REVIEW

on a competition for an occupation of the academic rank "Associate Professor" in the research area 4.5 Mathematics (Mathematical analysis), for the needs of Sofia University "St. Kliment Ohridski" (SU), Faculty of Mathematics and Informatics (FMI), published in DV, No. 30 from 15.04.2022.

The review is prepared by Assoc. Prof. DSc. Ognyan Borisov Christov, FMI, SU, as a member of the scientific juri of the competition, according to order No RD - 38-286 from 14.06.2022 of the Rector of the Sofia University.

Only one candidate - Principal Assistant Professor Nikolay Antonov Ivanov, PhD, FMI, SU has applied for participation in the competition.

1. A general description of the presented materials.

D-r N. Ivanov has submitted 7 papers for the participation in the competition, as well as all necessary other documents such as: CV, certificate for higher education, certificate for a scientific degree, the list of all publications, author's reference, citation reference, reference for meeting the minimum national requirements under Article 2b, paragraphs 2 and 3 of ZRASRB etc.

The scientific publications presented for the competition do not repeat the works presented for the educational and scientific degree "Doctor" . The author's reference accurately reflects the applicant's achievements. No plagiarism is found in the works submitted to the competition.

2. Applicant details.

M-r Nikolay Ivanov has a degree of mathematics from Sofia University. In the period 2001 - 2007 he was a PhD student at Texas A & M University, USA.

He was a visiting fellow in Westfälische Wilhelms-Universität Münster during September 2004 - July 2005.

In 2007 he successfully defended his PhD thesis under guidance of Ken Dykema.

He was granted post-doc positions

– in Fields Institute, between July 2007 - December 2007;

– in Queen's University, Canada between January 2008 - June 2009.

He was an assistant-professor at the University of Veliko Tarnovo from 2009 till 2016.

From 2016 till now he is assistant-professor in the Section of Mathematical analysis in FMI, SU.

3. A general description of the applicant's scientific works and achievements.

The scientific interests of N. Ivanov are in the field of operator algebras. This mathematical field is a combination of functional analysis and algebra at a minimum and originated in research related to quantum mechanics. Undoubtedly, such studies are relevant and promising.

The applicant has submitted for the competition 7 articles, not used in other procedures. He is the sole author in five papers and the rest works are co-authored. I regard as true, that the contributions of the co-authors are equivalent.

Some of the applicant's articles are published in extremely prestigious journals: Journal of Functional Analysis (2009, **IF 1.247**) and (2017, **IF 1.326**), Transactions of the American Mathematical Society (2010, **IF 1.100**), Annales de L'institut Fourier (2020, **IF 0.968**). All these works are on a very high technical level. The total impact factor of the presented publications is **4.641**.

In my opinion, the results of presented papers are more than enough in quality for obtaining the academic degree "Associate Professor".

4. Description and evaluation of the applicant's teaching activity

The teaching activity of D-r Ivanov includes lecturing and seminar classes on Calculus and Mathematical analysis in Faculty of Mathematics and Informatics and Faculty of Physics. I personally have no direct impressions.

5. Analysis of the applicant's scientific achievements.

In order to make a detailed analysis of the candidate's results in such a complex field in which he works, it would be necessary to define many notions and results, which, as a rule, are very complicated and technical and inevitably involve formulas. To avoid this, we will briefly recall only the relevant concepts.

A C^* - algebra A is a Banach algebra $B(H)$ on the Hilbert space H equipped with an involution, which satisfies the properties of adjoint.

A C^* - algebra is simple, if it does not contain nontrivial, two-sided, closed ideal.

A state on C^* - algebra A is a positive linear functional $\phi : A \to \mathbb{C}$, $\phi(aa^*) \geq 0$ for every $a \in A$ and $\phi(1) = 1$. A state is called a trace, if additionally satisfies the cyclic property $\phi(ab) = \phi(ba)$ for all $a, b \in A$.

Important questions in the research on the C^* - algebras in general are to find the conditions under which the C^* - algebra is simple, the trace is unique and to study the relations between these properties.

I will accept the numbering from the applicant's list of the works, presented in the competition.

In [1] the reduced free products of finite-dimensional C^* - algebras are investigated. Such C^* - algebras are introduced by Voiculescu in investigations of their connections with the topology and the ergodic theory. Earlier Dykema gives necessary and sufficient conditions for simplicity of the reduced free products of finite-dimensional commutative C^* -algebras.

Then Dykema makes a conjecture about the necessary and sufficient conditions for simplicity of the reduced free products of finite-dimensional C^* - algebras with given states on them. The applicant proves this conjecture in the case, when these states are traces. It is proven also that under these conditions the trace is unique.

