REFEREE REPORT

On the dissertation for awarding the scientific degree “Doctor of Science (DSc) in Physics”
Author of the dissertation: Dr. Peter Aleksandrov Ivanov, Associate Professor at the Faculty of Physics, Sofia State University”St. Kliment Ohridski”
Title of the dissertation: *Critical Phenomena and Quantum Metrology with Strongly Correlated Quantum- Optical Systems*
Code: 4.1 “Physical Sciences”

1. Description of the submitted documents

Peter Ivanov received his bachelor degree at the Faculty of Physics of Sofia University “St. Kliment Ohridski” in 2002. During the period 2004-2008 he was a Ph. D. student there and got his Ph. D. degree. His Ph. D. thesis is based on 9 papers published in *Physical Review A* and *Optics Communications*.
During the period 2012-2015 he was Assistant Professor at the Faculty of Physics and in June, 2015 he was elected Associate Professor there. He was a lecturer and tutor in the bachelor and Master programs in several courses in Physics: quantum mechanics, theoretical mechanics, quantum phase transitions, quantum simulations and quantum metrology. He was the supervisor of one B.A. and one M.S. theses.
Dr. Ivanov has submitted the following documents: Dissertation in English (302 pages); Extended Summary of the dissertation in Bulgarian (130 pages); electronic texts of the 23 articles included in the dissertation; Extended Summary of the dissertation in English (110 pages); copies of the B.A., M.S. and Ph. D diplomas; citation list; reference on the fulfillment of the requirements by the law for professional development of scientists; declaration for authorship; CV.

The Extended Summary of the dissertation reflects correctly the content of the dissertation

2. Importance of the Topics in the Dissertation

Classical phase transitions are driven by temperature fluctuations. Quantum phase transitions differ substantially from the classical ones. They are transitions between different phases of matter *at zero absolute temperature*. Various quantum phases differ by the domains of their physical parameters entering the quantum Hamiltonian (characteristic frequencies and masses of the particles as well as the coupling constants) and the parameters of the ground state of the system at zero temperature (e. g., the characteristics of the topology of the ground state). These parameters could only be changed by quantum fluctuations and the system could be driven to another phase with different values of the parameters, thus undergoing quantum phase transition. Example of such change is the transition of polaritons from the phase of Mott insulator to superfluid phase.
In the projective Hilbert space of the wave functions, or more generally, in the space of density matrices of the quantum system, one could introduce a Riemannian metric, connected with the quantum Fisher information. Both variables are important for the precision of the estimation of one or more physical parameters.
The experimental study of quantum phase transitions needs controls of the system at very low temperatures. This can be achieved in optical systems by the use of laser cooling and control on the parameter by laser fields.

The quantum Hamiltonian may possess different symmetries and properties in the various phases. Their investigation could lead to deeper understanding of the properties of matter and development of important new technologies and other applications.
The topics and scientific contributions in the dissertation belong to this interesting modern hot and competitive field.

