REVIEW

in a competition for the academic position of "Associate Professor"
in professional field 4.5 Mathematics (Mathematical Logic)
for the needs of Sofia University "St. Kliment Ohridski" (SU),
Faculty of Mathematics and Informatics (FMI),
announced in State Gazette, No. 87 of 2021,
and on the websites of FMI and SU

The review was prepared by Prof. Dimiter Genchev Skordev, retired, in his capacity as a member of the scientific jury on the competition according to order No. RD 38-591/10.12.2021 of the Rector of SU.

Documents for participation in the announced competition have been submitted by only one candidate – Dr. Ivan Dimitrov Georgiev, who held at the "Prof. Dr. Asen Zlatarov" University – Burgas the position of "Assistant" from September 2009 till June 2016 and the position of "Chief Assistant" from July 2016 till September 2021.

The documents submitted for the competition by the candidate comply with the requirements of the Law on the Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its implementation and the Regulations on the Terms and Conditions for Acquiring Scientific Degrees and Holding Academic Positions at SU. The submitted list of all publications of the candidate contains a total of 20 titles. Of these publications, 7 (4 single-authored and 3 joint) are in impact factor journals, and 5 ones (one single-authored and 4 joint) have SJR without having impact factor (of the remaining 8 publications, 4 are single-authored and 4 are joint). The list of publications submitted for review by Dr. Georgiev contains 8 of the mentioned 20 (4 single-authored and 4 joint), all of which were published after 2015 and were not used in his dissertation. There are no members of the current jury among the co-authors of the joint publications submitted for review. Also presented: CV, diploma from SU for Master of Mathematics (Mathematical Logic), diploma from SU for Doctor of Mathematics (Mathematical Logic), employment record book, contracts with SU, with University "Prof. Dr. Assen Zlatarov" – Burgas and with the Union of Bulgarian Mathematicians, diploma from FMI of SU for the award "Prof. Ivan Soskov" for 2014, medical certificate for employment, certificate that the candidate has not been convicted, reference for the implementation of the minimum national requirements under Article 2b of the Law on the Development of the Academic Staff in the Republic of Bulgaria, documentation for citations of Mr. Georgiev's publications, reference to his original scientific contributions in the eight publications submitted for review and relevant confirmations by his co-authors, reference for his participation in seminars, conferences and scientific projects, description of his educational work in the period 2009-2021, abstracts

of the eight articles submitted for review, the issue of the State Gazette, containing (on page 122) the announcement for this competition, and an application for taking part in the competition. There is also information about four articles proposed for publication by other authors and peer-reviewed by him, but in my opinion the inclusion of this information in the competition materials is not in accordance with the anonymity of reviewers in the respective editions (this, however, is not a problem for the fourth of the mentioned articles because, as it is clear from the candidate's contribution reference, its authors published later an improved version of it together with him).

Mr. Ivan Georgiev was born in 1983 in Burgas. He graduated with honors in 2009, defending a MSc thesis on "Subrecursive Computability in Analysis". On the same topic is his doctoral dissertation, defended in February 2016 after part-time doctoral studies in the FMI of SU at the Department of Mathematical Logic and its Applications. In 2018 and 2019, in parallel with his work at the University "Prof. Dr. Asen Zlatarov" - Burgas, he was a part-time lecturer at Sofia University.

The main scientific contributions in the presented eight publications are in the theory of computability, and in particular in its field "Computability in analysis". In this direction are the four single-authored among the publications in question and two, which are joint with Lars Christiansen and Frank Stefan. The subject of the other two publications is quite different - in co-authorship with Krassimir Atanasov and two Polish authors, some attempts made by Atanasov and his followers to generalize the operations of predicate calculus are considered.

I move on to consider the candidate's contributions on both topics, and for each of them I will stick as much as I can to the chronology of writing the works (unlike the list in the materials of the competition). In rating the candidate's share in the joint publications, I will be based on the above-mentioned reference to his original scientific contributions.

(A) Contributions to Computability theory

Work No. 5: Georgiev, I., Characterization theorem for the conditionally computable real functions, Logical Methods in Computer Science, 2017, Vol. 13 (3), 1-17.

