Group of indicators B

Indicator B.3. Habilitation work – a monograph

"Theorems, Analogy, Heuristics or Theorem – Hypothesis – Theorem Prim"

Summary

The research reveals a heuristic role of theorems in mathematical creativity. It develops the thesis that apart from being a "building material" in proving theorems and solving problems, theorems are a "reliable source" of new hypotheses and theorems in mathematical courses. The idea of the general "origin" of mathematical knowledge gives rise to the concept that much of the new knowledge is obtained as a result of analogy and generalization of knowledge that has already been confirmed (by proof). The study brings us back to the classical analogy in geometry "plane-space", but realized through vector-algebraic modeling, by changing the dimension. From a formal point of view, the structure of the monograph is composed in four chapters and thus reflects the logic of the study. But, from a substantive point of view, the study consists of two main parts, which we could conditionally call the theoretical part and the practical part. The theoretical part combines the idea of theorems as a prototype of new hypotheses with a high degree of plausibility and the idea of the natural heuristic possibilities of vector-algebraic modeling in geometry. This unification led to the derivation and formulation of a strategy for a "heuristic transition" from the plane to spaces of three or more dimensions by "upgrading" plane theorems and constructing their spatial analogues. In the practical part the approbation of the hypothesis is made and the effect of the theoretically derived strategy for formulating hypotheses on the theorem for a quadrilateral face from the school course in geometry is confirmed. In the third chapter generalizations of the theorem are made and new theorems related to the octahedron, considered as a spatial analogue of the quadrilateral, are formulated and proved. In the fourth part, successful attempts were made to "exit" in spaces with three or more dimensions. For this purpose, new concepts have been introduced as an elementary and diagonal point configuration - analogues of the concepts triangle (tetrahedron) and quadrilateral (octahedron). Theorems for invariants of point configurations - analogues and generalizations of the concepts face and volume (triangle-tetrahedron and quadrilateral-octahedron) are also formulated and proved. At the end of the study a reflection was made on the applied part of the monograph and it

was found that the stages through which the authors went through the discovery and study of the octahedron theorem are similar to the stages of development of thinking skills in studying a particular subject. by psychologist Bloom.

Group of indicators Г

Indicator Г.4. A monograph, which is not presented as the main habilitation work

"Vector-Algebraic Method of Solving Geometric Problems of Collinearity and Competitivenes"
Summary

In many geometric situations, problems arise related to the collinearity of points or the competitiveness of lines, which with the help of elementary-geometric means are often solved on their own. Vector algebra makes it possible to approach problems of this type in a standard way, to avoid additional constructions, as a result of which in many cases the problems become a simple application of vectors. The aim of the monograph is to show this possibility only by means of affine vector algebra, which makes it applicable in school courses in geometry. The study consists of two parts. The first part plays a preparatory role and aims to propose a methodology for the formation of skills for working with vectors in solving problems of collinearity and competitiveness. The material is presented in the form of problems and their vector solutions. The tasks are systematized and arranged in didactic systems on the principle of "tasks-components". Two main stages of the process of forming skills for working with vectors are outlined - without using the term base and using the term base. Each of the stages is divided into sub-stages, according to the methodological understandings of the authors for gradual increase in the complexity and difficulty of the tasks. The methodology proposed in the first part of the monograph can be successfully applied by the teacher in the compulsory preparation classes (if the curriculum provides him with the necessary study time). In the second part of the monograph, research on the topic of collinearity and coplanarity focuses on the theorems of Menelaus and Cheva in space. (Previous research in this direction has been done by introducing barycentric coordinates and vector work - mathematical tools that are not suitable for school courses) With the help of vector algebra, only on the basis of affine operations (without the use of metric operations) are achieved spatial analogues of the theorems of Menelaus and Cheva. It is also established vector that if there is a common point of the Chevians (first point of Cheva), then there is also a common point of the transfers (second point of Cheva) and vice versa. A formula for the radius vector of the Cheva point is derived. The monograph is intended for mathematics teachers and students-future students in natural sciences.

