Long report

according to the procedure for the defense of a dissertation on a topic:

"Branching processes - optimization and applications"

to acquire educational and scientific degree "Doctor"

from

candidate: Kaloyan Nikolaev Vitanov,

Area of higher education: 4. *Natural sciences, mathematics and computer science*
Professional field: 4.5. Mathematics
Doctoral programme: „Probability theory and mathematical statistics“, department: „Probability, operations research and statistics“,
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n my capacity as a member of the scientific jury, according to Order No. RD-38-308/1.07.2022 of the Rector of Sofia University.

1. General characteristics of the dissertation work and the presented materials

The dissertation is written in English and contains 189 printed pages and consists of an introduction, three chapters, an appendix with proofs and a description of the main scientific contributions. The bibliography is correctly formatted and contains 205 titles, including three of the candidate himself. All other materials are prepared and presented according to the requirements.

2. Data and personal opinion about the candidate

The Candidate Kaloyan Nikolaev Vitanov graduated in 2015 the bachelor's program in "Applied Mathematics"of FMI-SU. In the same year, he enrolled in the Master's program "Probability, actuarial science and statistics defending his thesis in 2017. In 2018 he successfully passed the entrance doctoral exam and he was enrolled as a full-time doctoral student at the PORS Department of FMI. Since 2016 until now he also works at the company FactSet. During his doctoral studies, he published 4 articles, all co-authored with the supervisor Prof. M. Bozhkova. Three of these articles have an impact factor. He also participated in seven scientific forums. I know the candidate's work from two of his talks that I have attended to. I have also been on his entrance doctoral examination committee. My general impression of

Kaloyan Vitanov is good and I would highlight his independence, something that is not found too often at the doctoral level.

3. Content analysis of the candidate's achievements in theoretical and applied science, contained in the dissertation and the publications, included in the procedure

The results in this dissertation concern problems related to classical branching processes. Their nature varies from new, original contributions to new proofs of well-known claims. In general, there is some repetition of the techniques and proofs, but with the exception of Theorem 2.4, they are conducted correctly. A comprehensive review of the literature is done and "scientific honesty"is noted in the discussion of one's own contribution, i.e. the achievments of others are correctly described and some weaknesses in the thesis are clearly discussed. I will take a closer look at the results chapter by chapter.

3.1. **Chapter 1.** The introductory chapter provides a comprehensive review of the literature and the history of branching processes and briefly discusses the achievements of the founders of this classical field of stochastic processes theory, including N. Dmitriev. There is also a detailed exposition of the main titles in the literature, with attention paid to the nature of the results of each of them. In addition, the candidate has clearly described the development of the tasks he is dealing with in this dissertation work.

In the introductory chapter, the main terms that will be used in the dissertation are introduced and the main quantities that describe Sevastianov's multi-type processes are presented. Optimization problems with sequential decision making are also examined, mostly from the perspective of basic objectives, concepts, and domains in which they occur.

3.2. **Chapter 2.** In Chapter 2, multitype branching processes via mutation probabilities (MRPVMs) are considered. Let $\mathbb{W} = \{1, 2, \cdots, n\}$ denote the possible types of particles, each type having a lifetime of the random variable $\tau^i, i \in \mathbb{W}$, with distribution function G^i. At the end of its lifetime, the particle reproduces descendent particles that can be of any possible type, and the distribution of the offspring can be random over $\mathbb{N}^n$. In addition, the distribution of the successors may depend on the age of the particle. The particles evolve independently of each other. In general, we will call for brevity these processes Sevastianov processes (PS). Then MRPVMs are a special case of PS, the particular features of which are determined by the following generation scheme: depending on the age of reproduction, the total number of particles in the offspring has a specific distribution for each type; conditional on a fixed total number of successors, say $N \geq 0$, the distribution among the N-successors is multinomial with given fixed probabilities for success. The MRPVM model naturally allows offspring that do not retain the type of the parent to be considered "mutants". This subclass of processes is then studied by the well-established methods in theory of branching processes, but to the best of my knowledge in the field many of the investogated quantities do not seem to fall within the standard theory. Thus, for example, the number of generated particles of given subtypes, the time to the first successful process of the successors of particles of a given subtype, and others are considered.