The paper studies the computability of real functions by means of a subrecursive class of functions in the set of natural numbers, such as the class $\mathcal{M}^2$, consisting of the total functions with Δ_0-graphs, which are majorized by polynomials. One possible approach to defining such computability is to use operators that convert argument representations to representations of the corresponding function values and in a certain sense are accordant with the given subrecursive class, and another is to work more directly with rational approximations of the arguments and their transformation into rational approximations of the corresponding function values. Unlike the case of the general notion of computability, in the case of computability by means of a subrecursive class of functions an unlimited search for a natural number with an effectively verifiable property is

generally not feasible with the means of the class in question. This leads to the stratification of the computability of real functions by means of this class into two types of computability – one in which only the arguments of the function are used in the computation (uniform computability), and another allowing the additional choice of a natural number as a parameter of the computational process (conditional computability). In the case of uniform computability, the reviewer proved in 2011 that, under certain assumptions, these two approaches are equivalent. In the currently reviewed work, Dr. Georgiev, overcoming a number of technical difficulties, proved a similar theorem for conditional computability and obtained important corollaries from this theorem.

I have the following small remark about terminology: since two different notions of an acceptable pair are used in the paper (the one introduced by the reviewer and another one), it would be better if different terms were used for them.

Work No. 3: Georgiev, I., On subrecursive complexity of integration, Annals of Pure and Applied Logic, 2020, Vol. 171 (4).

The complexity of integrating analytic real functions has been studied. Unlike previous work by other authors, where this has been done within the framework of discrete complexity theory, issues of subrecursive computability are explored here. This was done with the skilful use of the properties of the trapezoid method considered in a work by Trefethen and Weideman from 2014. It is proved that the definite integral of an analytic uniformly $\mathcal{M}^2$-computable real function in $\mathcal{M}^2$-computable limits is an $\mathcal{M}^2$-computable real number. A generalization is also given for integrals with parameter, for integrals with variable limits and for improper integrals. Using the fact that the Euler-Mascheroni constant is the opposite number of the value at 0 of the Laplace transform of the logarithmic function, the $\mathcal{M}^2$-computability of this constant is proved as a corollary. This provides an answer to a long-standing difficult question (posed, for example, in a talk by the reviewer and Andreas Weiermann at the Workshop on Computability Theory 2009 in Sofia).

Noticed oversights:

In clause 5 of Definition 3.1, an error was made when typesetting the formula defining the operator F – on the right-hand side instead of $[F_0(\vec{f})(\vec{x}, z) = 0]$ should only be $F_0(\vec{f})(\vec{x}, z)$. It should be noted in the references that the article [2] is in Bulgarian. The internet link provided for information on this article actually leads to the SCOPUS homepage. This is also the case with the links for the articles [10], [11] and [15].

Work No. 8: Georgiev, I., Fast converging sequence to Euler-Mascheroni constant, Annuaire de l'Univ. of Sofia "St. Kliment Ohridski" , Fac. de Math. et Inf., 2017, Vol. 104, 185-191.

By carefully tracing the proofs in the paper No. 3, a computable sequence of real numbers is extracted explicitly from them, for which the absolute value of the difference between the n-th term and the Euler-Mascheroni constant is $O\left(\frac{\sqrt{n}}{\sqrt{e^{\sqrt{n}}}}\right)$, and the n-th term can be written as $\sum_{k=-n}^{n} \Theta(k, n)$, where Θ is

expressed by elementary functions of analysis. Although, as noted in the article, the sequence in question is not convenient enough to easily find good decimal approximations of the constant, some results from experiments in this direction by using software that allows high-precision calculations are reported.

Work No. 2: Georgiev, I., Uniform and conditional $\mathcal{M}^2$-computability of some nonelementary real functions, Comptes rendus de l'Académie bulgare des Sciences, 2020, Vol. 73 (3), 306-314.

In a joint article by the reviewer and Ivan Georgiev in 2011, it was shown that the elementary functions of analysis are conditionally $\mathcal{M}^2$-computable. In the currently reviewed paper, Dr. Georgiev proves that two other important functions have this property, namely the Γ function and Riemann's ζ function – respectively in the interval $(0, +\infty)$ and in the interval $(1, +\infty)$. This is achieved by inventively using the results for integration from work No. 3, a lemma for preserving the uniform $\mathcal{M}^2$-omputability for a certain type of one-point extensions and a theorem of a technical nature, which is also useful for establishing uniform $\mathcal{M}^2$-computability. First one proves the result for the function Γ, and then the integral representation of the product $\zeta(s)\Gamma(s)$ for $s > 1$ is used.