Indicator Г.6. Articles, published in scientific journals, referenced and indexed in internationally-recognized databases of scientific information

Г.6.1. *"Entertaining Problems on the Topic "Picture Gallery"*

Summary

The subject of the article is the inductive approach to solving entertaining tasks related to the movement of a light beam in a rectangular gallery with mirrored walls, in three of the corners of which are placed paintings. A number of cases are considered and geometric solutions of three groups of problems are presented. As a result of geometric constructions, data were collected (galleries of different sizes were studied), regularities were established and conclusions (hypotheses) were formulated. Based on the conclusions, the trajectory of the light beam, the number of its reflections and the number of the illuminated picture can be predicted. Methodologically, the study is a continuation of Perelman's geometric method for solving the Poisson problem.

Г.6.2. *"Arithmetical or Algebraic Method of Solving Primary School Mathematical Problems"*

Summary

The purpose of this article is to outline the boundaries of the mathematical "coverage" of arithmetic methods for solving problems in the initial course of mathematics. Also to answer the question in which cases the models of the classical methodology of school arithmetic are "powerless" and therefore it is necessary to seek "help" from the methodology of school algebra. In other words, the aim is to show when the algebraic approach to solving arithmetic problems is logically unavoidable or technically justified. Based on the MZ-map of the problem, the authors formulate a criterion for the need to introduce the unknown and use a mathematical model of compound equations in solving arithmetic problems in primary school mathematics.

Г.6.3. *"Elementary Arithmetic Problems. Structure and Mathematical Model. Classification. Word Problems"*

Summary

The development focuses on elementary arithmetic problems in primary school mathematics. For theoretical and practical purposes, "definitions" of the concept of elementary arithmetic problem are proposed and the category structure of the problem is introduced. Based on a "list" of 12 types of problems, a logical-mathematical classification of elementary arithmetic problems is made, which covers 24 classes. Each class is presented through a specific example, which shows the structural and mathematical models of the problem. Mathematical and practical problems are considered in unity. The development is a reflection of the constructivist approach in the teaching of mathematics and represents the first stage of research on the topic "Arithmetic problems in primary school mathematics".

Г.6.4. *"Two Approaches to Study Equations in Primary School Mathematics"*

Summary

The authors introduce the concept of arithmetic equation and show that there is a mutual correspondence between elementary arithmetic equations and elementary arithmetic transformations, where the rules for finding an unknown component correspond to the inverse arithmetic transformations. Based on this correspondence, a mathematical and methodological parallel is made between the traditional method of the unknown component and the method of arithmetic transformations (inversion) for solving compound arithmetic equations in primary school mathematics. It is emphasized that in the method of the unknown component the unknown is in the "center" and the decision is made by "moving from the periphery to the center". While in the inversion method the unknown is at the "beginning" and the decision is made by "moving from end to beginning". From a mathematical point of view, the two approaches are equivalent, but from a methodological point of view, the inversion method is preferable because its "steps" can be visualized by a "chain diagram" and thus have a lower degree of abstraction.

Г.6.5. *"Ceva's Theorem for a Tetrahedron"*

Summary

The task of competitiveness of two or more lines is an important task not only for the science of geometry, but also for school courses in geometry. Many theorems that became classical in planimetry follow from Cheva's theorem for the triangle. Since the connections between the geometric concepts affected by Cheva's theorem are essential, it is natural to expect that the theorem can be generalized. Indeed, spatial analogues of the theorem have been discovered, but the formulations and proofs have been made by means beyond fictional mathematics. In this way, they remain accessible only to professional mathematicians. In 1979, Prof. G. Stanilov, from the rostrum of the conference of the Union of Mathematicians in Bulgaria, set the task of finding spatial analogues of Cheva's theorem, using only vector tools included in the school course in mathematics. The present work is provoked by Stanilov's task. In it, a successful attempt was made to present Cheva's two tetrahedron theorems (the Chevian common point theorem and the common point theorem for transversals) using only affine vector operations. The latter makes Cheva's theorems provable and only with mathematical means from the school mathematics course. The uniform approach used in the exposition of the evidence allows easy comparability of the results and establishing the fact that the first point of Cheva and the second point of Cheva coincide, ie. that these are two different ways of representing the same point in the tetrahedron configuration. A formula for the radius vector of the Cheva point in the tetrahedron is also derived.