Given that the MPVM$\subsetneq$ PS, some of the theoretical results are not original and can be derived from those for PS. The novelty lies in the interpretation with mutation probabilities, which gives a bit more structure to the process and, accordingly, to the associated equations and variables, and in the introduction of the aforementioned quantities, which can actually conveniently be given different meanings depending on the applications, e.g. cancer cells, their extinction, mutation and so on. I will continue the discussion by sub-chapters.

3.2.1. *Subchapter 2.1.* In this part, the general results for the MRPVM are given. If $Z_j(t)$ is the number of particles of type $j \in \mathbb{W}$ at time t, the main integral-differential equations are obtained, see Theorem 2.1, for

$$F_i(t;\mathbf{s}) = \mathbb{E}\left[\prod_{j\in\mathbb{W}} s_j^{Z_j(t)} | \mathbf{Z}(0) = \delta_i\right],\ i \in \mathbb{W},$$

where, following the notation in the thesis, bold text denotes vectors and δ_i denotes a particle of type $i \in \mathbb{W}$.The case when the initial particle has an initial age $a > 0$ is also studied, see Corollary 2.1. Using these equations, the candidate derived equations for the extinction probabilities up to time $t \in (0, \infty]$ of a MPVM started with a single particle of type $i \in \mathbb{W}$ and an arbitrary initial age $a \geq 0$, see Theorems 2.2, 2.3; Corollaries 2.2, 2.3. For $t = \infty$, Theorem 2.4 states that the extinction probabilities do not depend on a. I think there is an error in the proof and will provide a potential counterexample below in the review. Except for the last one, the proofs appear to be correct. Given the MRPVM$\subsetneq$ PS, one can hardly speak of any significant originality or challenge, but given the additional structure of the MRPVM the equations have some advantages.

The novel research starts with the examination of the contribution of particles of subtype $\mathbb{W}_e \subseteq \mathbb{W}$ to the total number of particles of each type $j \in \mathbb{W}$ produced up to time $t\ in\,(0, \infty]$. If

$$h_i^{\mathbb{W}_e}(t;\mathbf{s}) = \mathbb{E}\left[\prod_{j\in\mathbb{W}} s_j^{I_j^{\mathbb{W}_e}(t)} | \mathbf{Z}(0) = \delta_i\right],\ i \in \mathbb{W},$$

is the generating function of the number of particles of type $j \in \mathbb{W}$ descendants of a particle of subtype $\mathbb{W}_e$, then Theorem 2.5 furnishes an integral equation for these functions, with the form of the equation depending on whether $i \in \mathbb{W}_e$ or not. Corollary 2.6 considers the case when the first particle that initiates the process is at age $a > 0$. Theorem 2.6 and subsequent corollaries specialize the above results to the case $t = \infty$ and to $\mathbb{W}_e$ of a certain kind. I will emphasize that the originality consists in considering relevant quantities and their accompanying equations, which makes them directly applicable. Otherwise, the proofs follow a well-known method, also applied in the results discussed above.

The research continues by considering the random time $T_{\mathbb{W}}^{\mathbb{W}_e}$, which describes the moment of production of the first particle, a successor of $\mathbb{W}_e$, which will give rise to a non-vanishing process, provided that the initial particle configuration consists only of particles of subtype $\mathbb{W}_e$. Results concerning the tail of $T_{\mathbb{W}}^{\mathbb{W}_e}$, as well as for the conditional expectation of $T_{\mathbb{W}}^{\mathbb{W}_e}$, are obtained in Theorem 2.7. Theorem 2.8 extends Theorem 2.7 by allowing the particles in the initial configuration to have non-zero starting age. The results are new. The proofs are correct. They fit into the research line of studying MRPVM in order to apply them.

Also the candidate offers numerical schemes for the approximation of the equations derived in the previous sections, see section 2.1.7, and he considers particular cases of MRPVM, see section 2.1.8. For the latter it is discussed whether they are sub-, super- cirtical.

3.2.2. *Subchapter 2.2.* In this subchapter the decomposable MRPVM (DMRPVM) are considered. They have some additional structure. If $\mathbb{W}_e \subsetneq \mathbb{W}$, then the particles of type $\mathbb{W}_0 = \mathbb{W} \setminus \mathbb{W}_e$ can only give offspring of type $\mathbb{W}_0$. The results regarding the probability generating functions and the probabilities for extinction from subchapter 2.1 are specialized to this case, see Corollaries 2.12-2.17. Corollary 2.18 uses Теорема 2.4 and is therefore most likely wrong.

