REFEREE REPORT

on the procedure for the defence of a thesis for the award

of the educational and scientific degree

„Doctor“

in professional direction 4.1. Physical sciences

Scientific specialty: 01.03.01 „Theoretical and mathematical physics“

Title: “Structure and astrophysics of self-gravitating objects in multiscalar theories”

Author: Radostina Zhekova Zheleva, full-time PhD student at the Department of Theoretical Physics at the Faculty of Physics of Sofia University „St. Kliment Ohridski“

Supervisor: Prof. D.Sci. Stoytcho Yazadjiev (Faculty of Physics, SU)

Рецензент: Assoc. Prof. PhD Galin Gyulchev (Faculty of Physics, SU)

1. Brief biographical data about the PhD student

Radostina Zheleva acquired the educational and qualification degree “Master” in the specialty “Physics” with the professional qualification “Master in Physics - Theoretical and Mathematical Physics” in October 2018 at SU “St. Kliment Ohridski”. In January 2019, she was enrolled as a full-time doctoral student in the "Theoretical Physics" department of the Faculty of Physics of the SU, supervised by Prof. D.Sci. Stoytcho Yazadzhiev. After successfully implementing the planned individual plan for education and research, Radostina Zheleva was assessed with the right to defence. On 17.01.2023, by decision of the Faculty Council of the Faculty of Physics, she was referred to a scientific jury for an official defence procedure.

2. General description of the dissertation and the presented materials

Master's degree Radostina Zheleva has presented a dissertation for acquiring the educational and scientific degree “doctor” written in Bulgarian, in a volume of **80** pages. **Seventeen** figures are given in the dissertation. The dissertation begins with a summary and is structured into **four** main chapters, followed by a list of the doctoral student's publications in refereed journals and ends with acknowledgements. The bibliography of the dissertation contains **136** titles.

Chapter 1 discusses quasi-periodic oscillations from an accretion disk gravitating around rotating traversable wormholes. A general class of geometries that describe a stationary and axisymmetric wormhole is introduced, the stability of circular orbits in the equatorial plane is studied, the properties of the epicyclic frequencies of the particles at the accretion disk in the considered geometry are studied, and the nonlinear resonances are studied. The results are

compared with those of a rotating Kerr black hole. Chapter 2 proves the existence of spherically symmetric black holes in some classes of multi-scalar Einstein-Gauss-Bonnet (MSAGB) theories of gravity for linear and exponential coupling functions. Black hole characteristics such as horizon area, entropy, and photon sphere are also studied, and observational perspectives are discussed. In Chapter 3, secularised non-topological neutron stars are investigated in the MSAGB theory of gravity. Static and spherically symmetric solutions for neutron stars satisfying the MSAGB theory of gravity equations are considered. The numerical methods used are presented, and the obtained results are analysed. The investigated models have a zero scalar field in the star's centre, and the presented results are for a coupling function allowing scalarisation. In Chapter 4, the scientific contributions of the dissertation work are formulated.

The summary of the dissertation has a volume of **41** pages, and it correctly describes the content and objectives of the dissertation.

The thesis is based on three scientific publications attached in full text to the presented dissertation materials. The PhD student's publications have been co-authored in peer-reviewed ***Q1*** quartile journals with an impact factor above **4.6**.

3. Objectives and actuality of the thesis

Current gravity models, which are based on the fully experimentally confirmed General Theory of Relativity, allow the existence of new, hitherto undiscovered scalar fields. In these alternative theories aimed at searching for new fundamental physics, gravity is described by the metric tensor and one or more scalar fields. Of considerable interest for modern physics are gravitational theories with scalar fields coupled to the curvature of spacetime, including multiscalar Gauss-Bonnet theories. These theories predict the existence of new objects, such as spontaneously scalarized black holes, neutron stars, and traversable wormholes without exotic matter. This sheds light on new hitherto unexplored astrophysical effects that may serve as a tool for the observational manifestation of these gravitational objects in the future. X-ray spectroscopy is a promising tool for testing gravity in the regime of strong gravitational fields, which can study various features of the emission flow of accretion disks around compact objects using next-generation X-ray satellites such as LOFT eXTP или STROBE–X.