Indicator Г.7. Articles, published in unreferred journals with scientific review or in peer-reviewed journals

Г.7.1. *"Conception for Contemporary Mathematical Education of Students – Future Primary School Teachers"*

Summary

This paper presents the main points of the authors' concept for modern university teaching of mathematics to students-future primary teachers. The concept reflects the authors' views on the nature of mathematical knowledge at the present stage and the methods of teaching mathematics to future primary school teachers. The opinion is argued that mathematical modeling is a method and the inductive-constructive approach is leading in the approach in teaching mathematics to the primary school teacher. It is emphasized that teaching by teaching is ineffective at this stage and needs to be replaced by learning by recreating mathematical knowledge. An innovative form of education, called an academic lesson, has also been proposed.

Г.7.2. *"Perelman's Geometric Method of Solving Liquid Pouring Problems*

Summary

The article presents a method for solving problems for overflow of liquids with three vessels (Poisson's problem), as the largest vessel with volume c is full, and the other two, with volumes *a* and *b*, are empty and. The method is based on elementary geometric constructions used by JI Perelman, which are based on the principle of "reflection of the billiard ball from the walls of a rhomboid mass." The main research tool is a rhomboid grid built on a clinogonal coordinate system. The problem situation is modeled geometrically and the solution of the problem is sought by constructing the trajectory of the "overflow beam" on the network. The method is described in detail with the help of elementary tools of analytical geometry and is presented through concrete overflow problems. Comments are made in the direction of optimizing the solution. The development is built on one of the topics of the entertaining mathematics course, designed for students-future teachers. It can be successfully applied in "school" conditions, in the classroom.

Г.7.3. *"The Reflective Property of Binumeric Relations"*

Summary

The report examines the role and place of two-part relations in the school mathematics course. The authors propose a classification of these relations. It is based on the ideas and theorems for the consolidation of mathematical knowledge, which reveals

the property of "reflexivity" of two-membered relations. Four groups of relations are formed. Study methods are proposed for each of the groups. Theoretical ideas are concretized through examples.

Г.7.4. *"Vectors as a Basis of Subject Interrelations in School Mathematics"*

Summary

The report promotes the idea that vectors can be the basis for making interdisciplinary connections through mathematics education. For this purpose, tasks from the school courses in algebra, geometry, physics, chemistry and practical tasks are presented, the solutions of which pass through vector-algebraic models. It is shown that vector algebra allows for a new type of mathematical modeling and discovery of new interdisciplinary connections in the natural sciences studied in high school.

Г.7.5. *"Overview of Mathematics and Mathematical Education in Byzantium"*

Summary

The great mathematician and historian of mathematics Van der Warden, in 1954 at the end of his famous book on the history of mathematics "Awakening Science", after mentioning the latest achievements of Hellenic mathematics in the IV and V century wrote: "After these last lights the flame of Greek mathematics extinguished like a burning candle." But whether the scientist has the right to such a definite assessment. Recent research on the history of science and education reveals facts that do not fit into the generally accepted picture of Byzantine mathematics and mathematical education and in a sense refute the assertion of Van der Warden. The aim of the present study is to give a brief overview of Byzantine mathematics and to show that the fire of ancient Greek mathematics burned for the next thousand years. The Byzantine school of mathematics not only inherited and preserved the ancient Greek mathematical thought, but also had a strong influence on the mathematical knowledge of the peoples of Eastern Europe, including the Bulgarians.

Г.7.6. *"Volume of Octahedron"*

Summary

The presented article presents an unconventional way to find the volume of an octahedron. For this purpose, a new stereometric theorem is formulated and proved. The proof uses elementary results from analytic geometry and linear algebra. The proven theorem allows the volume of the octahedron (and other polyhedra) to be calculated directly (without addition or division). For this purpose, it is sufficient to find the volume

of a tetrahedron whose defining vectors are given by the diagonals of the octahedron. The material can be taught in schools with intensive study of mathematics.