Work No. 4: Georgiev, I., Kristiansen, L., Stephan, F., On general sum approximations of irrational numbers, In: Manea F., Miller R., Nowotka D. (eds) Sailing Routes in the World of Computation. Cmputability in Europe, 2018. LNCS, Springer, Cham, Vol. 10936, 194-203.

Work No. 1: Georgiev, I., Kristiansen, L., Stephan, F., Computable irrational numbers with representations of surprising complexity, Annals of Pure and Applied Logic, 2021, Vol. 172 (2).

As is known, various representations of real numbers through rational ones are considered in the literature, for example by Cauchy sequencees, by Dedekind cuts in the set of the rational numbers, by infinite fractions in a given base of the number system, by continuous fractions. It is known that if all real numbers are considered, these representations are generally not computationally equivalent (for instance, a Mostowski theorem shows that there is a computable operator transforming any base-b infinite fraction representing a real number into an infinite base-a fraction representing it iff all prime divisors of a are divisors of b). The situation changes if we limit ourselves to irrational real numbers. Then the usual representations in question are computationally equivalent to each other, but it turns out that in the general case the computable transformations of these representations into each other require an unlimited search in the set of the natural numbers (for example, this is the case for the transformation of infinite binary fraction representing an irrational number into infinite decimal fraction representing the same number). In the two works under consideration, the impossibility of carrying out some of these transformations in the general case without unlimited search is established by showing that this transformation does not always retain the membership in appropriately selected natural subrecursive classes.

According to Dr. Georgiev's reference for his original scientific contributions in the papers submitted for review, his scientific contribution to the two papers

currently under consideration can be described as follows. In 2017, he was a reviewer of a preliminary version of work No. 4 with authors Christiansen and Stefan. There was a number of inaccuracies in the proofs, and for some it was not at all obvious how they could be corrected. The following year, Dr. Georgiev succeeded to fill all the gaps in these proofs, and the two original authors agreed to submit a new version of the article with Dr. Georgiev as co-author. The reference points out as its most important contributions in the article the following ones: modification of Definition 1 with the assumption of an appropriate condition for the growth rate of the function used, new expressions for $M(j)$ and $M'(j)$ in Lemma 3, justification of step 3B in Theorem 4, a new correct algorithm for the tracking function of Theorem 6. In 2021, the same three authors published work No. 1 as an extended journal version of work No. 4, supplemented by a number of new results. The proof of Theorem 4.7 is stated in the reference as the most outstanding contribution of Dr. Georgiev in work No. 1, according to which none of the considered representations, with the exception of Cauchy's series, is closed under addition. This proof required a separate nontrivial study of the complexity of a number constructed in Theorem 4.3. In addition, Dr. Georgiev proved that the sum of the series from Definition 5.3 is a Liouville number and therefore a transcendental one.

Since I do not know the above-mentioned preliminary version of work No. 4, I cannot evaluate in detail most of the changes described above, made by Dr. Georgiev, but I see that they concern very non-trivial things and their implementation requires great skill and great ingenuity. I also see that he has made a very significant contribution to the new results added to the writing of the paper. Both works speak convincingly of the great potential of Dr. Georgiev as a researcher.

Noticed oversights in work No. 4:

The base b is mentioned in an inappropriate place in the statement on the initial page about the uniqueness of the representation – the sentence should start with the assumption that $b \in \mathbb{N} \setminus \{0, 1\}$.

Noticed oversights in work No. 1:

In the explanation of how subrecursive classes can be used in considering the problem of converting Cauchy series into Dedekind sections, there is no assumption about the computability of the number β, and such an assumption is needed for the computability of the function ψ_C. In the cited literature, the links provided for articles [7], [10] and [16] lead to the Scopus homepage, the link for [9] gives an error message, and the link for [11] opens a page with information about an article in a medical journal.

