm operators in a class groupoid C-algebras, to appear in Ann. Univ.Sofia, Fac. Math. Inf.]. The main result is Theorem 3.1 where a necessary and sufficient condition for an operator to be Fredholm is found. Sufficient conditions are obtained for the image* $\psi(b)(x,n)$ *to be a continuous cross-section. In Theorems 3.2-1 and 3.2-2 we consider two similar (the image from Theorem 3.2-2 is generated from the images studied in Theorem 3.2-1), but differently defined function* $\psi(b)(x,n)$*. In chapter four, the author refers to the already defined function* $\psi(b)(x,n)$*, which confuses the reader. Perhaps it would be more appropriate to use the notations* $\psi_1(b)(x,n)$ *and* $\psi_2(b)(x,n)$ *in Theorems 3.2-1 and 3.2-2.*

In the fourth chapter, a new algebra of operators is defined, which is used in the proofs of the main results in the chapter. A formula was found for calculating the index of the Fredholm operator T under the condition that $\gamma(T), (\gamma(T))^{-1} \in T^{\infty}$*. The results in this chapter are based on a manuscript accepted for a publication [Buyukliev N., An index formula in a class of groupoid* $C*$*-algebras, to appear in Ann. Univ.Sofia, Fac. Math. Inf.].*

In the fifth chapter, the concept of the K-theory of the groupoid algebra $B(R^n, P)$ is considered, for the semigroup P, which is a polyhedral exhaustible cone. The results in this chapter are based on the publication [Buyukliev N., K-theory of the $C*$-algebra of multivariable Wiener-Hopf operators, associated with some polyhedral cones in Rn, Ann. Sofia Univ., 1, (1997), 115-119]. A Fredholm operator with index 1 is constructed in the considered set $B(R^n, P)$ (Theorem 5.2). This result has been cited in two publications, one of them in a journal with Q2 from WoS. Both publications are by authors from abroad.

In chapter six, a representation of the algebra $T(H_3(\mathbb{Z}))$ as a groupoid C-algebra is obtained. This result is based on the manuscript [Buyukliev N., The $C*$-algebra of Toeplitz operators of the discrete Heisenberg group H_3, C. R. Acad. Bulgaria Sci], submitted for a review.*

The author's contributions are correctly described in the PhD thesis.

4. Approbation of the results

The PhD thesis is based on 4 publications in journals indexed in zbMATH Open (Zentralblatt MATH) u MathSciNet. One of the publications will indexed in SCOPUS and WoS with Q4. The results were reported at three scientific forums and three seminars. The three scientific forums are held abroad. The thre seminars are the FMI spring scientific session at SU, which has become a tradition over the years and is a nice one, where specialists in mathematics, informatics and education in mathematics and informatics can share their new ideas or results

a) *The PhD student has submitted a thesis and thus satisfies the group of indicators "A" with 50 points. The total number of points with which the candidate participates in the group of indicators "Д" in the procedure for acquiring the PhD degree will be 90 points. It follows that Nikolay Buyukliev satisfies the minimum national requirements. The PhD student has successfully passed the required examinations for the training to acquire the PhD degree and accordingly satisfies the requirements of SU "St. Kliment Ohridski" for the acquisition of an educational and scientific degree "Doctor" in the Professional field: 4.5. Mathematics.*

   As I mentioned in the presentation, so far the author has two citations to one of his publications.

   The PhD student participates with four publications in which he is a single author. It is therefore inapplicable to comment on his contribution to the results obtained.

b) *The results presented by the candidate in the PhD thesis and scientific works to it do not repeat such from previous procedures for acquiring a scientific title and academic position as it is a first procedure for academic growth of the candidate*

c) ***there is no plagiarism proven in the legally established order in the submitted dissertation work and scientific papers under this procedure in the sense of the "Law on the Development of the Academic Staff in the Republic of Bulgaria" in the Republic of Bulgaria.***

Qualities of the abstract

The presented abstracts in Bulgarian and in English meet the requirements of the SU and correctly systematize the results of the presented thesis.

5. Critical notes and recommendations

I have no significant critical remarks about the PhD work. Nikolay Buyukliev has another publication [Buyukliev, Nikolay P. On the C-algebras of multivariable Wiener-Hopf operators. (English) Zbl 0735.46040, MR1088000 Math. Balk., New Ser. 4, No. 2, 129-138 (1990)], which is related to the thesis topic and is indexed in zbMATH Open (Zentralblatt MATH) and MathSciNet. If this article were included in the procedure, it would have the points available according to the minimum national requirements, without the need to wait for the printing and subsequent indexing of the other three publications.*

In the submitted reference for satisfying the minimum national requirements, only the two publications accepted for printing are mentioned. The article on which chapter five is based is omitted, and it is indexed in zbMATH Open (Zentralblatt MATH) and MathSciNet and carries 18 pts.

6. Conclusion

Having become acquainted with the PhD thesis presented in the procedure and the accompanying scientific papers and on the basis of the analysis of their importance and the scientific and applied contributions contained therein, **I confirm** that the presented PhD thesis and the scientific publications to it, as well as the quality and originality of the results and achievements presented in them, meet the requirements of the "Act on Development of the Academic Staff in the Republic of Bulgaria", the Rules for its Implementation and the corresponding Rules at the Sofia University "St. Kliment Ohridski" (FMI-SU) for acquisition by the candidate of educational and scientific degree "Doctor" in the Scientific field 4. Natural Sciences, Mathematics and Informatics, Professional field 4.5. Mathematics. In particular, the candidate meets the minimal national requirements in the professional field and no plagiarism has been detected in the scientific papers submitted for the competition.

Based on the above, **I strongly recommend** the scientific jury to award **Nikolay Petrov Buyukliev**, the educational and scientific degree "Doctor" in the Scientific field 4. Natural Sciences, Mathematics and Informatics, Professional field 4.5. Mathematics, Doctoral program: "Mathematical analysis ".

May 26, 2023

Reviewer:
/Boyan Zlatanov, Professor, Dr.Sci. /