

REPORT

according to the procedure for the defence of a PhD Thesis entitled

“A CLASS OF TÖPLITZ C^* -ALGEBRAS”

for the acquisition of PhD degree of the Sofia University

by the PhD candidate: **Nikolay Petrov Buyukliev**

in the field of higher education: **4. Natural sciences, mathematics and informatics**

professional direction: **4.5. Mathematics**

doctoral program: **Mathematical analysis of the Faculty of Mathematics and Informatics (FMI) of the SU "St. Kliment Ohridski"**

The referee report is prepared by D.Sci.Stanimir Troyanski, associated member of the Institute of Mathematics and Informatics, Bulgarian Academy of Science (e-mail troyanski@math.bas.bg).

The PhD thesis, is written in Bulgarian, and is a text of 56 pages, A4 format, 61 titles are cited. It consists of an Introduction, and six chapters, divided into paragraphs, in which the results of the dissertation are presented.

The PhD thesis is in the field of Classical and Functions Analysis. More precisely Operator Theory, which are part of Banach Algebras, especially C^* -algebras. The topic of the dissertation rapidly developed in the 60s and 70s of the last century. These studies are relevant today. The index problem for elliptic differential operators was posed by I.Gel'fand. He noticed the homotopy invariance of the index, and asked for a formula for it by means of topological invariants. In 1963 M.Atiyah and I.Singer proved the famous theorem states that for an elliptic differential operator on a compact manifold, the analytical index related to the dimension of the space of solutions, is equal to the topological index, defined in terms of some topological data. It includes many other theorems, such as the Chern–Gauss–Bonnet theorem and Riemann–Roch theorem, as particular cases, and has applications to Theoretical

Physics. For his research related to the index theorem, M. Atiyah received the Fields medal in 1966.

N. Buyukliev gets acquainted with this range of questions under the influence of the late Prof. R. Levy, who "brought" this topic from the Moscow School of Functional Analysis and its applications. The dissertation examines operators appearing in integral equations. Generally speaking, author study integral operators acting on functions defined on a solid semigroup P of a locally compact group G , which may be non-commutative, with a left-invariant Haar measure. In the simplest case, when the group G is the real line \mathbf{R} equipped with the Lebesgue measure, and the semigroup P is \mathbf{R}_+ , the Wiener-Hopf operator $W_f : L_2(\mathbf{R}_+) \rightarrow L_2(\mathbf{R}_+)$, with symbol $f \in L_1(\mathbf{R})$ is defined as follows $u = W_f v$, where

$$u(x) = \int_0^{\infty} f(x-y)v(y)dy.$$

The classical Wiener-Hopf integral equation is $(I + W_f)v = w$. The corresponding C^* -algebra generated by the operators W_f is the Wiener-Hopf algebra. Another classic case is when the group G is the group of integers \mathbf{Z} and the semigroup is \mathbf{Z}_+ . In this case, the corresponding C^* -algebra is called C^* -algebra of Töplitz operators.

The author studies various C^* -algebras of operators corresponding to various semigroups of the above type, which we will denote by \mathcal{B} . The main achievements of the author are: Compute K -theory of ideals of \mathcal{B} and corresponding quotients and whole \mathcal{B} . Find Fredholm criteria for operators in \mathcal{B} . Under the influence of the results of A. Connes and E. Park author give a formula which express the Fredholm index in topological terms. Construct a Fredholm operator with index 1.

Parts of the thesis have been reported in various scientific forums. From the direct analysis of thesis, it can be seen that the author has mastered a significant mathematical knowledges in Analysis (Classical, Functional, Differential Geometry), Algebra, Algebraic Topology and has skilfully applied it in solving specific Operator Theory tasks. In my opinion, the results of the thesis, both in terms of quantity and quality, meet the standard requirements for a PhD thesis.

Based on the above, I strongly recommend that the scientific jury award Nikolay Petrov Buyukliev the educational and scientific degree "doctor" in a scientific field: 4.Natural sciences, mathematics and informatics, professional direction: 4.5. Mathematics, doctoral program "Mathematical Analysis".

Sofia, 1 June ,2023

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(Stanimir Troyanski)