The PhD thesis uses resonance models to study quasi-periodic oscillations in accretion disks around rotating traversable wormholes. It is shown that the astrophysics of quasi-periodic oscillations can be significantly different from that of rotating Kerr black holes, which can help to distinguish those objects via various observational methods. Another main task of the PhD thesis is to numerically prove the existence of solutions describing black holes and neutron stars in MSAGB gravity with a maximally symmetric scalar space for several coupling functions, including the case

of spontaneous scalarization. A systematic study of the horizon's area, the entropy of black holes, and the radius of the photon sphere have also been made.

Precisely because of this, there is no doubt about the relevance of the scientific problems investigated in the thesis, which are at the forefront of modern gravitational theory.

4. Theoretical formulation and methods

Wormholes can mimic a Kerr black hole in astrophysical effects such as shadow images and accretion disks, iron emission lines, quasi-normal modes and perturbations of stellar orbits. Therefore, it is necessary to investigate additional observable effects in the electromagnetic spectrum that can distinguish wormholes from other compact objects by studying the high-frequency quasi-periodic oscillations in the space-time of the wormholes. For this purpose, Chapter 1 of the thesis examines a class of geometries constructed by Edward Teo, which describes a stationary and axially symmetric wormhole, which is a generalisation of the static and spherically symmetric Morris-Thorne wormhole. In particular, a class of metrics with integrable geodesic equations is considered in Teo's geometry. Subsequently, a study has been made on the existence and linear stability of circular geodesic lines in the equatorial plane and expressions were derived for the specific energy, angular momentum and angular velocity of the orbits as well as the radial and vertical epicyclic frequencies. In addition, for a more realistic description of the processes in the accretion disc, additional non-linear terms are included in the perturbative equations. A detailed analysis of the quasi-periodic oscillations within the resonance patterns compared to the Kerr black hole is made. It was established that in the geometry of the wormholes, the resonances have a richer structure, which makes it possible to more effectively model the observational data from the X-ray spectroscopy of their accretion disks.

Gravitational-wave astronomy may provide an opportunity to test gravity in the strong-field regime and, in the foreseeable future, lead to constraints on the parameters of modified theories. The scalar-tensor theories of gravity are engaging and admit scalar degrees of freedom. An important example of a gravitational theory modifying GR is Gauss-Bonnet gravity. The scalar field is coupled to the Gauss-Bonnet invariant, and the field equations are second-order, as in the GR. Chapter 2 of the thesis considers a multi-scalar extension of the standard Gauss-Bonnet gravity. In multi-scalar-tensor theories, N dynamic scalar fields are introduced instead of one, coupled in the Gauss-Bonnet invariant and taking values in an abstract Riemannian space. Introducing multiple scalar degrees of freedom leads to a new type of compact object in the non-trivial mapping between space-time and the target space. The dissertation numerically proves the existence of spherically symmetric black holes in some MSAGB theories of gravity for linear and exponential coupling functions and coupling functions leading to spontaneous scalarization. More specifically, MSAGB theories of gravity, whose space of scalar fields is a three-dimensional maximally symmetric space

and in cases of non-trivial mapping between space-time and target space, are considered. For this purpose, after the mathematical formulation of Einstein-Gauss-Bonnet gravity, the reduced field equations were derived, and the condition for the existence of black holes was discussed. The numerical solution of the system of equations is based on the shooting method applying appropriate infinity boundary conditions and regularity conditions on the horizon. The calculations were performed using a linear and exponential coupling function, which allows the existence of black holes with scalar hair, including spontaneously scalarized black holes.

In Chapter 3 of the dissertation, the static and spherically symmetric solutions for neutron stars satisfying the equations of MSAGB gravity are considered. The geometric formulation completely follows the formulation for black holes with the same scalar fields. The dimensionally reduced field equations have been obtained, to which the equation for the hydrostatic equilibrium of a fluid has been added. The realistic equation of state (EoS) allows maximum neutron star masses greater than two solar masses. For the numerical calculations, its polytropic approximation was used. Since numerical solving is complex and time-consuming, the studies are limited to one EoS and one coupling function. The reduced and hydrostatic equilibrium equations are solved numerically by imposing the natural boundary conditions for regularity at the star's centre and an asymptotic plane at infinity. The scalarized solutions' stability has been investigated by studying the star's binding energy.