For DMRPVM the candidate studies the probability generating functions

$$h_i^{\mathbb{W}_e}(t;\mathbf{s}) = \mathbb{E}\left[\prod_{j\in\mathbb{W}} s_j^{I_j^{\mathbb{W}_e}(t)} | \mathbf{Z}(0) = \delta_i\right],\ i \in \mathbb{W},$$

under the conditions, that only the mutants, i.e. $\mathbb{W}_e \to \mathbb{W}_0$, are studied. The results are Corollaries 2.19- 2.22, which contain integral equations for $h_i^{\mathbb{W}_e}(t;\mathbf{s})$. The proofs, with minor modification due to the counting of the mutants only, follow the steps of those of subchapter 2.1. The candidate has obtained results on the probabilities of extinction until moment $t \in (0, \infty]$. In all results the first particle can be age dependent. In the spirit of subchapter 2.1 the time to the production of the first mutant, which starts a successful process, is considered. The statements are a direct corrolary to the previous ones. Corollary 2.23 relies on Theorem 2.4 and is probably wrong.

The candidate has also studied the probability that the first successful mutant has appeared in the interval $(t, t + dt)$ conditionally on the moment being larger than t and at time t there are survivors from $\mathbb{W}_e$. Given the numerical results I cannot confirm substantial improvements in this direction.

The last part of this subchapter is dedicated to processes, whose reproduction is age independent, i.e. Bellman-Harris processes. They are special case of DMRPVM and the presented results are substitution in an easier context.

3.3. **Chapter 3.** In Chapter 3, optimization problems with sequential decision making with dynamics based on branching processes are considered. The idea is not new, but it seems to be rather poorly studied in the literature. Conceptually, things go like this: we have a branching process whose parameters we can control; we have discrete times $t = 0, 1, \cdots, T$ in which we receive a reward depending on the current configuration of the branching process and we can change the parameters (control) that determine the evolution of the branching process; we are looking for such a management where the expected reward collected up to time T is maximal.

The first results are to embed the above task in the so-called „Universal Modeling Framework", which was introduced for tasks related to sequential decision making. This allows the Bellman equation to be used to find an optimal solution. Unfortunately, this equation almost always does not allow a direct solution to the problem, and an additional structure is sought to simplify it or allow its approximate solution.

After the successful embedding of the problem in the general formulation, Theorem 3.2 is obtained, which gives an equation for finding the optimal policy, i.e. at any given configuration of the branching process (BGW type only) what change to its parameters should be made to obtain the maximum reward. This is related to the introduction of the so-called maximum return operator. Restricting the branching process to BGW type is necessary to preserve the Markov structure of the process in the decision times t. The same has been achieved for MRPVM processes with exponential lifetime of the particles, see Theorem 3.3. For more general MRPVM processes, in order to preserve the Markov structure in the decision times t it is necessary to expand the state space, which complicates the task significantly, and the candidate has only managed to discuss some possible algorithms for finding the optimal policy.

Chapter 3 concludes with a specific example related to the acquisition of educational degrees. I will not dwell on it because as far as I understand it is just a test case for the models and results obtained in this chapter.

The new result in Chapter 3 is the reduction of the sequential decision optimization problem with branching process-based dynamics in the context of a well-known theory. Theorem 3.2 is not new, but the proof is a consequence of the general theory. Theorem 3.3 is new, but essentially there is no difference in the derivation. Both theorems concern cases where the Markov structure is preserved in decision times t.

4. Апробация на резултатите

The candidate has given 4 publications on which the dissertation is based. It should be noted that some results are beyond these articles and will likely be submitted for publication. Three of these four articles have an impact factor and they are distributed as follows: 2 in Stochastic Models (Q4) and 1 in Comptes rendus de l'Academie bulgare des Sciences (Q4). One article is for a conference issue. I will not comment on impact factor rankings and numerical values, as this is often a misleading and pointless exercise. Stochastic Models journal has a good reputation in expert circles. All articles are co-authored with the supervisor, and a protocol for equal contribution to them has been provided by the candidate. These articles have one independent Scopus citation. The applicant also has a number of other publications and citations outside the dissertation topic.