3. Brief analysis of the main scientific contributions

The dissertation contains 302 pages and consists of an introductory chapter, 12 chapters in which the scientific results are presented, 25 appendices with specific theoretical calculations used in the main text - a total of 32 pages, 315 bibliographic titles and a list of 23 applicant's works, on which the dissertation is based.
All of these 23 works were published after the defense of the applicant’s Ph. D. thesis in some of the most renowned international scientific journals within the field of condensed matter theoretical physics with high impact factors. In 7 of these publications the applicant is a single author, in the rest of them (with one exception) Dr. Ivanov had a decisive contribution for the success of the research both conceptually and technically in the calculations.
In the introductory **chapter 1**, consisting of 15 pages, the applicant outlines the main concepts and theoretical results in the physics of trapped ions in a Paul trap. The theoretical and experimental platform of the Paul trap (for which Wolfgang Paul received the Nobel Prize in Physics in 1989) is the basis of most of the author's dissertation research. The equation of motion of a charged particle under the action of two harmonic potentials was considered: one with time-independent amplitudes, the other with rapidly oscillating amplitudes in the radio frequency range. The resultant potential has a stable minimum.
The following cases were considered: determination of the equilibrium positions of N ions in a Paul trap, interacting pairwise with a Coulomb potential. In a harmonic approximation (after developing the potential in a Taylor series around the equilibrium positions and preserving the terms up to second order), 3N modes of collective oscillations (vibration modes) are obtained. Their type depends on the location of the equilibrium positions - whether the ion crystals in equilibrium is one-, two- or three-dimensional. All 3 cases are considered separately. The case of an ion with 2 internal metastable states and any number of external (vibrational) degrees of freedom in a Paul trap interacting with a laser field propagating along the axis of the trap is also considered. The transitions between the two internal energy levels can occur with or without changing the number of vibrational degrees of freedom, depending on the frequency of the laser field.
The presentation of the material in this chapter demonstrates that the applicant has a thorough knowledge of the problems in the physics of trapped ions, as well as in the methods for solving them.

Chapter 2, consisting of 30 pages, examines the Jaynes-Cummings-Hubbard quantum model, describing a system of: atoms, each with two energy levels; quantized boson (phonon) modes; (tunneling) of phonons along the lattice nodes. A physical realization is proposed with a lattice of N ions with two internal spin degrees of freedom. The role of the bosonic degrees of freedom is implemented by the oscillations of the ion around its equilibrium position in a Paul trap.
It is assumed that the transverse ion trapping frequencies are much larger than the frequency parallel to the trap axis. Under additional proper approximations in the quantum-mechanical

interaction picture the Hamiltonian consists of an interaction term between them of the simplest possible form and a term describing the tunneling. All terms together represent the Hamiltonian of the Janes-Cummings-Hubbard model.

The Janes-Cummings-Hubbard model is not exactly solvable (unlike the Janes-Cummings model), so finding theoretical methods to study the quantum phase structure is of particular interest. In the author's publications [2] , [3] and [4] from the list of publications, the following results were obtained:

• In the proposed implementation of the model, the observables responsible for a polariton quantum phase transition between a Mott insulator phase and a superfluid phase are investigated in detail (polaritons are states composed of spin and boson excitations). It is important to note that upon using this theoretical result of the author K. Toyoda et. al. published the first experimental work demonstrating the quantum phase transition [Phys. Rev. Lett. 111, 160501(2013) by K. Toyoda et. al.].

• An analytical study of the phase diagram of the Jaynes-Cummings-Hubbard model was proposed and carried out using effective models - the anisotropic XYZ model and the Heisenberg model in an external magnetic field. These approximating models make it possible to determine the relationship between the tunneling parameter and the spin boson interaction parameter at the critical point. Analytical results are compared with numerical simulations in the Janes-Cummings-Hubbard model, which show very good agreement.

Chapter 3, consisting of 26 pages, is based on the results in publications [5] , [6] and [7]. In this chapter the collective and cooperative Jahn-Teller-Dicke models, realized with ions in a Paul trap, are considered. The Jahn-Teller-Dicke model describes an U(1)-symmetric interaction between spin-1/2 fermions and bosons. The Jahn-Teller-Dicke collective model describes the interaction of an ensemble of N spins interacting with a single vibrational mode. It is shown that in the thermodynamic limit, when the collective spin approaches infinity, the model is exactly solvable and describes a *quantum phase transition between normal and super-radiant phases*. When the number of nodes is finite, a numerical diagonalization of the Hamiltonian was performed and compared with the corresponding analytical results. The cooperative Jahn-Teller-Dicke model, in which an ensemble of N spins interacts with an ensemble of N bosons, is also considered. The main contributions of the author in this chapter are:

• This model is shown to describe a *magnetic structural quantum phase transition* in which the equilibrium positions of the ground state particles rearrange from linear into a zig-zag configuration and the spins form an antiferromagnetic arrangement. The ground state corresponds to a spin-boson quasi-condensate. This quantum phase transition occurs as a result of the quantum fluctuations in which the continuous U(1) symmetry is spontaneously broken.