In [2] a class of C^* -algebras, generated by Toeplitz operators, satisfying certain conditions are considered. Their K-theory is found and it is proven that these algebras are nuclear, belong to a small bootstrap class, in particular, satisfy the Universal Coefficient Theorem (such algebras are known as Kirchberg's algebras). Making use of already found K-theory and known classification theorem the applicant proves that these algebras are isomorphic to tensor products of Cuntz algebras.

In the works what follow only discrete groups are considered.

Let G be such a group. The Hilbert space $\ell^2(G)$ has a standard orthonormal basis $\{\delta_g\}_{g\in G}$. Define the left regular representation λ of G on $\ell^2(G)$ via $\lambda(g)\delta_h = \delta_{gh}$. Then the reduced group C^* - algebra of G, denoted by $C_r^*(G)$, is C^* -subalgebra of $B(\ell^2(G))$, generated from $\lambda(G)$.

The group G is called C^*-simple, if $C_r^*(G)$ is simple. It can be defined a canonical trace τ on $C_r^*(G)$. It is said for the group G that it satisfies unique trace property, if τ is the only trace on $C_r^*(G)$.

A free product of the groups G_0 and G_1 with amalgamation over a common subgroup H (group amalgamations) is a group $G = G_0 *_H G_1$, together with homomorphisms $\phi_i : G_i \to G$, $i = 0, 1$ which agree on H, universal in sense that every other group with the same property is homomorphic to G.

It turns out for the considered groups that C^*-simplicity implies the unique trace property. The reverse implication does not hold – Boudec, Invent. Math. (2017) gives examples for that.

In [4] and [6] group C^* - algebras of group amalgamations and HNN-extensions (Higman, Neumann, Neumann), respectively, are studied for the above defined two properties: simplicity and uniqueness of the trace. The novelty in these works is the introduction of quasi-kernels, with whose help necessary and sufficient condition for C^*-simplicity is given. In addition, concrete examples of groups are given, whose C^* -algebras are with unique traces, but they are not C^*-simple.

In [5] and [7] a broad class of examples of group amalgamations and HNN-extensions, correspondingly, in the spirit of the examples from [4] and [6] is introduced. These new examples satisfy the unique trace property.

Besides, necessary and sufficient conditions for these groups to be C^*-simple are given.

Finally, in [3] some questions concerning non-commutative instantons and ADHM (Atiyah, Drinfeld, Hitchin, Manin) construction, related with them are considered.

The non-commutative instantons are introduced by Nekrasov and Schwarz as an analogue to the conventional ones.

In general, the article [3] gives a presentation of the results of the above work, as well as

the development of the topic in the following years.

More stringent definitions are given. At the end, some remarks on the computation of the topological index non-commutative ADHM instantons are made.

Summarizing, Nikolay Ivanov has fit in very well in this interesting, complex and forward-looking research topic and alone or with co-authors has achieved non-trivial results.

D-r N. Ivanov has provided data for 8 citations of his papers. In fact, the citations of his works are above 20. Let us mention only three of the citers in the presented list – Ara P., Dykema K., De La Harpe, P. These are world-renowned experts in the field with over 2,000 citations each.

The applicant has reported his results at many seminars and conferences, to notice a few:
Great Plains Operator Theory Symposium, 2006 and 2007
Fields Institute, 2007
International Colloquium on Differential Geometry, 2010, 2012 and 2014.
Spring Scientific Session of FMI, 2016, 2017, 2018 and 2021

D-r Ivanov has participated in several scientific projects with Sofia University.

6. Critical notes and recommendations.
I have no critical remarks. The CV could be a bit more detailed.

7. Personal impressions.
I have known Nikolay since 2016 and I highly evaluate his professional activity. His scientific articles present him as a serious researcher. As a person he is modest and reliable.

Conclusion.
After I got acquainted with the submitted materials and scientific works and on the basis of an analysis of their importance and the scientific achievements contained therein **I confirm** that the scientific achievements meet the requirements of ZRASRB, Regulations for its implementation and the corresponding Regulations of SU "St. Kliment Ohridski" for taking of the candidate the academic rank "Associate Professor" in the scientific field and in the research area of the competition.

I strongly recommend to the scientific jury to offer to the Faculty Council of FMI, SU "St. Kliment Ohridski" to choose Principal Assistant Professor Nikolay Ivanov, PhD, to take the academic rank "Associate Professor" in the research area 4.5 Mathematics (Mathematical analysis).

July 30, 2022

Prepared the report :
Assoc. Prof. DSc Ognyan Christov