I confirm that a) the scientific works **meet** the minimum national requirements (according to Art. 2b, paras. 2 and 3 of LDAMRB) and accordingly to the additional requirements of SU "St. Kliment Ohridski"for the acquisition of an educational and scientific degree "doctor"in the scientific field and professional direction of the procedure; б) the results presented by the candidate in the dissertation work and related scientific works **do not repeat** those from previous procedures for acquiring a scientific title and academic position; в) theres **no** proven plagiarism in the submitted dissertation and scientific works under this procedure.

5. Quality of the summary

The summary meets all the requirements for its preparation and correctly presents the results and content of the dissertation work.

6. Critical notes

The discussion on page 34 regarding the relation of mutation probability models to multitype Sevastianov processes is unclear about the results in paper [8]. Does the candidate think that there is an inaccuracy in [8] about the necessary assumptions or that we do not need to have indecomposability? This needs to be clarified.

I expect Theorem 2.1 and Corollary 2.1 to be a direct consequence of [8] and it is good to comment on this.

The discussions about the double limits on pp. 40, 51 and the accompanying definitions and remarks are puzzling to me. After all, everywhere in the proofs and statements we have a result of the type

$$\lim_{t\to\infty} f(t) = \cdots + \int \lim_{t\to\infty} f(t-y)G(dy).$$

Everywhere the limit is at a fixed y, but even if we had an expression of the following form $\lim_{y\to\infty}\lim_{t\to\infty} f(t,y)$ (a general case of the one in the thesis), then this is always understood as $\lim_{y\to\infty}\lim_{t\to\infty} f(t,y) = \lim_{y\to\infty} f(\infty,y)$. If $y = t-b$ then the limit is only along t, and if we take both at the same time $\lim_{y,t\to\infty} f(t,y)$ is written.

Regarding Theorem 2.4, let us take a one-type process that has 3 successors if its age is less than 1 and otherwise, 0 with probability q and 1 with probability p . Let the probability of a particle with starting age to give a generation by time 1 be h. Then $q_2 \geq q$ and $1-q_0 \geq h(1-q_0^3)$, which gives $1 \geq h(1+q_0+q_0^2)$. If we assume that $q_0 = q_2$, then

$$1 \geq h(1+q+q^2),$$

which is not possible for all q, h, which we have the freedom to choose. Corollary 2.18, Corollary 2.23 follow this theorem.

The numerical schemes in 2.1.7 have not been investigated for their error. I admit that it is difficult, but somehow the arguments for this are not very convincing. It would be good to explain more clearly why this is untenable at this stage.

Parts 2.2.1.6 and 2.2.2 contain practically no new results.

It should be clarified in the thesis why the infinite horizon of the SDP in Chapter 3 is excluded from the considerations. It seems natural that the problem should be considered in a community, especially in view of discounting, and it is good for the reader to have an idea what the difficulties and/or novelties are in this case.

The maximum contribution operator $\mathcal{R}$ must be introduced mathematically correctly because it involves the operation of the maximum of a vector quantity. The results based on it seem correct, but the definition needs fixing. The important thing in this case is that we can tensor on the solutions.

7. Conclusion

Having familiarized myself with the dissertation work presented in the procedure and the accompanying papers and based on the analysis of their significance and the theoretical and applied contributions contained in them, **I confirm** that the presented dissertation work and the scientific publications to it, as well as the quality and originality of the results and achievements presented in them, **meet** the requirements of LDAMRB, the Rules for its application and the relevant Rules of the SU "St. Kliment Ohridski"for the candidate's acquisition of the educational and scientific degree "doctor"in the scientific field 4. Natural sciences, mathematics and informatics and professional direction 4.5 Mathematics. In particular, the candidate satisfies the minimum national requirements in the professional direction and no plagiarism has been found in the scientific works submitted for the competition. Based on the above, **I recommend** the scientific jury to award Kaloyan Nikolaev Vitanov an educational and scientific degree "Doctor"in scientific field 4. Natural sciences, mathematics and informatics, professional direction 4.5 Mathematics (Probabilities and statistics).

30.08 2022г. Author of the long report: Prof. Dr. Sc. Mladen Svetoslavov Savov