• Using the mean field approximation the quantum fluctuations are found and the spectrum is shown to consist of three collective modes.

In **chapter 4**, consisting of 11 pages, Dr. Ivanov studies the quantum magnetic properties of a mixed ion crystal consisting of different spins in an external oscillating magnetic field with a non-zero gradient (the simplest case of such a field is considered, having a non-zero component along the axis of the ionic crystal: B_z = const z cos(ωt). The magnetic field induces an interaction between the different spins and the collective vibrational modes. The scientific contribution of Dr. Ivanov in this chapter is as follows:

• It is shown that when the magnetic field frequency ω does not coincide with the frequency of any vibrational mode, the phonon effect can be averaged out and the system can be described by an effective Hamiltonian of interacting spin degrees of freedom. If an additional oscillating magnetic field of the same frequency ω directed in the direction transverse to the z axis is applied, the model can effectively be described by an Ising model. The phase diagram for a system with spin 3 is obtained.
This chapter is based on the results obtained in the publication [8] of the list. These studies model an ion crystal with impurities.

Chapter 5, consisting of 7 pages, deals with emulations of the electric dipole moment of neutral relativistic particles. The results are published in article [9] of the list of publications.

Chapter 6, containing 15 pages, is based on the results in publication [10] from the list. Here the main contribution of Dr. Ivanov is:
• A method (stimulated Raman adiabatic transition) is proposed to transfer orbital angular momentum from a classical laser field to photons in an optical resonator with a non-planar geometry. Such geometry creates an effective magnetic field for the photons in the resonator with quantized Landau levels. The transfer of orbital momentum takes place through an atomic medium. A scheme for creating topological Laughlin states between Rydberg polaritons (states formed by Rydberg atoms and photons) is proposed. The technique of stimulated adiabatic transition of orbital momentum from the laser field to the photons inhabiting the lowest Landau level is implemented.

In **chapter 7**, covering 21 pages, a protocol is derived to eliminate the negative effect of the quadrupole interaction causing a decrease in the dipole moment of a trapped Rydberg ion. This effect can be removed by increasing the laser field intensity by a well-defined factor depending on the Rabi frequency. The content of this chapter is based on publication [11] of the list of publications.

In **chapter 8**, consisting of 7 pages, is a presentation of the basic concepts in quantum information geometry, quantum Fisher metric for one and multiple parameters in the case of pure states and in the more general case of mixed states when the system is described by a density matrix. The main quantity of quantum metrology is considered - the quantum Fisher information, providing an estimate of the precision (minimum variance) of the measurement of a given physical parameter.

Chapter 9, covers 48 pages. Its content is devoted to the results in publications [12], [13] and [14] of the list. In it, Dr. Ivanov examines quantum systems that can be described by another fundamental model of quantum optics – the Dicke model. In this quantum mechanical model of interacting electromagnetic field (light) with matter, the light component is represented by one bosonic mode, the matter - by N identical ions, each with two energy levels (i. e., a lattice, at each point of which there is a ½ spin) and an interaction term between the two types of degrees of freedom. The Dicke model is a generalization of the Rabi model where in the latter N=1. Both Dicke and Rabi models have discrete Z2 symmetry, to which corresponds a conserved quantity – the parity P of the total number of excitations Nex , $P= (-1)^{Nex}$. When at $N\to\infty$ the Z2 symmetry is spontaneously broken, the system undergoes a *quantum phase transition to a super-radiant* *phase*. The author considers a realization of the quantum Dicke model with ions in a Pauli trap forming a linear crystal. Two Raman laser fields are applied to the system in the transverse direction of the crystal, driving the interaction in the Dicke model. Besides them, two more laser fields are applied, creating a small perturbation (a perturbation with a small parameter) of the spins in the lattice. The goal is to determine theoretically and experimentally the most accurate small perturbing parameter by applying the laws of quantum metrology. Indeed, around the point of a quantum phase transition the quantum Fisher information increases. It is inversely proportional to the variance of the measured quantity, therefore the accuracy of the measurement is maximal around the point of quantum phase transition. The contributions of Dr. Ivanov in this chapter are:

• Quantum metrology is explored for strongly correlated quantum systems demonstrating a *quantum phase transition* on the example of the quantum Dicke model, in which there is a *quantum phase transition of second order between normal and superradiant phase*. A method based on an adiabatic transition between the two quantum phases driven by small perturbation which breaks a symmetry of the system, in this case – the discrete Z2 symmetry, is proposed. A metrological protocol is proposed that can be used to measure transition frequencies with Heisenberg precision.

Chapter 10, containing 32 pages, reflects the author's results from publications [15], [16], [17], [18], [19] in the list. Its main contributions include the following:

• Trapped in a Paul trap single-ion or system-of-ions quantum sensors are proposed to measure very tiny forces. In the proposed configurations, the information about the force is transferred to the spin degrees of freedom. The measurement of the force is performed by examining the time oscillations of the spin states of the trapped ion or ensemble of ions. The modeling Hamiltonian for quantum sensors is the integrable Jaynes-Cummings Hamiltonian. With such a sensor, oscillating forces of order 10^{-24} N can be measured.

• Methods are proposed for reducing the spin-dephasing effect by applying decoupling fields, which is an important advantage of the sensors.

Chapter 11, containing 22 pages, is based on the author's results in publication [20] of the list. The subject of discussion is multi-parameter quantum metrology whose task is to determine the maximum precision in the simultaneous estimation of several parameters in an open quantum system. A system is open if it is immersed in a medium and interacts with it by losing/gaining excitations to/from the medium. In this case, the system can undergo a *dissipative quantum phase transition.* Dissipative phase transitions are characterized by non-analyticity of the ground state parameters at the quantum phase transition point. The author chooses the open system in the vicinity of a dissipative quantum phase transition point as a subject for high-precision multi-parameter quantum metrology. In this chapter, bosonic excitation loss processes in the quantum Rabi model are considered. As main achievements of the author here, the following can be stated:

• Conditions are found for optimal measurement of two parameters - the magnitude and the phase of the shift operator in the phase space, using the laws of quantum metrology. It is shown that in the vicinity of the dissipative quantum phase transition, a significant improvement in the sensitivity of one or both of the mentioned parameters is achieved.

In **Chapter 12**, containing 27 pages, Dr. Ivanov proposes an adiabatic method for obtaining an optimal estimate of the vibrational temperature of ions in a Paul trap that can operate beyond the Lamb-Dicke regime. The method uses an external laser field that induces an interaction between the oscillations of the crystal and the collective internal states of the ions. The theoretical description is based on the non-linear Janes-Cummings model. At low temperatures in the adiabatic method, the information about the thermal distributions of the phonons is transferred to the spin-collective degrees of freedom, and in this way equality is achieved between the classical and quantum Fisher information (the thermal distributions are controlled by the classical Fisher information, and the spin-collective distributions – by the quantum one). This is the situation when the Kramer-Rao inequality is saturated, giving an estimate of the precision of the temperature measurement, and therefore the proposed quantum thermometer is optimal. This chapter reflects the author's results in publications [21] and [22] of the list. The main contribution of the author in this chapter is:

• A new method for optimal measurement of the temperature of an ion crystal (optimal quantum thermometer) is proposed.