5. Results and personal contributions of the candidate

The main original results placed in Chapter 1, Chapter 2 and Chapter 3 can be formulated as follows:

5.1) The quasi-periodic oscillations of particles in an accretion disk around rotating traversable wormholes have been investigated using resonance models. The linear stability of the equatorial circular geodesic orbits for a general class of wormholes is also examined, and analytical expressions for the epicyclic frequencies are derived. A comparison of the resulting quasi-periodic oscillations with that inherent for a rotating Kerr black hole and the critical distinguishing features distinguishing wormholes from black holes have been demonstrated. In contrast to the simple arrangement of the orbital and epicyclic frequencies for all moments of rotation of the Kerr black hole, wormholes have various ordinances of the frequencies in different regions of the parametric space. This property of the space-time geometry of the wormholes allows for many different types of resonances to occur, including lower-order parametric and forced resonances, which can lead to stronger visible signals. On the other hand, the results show that for the rotating orbits of a given type, the resonances can be excited in the vicinity of the wormhole's throat for a wide range of values of the spin angular momentum, which allows gravity testing in the regime of strong fields.

5.2) A numerical proof of the existence of black holes in multiscalar Gauss-Bonnet theories with a maximally symmetric scalar space is made for several coupling functions, including the case of spontaneous scalarization. Various characteristics of black holes and the space-time around them have been studied, such as the area of the black hole horizon, the entropy and the radius of the photon sphere. One of the most important properties of the resulting solutions is scalar charge cancellation, which is a sign of suppressed dipole emission, leading to negligible observational constraints compared to most modified gravity theories having a scalar degree of freedom. For one of the coupling functions, branches of scalarized black holes with a non-trivial structure expressed in the nonuniqueness of the scalarized solutions have been found.

5.3) Numerical solutions have been found describing spontaneously scalarized neutron stars in multiscalar Gauss-Bonnet theories with maximally symmetric scalar space. The dependences $M(\rho)$ and $M(R)$ have been constructed, as well as the dependence of the binding energy on the baryon mass, carrying information about the stability of neutron stars.

6. Publications and significance of the scientific results

The content of the thesis is based on three scientific papers, each of which was published in a leading global journal from the *Q1* quartile with a high impact factor: two of the articles have been published in the journal *Physical Review D* (*IF*: 5.131 for 2020 г. and *IF*: 5.217 for 2021 г.) and one paper has been published in *European Physical Journal C* (*IF*: 4.678 for 2021 г.). According to the provided author's reference for the contribution nature of the scientific works, the doctoral candidate has made a significant contribution to 1 publication and has presented one report at a scientific seminar, which meets the recommended criteria of the Faculty of Physics of the SU. Although two of the works of the PhD student are co-authored with her scientific supervisor, she undoubtedly made a significant contribution, especially in calculating quasi-periodic oscillations in the geometry of traversable, rotating wormholes.

The significance of the scientific results in the thesis is concluded in their relevance in the context of active and large-scale gravity research made by large international scientific collaborations in the regime of strong fields, including a contribution to the increasingly strongly developing gravitational-wave astronomy.

7. Critical remarks and recommendations

I have no critical remarks about the PhD thesis. The content is presented in a straightforward style and leaves an impression of a thorough understanding of the subject matter.

8. Personal impressions of the PhD student

The following qualities distinguish the PhD candidate:

8.1) The results achieved in the dissertation are innovative and represent a significant contribution to the problematic and non-trivial numerical study of the gravity-field equations in multi-scalar theories of gravity.

8.2) The dissertation solves current problems essential for developing gravitational physics in conditions of intense international competition.

8.3) The PhD student demonstrates good knowledge of the literature, in-depth insight and understanding of the main problems and tasks to be solved on the given topic.

8.4) The PhD student shows professional skills in handling complex mathematical techniques applicable to the non-trivial problems of modern theoretical astrophysics.

8.5) The PhD thesis results have been published in prestigious international physics journals.

Based on the presented point, the submitted dissertation is highly scientific. Its author is highly qualified to conduct independent scientific research and develop a successful future scientific career. I believe that all the requirements of the law on the development of the academic staff in the Republic of Bulgaria and the regulations for its implementation have been met and **recommend to the honourable scientific jury to award Master Radostina Zhekova Zheleva the educational and scientific degree „Doctor“** in professional direction 4.1 „Physical Sciences“ under the doctoral program „Theoretical and Mathematical Physics“.

28.03.2023 г. Report prepared by:..……

(Assoc. Prof. PhD Galin Gyulchev)