(B) Other Contributions

Work No. 7: Atanassov, K., Georgiev, I., Szmidt, E., Kacprzyk, J., Multidimensional intuitionistic fuzzy quantifiers, 2016 IEEE 8th International Conference on Intelligent Systems (IS), 2016, 530-534.

Work No. 6: Atanassov, K., Georgiev, I., Szmidt, E., Kacprzyk, J., Multi-

dimensional intuitionistic fuzzy quantifiers and level operators, In: Sgurev V., Piuri V., Jotsov V. (eds) Learning Systems: From Theory to Practice. Studies in Computational Intelligence, 2018, Springer, Cham, Vol. 756, 267-280.

The motivation of the research in these articles is related to some practical tasks of artificial intelligence. Within the so-called intuitionistic fuzzy logic proposed by Krassimir Atanassov, three groups of a certain type of multidimensional quantifiers are introduced, which act on predicates with a finite number of arguments, and the second article additionally defines a generalization of fuzzy fixed-level sets. In a team led by Atanassov, Dr. Georgiev worked on this topic as an expert in mathematical logic. From the report he presented, it can be seen that his contribution is approximately the following:

1. Since in the general case the conjunctions and disjunctions in question are neither commutative nor associative, he (in part 2.2 of work No. 7 and part 2.3 of work No. 6) proposed to assume, in case of a finite basic set, that a linear order is given in it, and thus to achieve unambiguity in determining the semantics of quantifiers by means of conjunctions and disjunctions.

2. In both articles he made numerous refinements of the definitions in order to achieve the maximum level of generality while maintaining their mathematical correctness, for example the use of supremums and infimums in the case of infinite sets and the explicit indication of the weights in the generality quantifier, given as an example of an analogue of a weight operator in part 2.2 of work No. 7 and part 2.3 of work No. 6.

Obviously, these contributions of Dr. Georgiev are not as impressing as the ones in the other presented publications, but in my opinion this is mainly due to the much more scarce mathematical content of the topics of both works. In particular, the coinciding conjunction and disjunction, in connection with which he mentioned the explicit indication of weights, obviously cannot be considered as continuations of the operations of the same name in two-valued logic. Unfortunately, both works have other weaknesses, for which Dr. Georgiev as a co-author bears some responsibility. For example, multidimensional quantifiers could be reduced to consecutively applied one-dimensional ones, and one could note the lack of significant connection with the intuitionistic approach to logic and the foundations of mathematics[1]. In both papers, only internet links are given for the last three sources cited. In work No. 6 the last of these links opens a text, which is obviously part of a book, but it is not clear which is this book[2] (in work No. 7 the same internet address is split in two lines and therefore does not work as expected).

[1]Cf. e.g. Dubois D., Gottwald S., Hajek P., Kacprzyk J., Prade H. Terminological Difficulties in Fuzzy Set Theory - The Case of "Intuitionistic Fuzzy Sets" (in: Fuzzy Sets and Systems, **156** (3), 2005, 485-491). To what has been said there, I add that all formulas of the type $A \vee \neg A$ turn out to be intuitionist fuzzy tautologies according to the definitions formulated in the first part of each of the two articles.

[2]By searching Google I found that it is "Mathematical Methods in Linguistics" by Barbara Partee, Alice ter Meulen and Robert E. Wall.

CONCLUSION

After getting acquainted with the materials and scientific papers presented in the competition and based on the analysis of their significance and the scientific and applied contributions contained in them, I confirm that the scientific achievements meet the requirements of the Law on the Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its implementation and the respective regulations of SU for the candidate to take the academic position "Associate Professor" in the scientific field and the professional direction of the competition. In particular, the candidate satisfies the minimum national requirements in the professional field and no plagiarism has been established in the scientific papers submitted at the competition. In the person of Dr. Ivan Georgiev I see a highly qualified mathematician with remarkable research potential. I give my positive assessment of the candidacy.

Based on the above, I strongly recommend the scientific jury to propose to the competent body for the selection of FMI at SU to elect Dr. Ivan Dimitrov Georgiev for taking the academic position of "Associate Professor" in the professional field 4.5 Mathematics (Mathematical Logic).

February 7, 2022 Reviewer:

(Prof. Dr. Dimiter Skordev)