Chapter 13, containing 9 pages, is devoted to a certain characteristics of the quantum chaos in the quantum Rabi model. (This model, named after the 1944 Nobel laureate in physics Isidore Rabi, is a basic ingredient in the theory of quantum technology and quantum computers.) It is well known as of 2015 that in the thermodynamic limit $\eta\to\infty$, at $g = g_c$ the model undergoes a *quantum phase transition from a state of chaos in the superradiant phase to equilibrium* (here $\eta = \omega_0/\omega$, ω_0 is the frequency of the internal (spin) degree of freedom, ω is the frequency of phonons, g is the coupling constant between them). The author studies the behavior of the “Out-of-time-order-correlator” (OTOC correlation function), which measures the distribution of quantum information among the degrees of freedom of the system and it is an indicator of the presence of quantum chaos. The main contributions of the author in this chapter are:

• It is shown that in the superradiant phase, the OTOC increases exponentially, which makes it possible to determine the quantum critical exponent - the quantum Lyapunov exponent.

• It is shown that the dimensionality of the time-averaged density matrix is larger than the dimensionality of the spin system and therefore the bosonic mode can be considered as an effective medium interacting with the spin. This leads to a reduction in the time oscillations for the spin states and induces a transition to equilibrium.

4. Publications and significance of results

The dissertation, as already noted, is based on 23 scientific publications. The nature of these works and their number fully satisfy the requirements of the Faculty of Physics, Sofia University “St. Kliment Ohridski” for obtaining the scientific degree “Doctor of Science (DSc) in Physics.

The main contributions of the dissertation can be formally categorized as:

(a) uncovering and establishing new scientific facts, enriching existing knowledge and in-depth understanding of processes and phenomena in the scientific field in question;

(b) presenting a theoretical basis for setting up experiments necessary both to confirm the theory and proposing insights for developing important new technological applications.

The above evaluation undoubtedly proves the scientific significance of the principal author’s results.

5. Impact of the scientific publications in the international scientific literature

The scientific works on which the dissertation is based are characterized as follows:

• 7 independent, 16 – in co-authorship. In each of them (except for the first [1]), Dr. Petar Ivanov has a significant contribution.

• 17 articles are published in journals with Q1, 6 – in journals with Q2.

• All papers have been published in the leading international condensed matter physics journals with high impact factor, among them 1 article in Phys. Rev. Lett. (ref.[6] of the list of publications) , 10 – in Phys. Rev. A, 2 in Scientific Reports, 2 in Optics Commun, and others.

The total number of published works of Dr. Petar Ivanov is 42. They have a wide Response among experts in the field, as is evidenced by the noticed over 450 independent citations by foreign scientists in the leading international scientific journals. The Hirsch index of Dr. Petar Ivanov is h=12.

The author has provided a separate list of 220 independent citations to the articles included in the Dissertation itself.

6. Critical Remarks

It is common for such an extensive work some minor technical errors to be present in the Bulgarian text of Extended Summary of the dissertation. However, this in no way diminishes the excellent impression from both the content and the structuring of the text.

7. Conclusion

After having been acquainted with all the materials for the competition and based on the present analysis of their significance and the scientific contributions of their author, I **confirm** that the submitted dissertation is of high scientific level and fully satisfies the requirements of the ŽRASRB (the Law for Academic Development of Academic Resources in the Republic of Bulgaria), the Regulations for its application and the relevant Additional Regulations of Sofia University "St. Kliment Ohridski" in the scientific field and professional field of the competition for obtaining the scientific degree "Doctor of Science (DSc) in Physics". I also **confirm** that Dr. Petar Ivanov satisfies and even exceeds the minimal national requirements for this scientific degree. For me, there is no doubt that he is an internationally recognized scientist of high expertise in his field, who has an indisputable contribution to the development of condensed matter physics and wonderful prospects for further successful development of his academic career.

Therefore, I give my categorical **positive assessment** to his application.

General Conclusion

On the basis of the arguments outlined in the above evaluation, **without hesitation I recommend the scientific jury to propose to the competent Selection Authority of the Faculty of Physics of Sofia University "St. Kliment Ohridski" to award Dr. Petar Alexandrov Ivanov the scientific degree "Doctor of Science (DSc) in Physics". code 4.1 Physical Sciences.**

Sofia, Aug 26, 2022 Referee:

(Prof., Dr. Sc. Svetlana Pacheva)