Γ.7.7. *"Inversion – a Method for Solving Problems in the School Course in Mathematics"*

Summary

The method of inversion for solving arithmetic problems has been known to Indians since the 5th century. With him, the reasoning goes "from end to beginning." This scheme is close to Pap's scheme, but with the difference that not sufficient conditions are found for the accuracy of the individual statements, and consequently in reverse order are applied actions opposite to those described in the text of the problem. The successful application of inversion in school as a method for solving problems requires a theoretical justification of the method. In the present work the statement that the essence of the inversion is in the arithmetic transformations is argued. A theoretical review of the elementary arithmetic transformations is made and their inverse transformations are specified. Composite transformations are considered as compositions of elementary ones. A way to graphically represent composite transformations using a "chain diagram" of squares and arrows is proposed. An attempt is made to classify text problems, whose mathematical model is a composition of arithmetic transformations. The development is aimed at the teacher in the primary grades.

Γ.7.8. *"Methodological Notes for Introduction and Study of Vectors in the School Geometry Course"*

Summary

In the first part of the article, the authors make a brief but relatively complete historical account of the role and place of vectors in university and school mathematics courses. An overview and evaluation of the interpretations of the concept is made - the vector as a quantity that has a "magnitude and direction" (physical interpretation), the vector as a class of equal directed segments (geometric interpretation), the vector as an element of vector space (algebraic interpretation). As a result of this review, it was concluded that the issue of introduction and study of vectors in the school course in mathematics is solved from a mathematical point of view, but is not solved from a didactic point of view. For this reason, the solutions to the "vector" question must be sought not by mathematical but by methodological means. In the second part of the article the authors offer methodological solutions for introducing and studying vectors in the school course in geometry. The possibility is justified when studying vectors to start from a specific vector space, such as the vector space of radius vectors, and then to proceed to the vector space of classes of equal directed segments. The vector space of radius vectors has a significantly lower degree of abstraction (its elements are concrete objects) than the vector space of "free vectors" (elements of which are classes of equal directed segments). The article outlines

two stages in the study of the main problem of vector algebra - the problem of the base and the representation of the vector as a linear combination of vectors. Finally, an attitude is taken on the issue of the two main views related to the introduction and study of vectors in the school course in mathematics.

Γ.7.9. *"Connecting Element"*

Summary

In the present study, it is heuristic in solving geometric problems in which the "path" from the condition to the conclusion is "not open" for the solver. In particular, it is possible to construct "intermediate stations" through which both the condition and the conclusion of the task can be "seen". The role of the "intermediate station" can be played by a suitable segment - a connecting segment, by a suitable angle - a connecting angle, by a suitable algebraic expression - a connecting expression. Based on specific tasks, the connecting role of these elements is shown.

Indicator Γ.8. Studies, published in scientific journals, referenced and indexed in internationally-recognized databases of scientific information

Γ.8.1. *"Diagonal Point Configurations. Rule of the Triangle. Invariants"*

Summary

In the present study, a specific continuation (by analytical means) of the idea of face and volume of the school course in geometry has been made. It is shown that the geometric figures quadrilateral and octahedron are concretizations of diagonal point configurations and whether the invariants of these configurations are analogs of the concepts face of a quadrilateral and volume of an octahedron. , 3-dimensional, 4-dimensional and n-dimensional (n> 4) space. In order to facilitate the summaries and simplify the evidence, compact notations have been introduced and a consistently applied "triangle rule" has been introduced.

Γ.8.2. *"Basic Configurations of Points. Diagonal Principle. Invariants"*

Summary

The concepts of triangle face and tetrahedron volume are interpreted as invariants of elementary point configurations in two-dimensional and three-dimensional space. Their analogue was found in four-dimensional space. The idea of an invariant of an elementary point configuration is continued in n-dimensional space. For this purpose, a

"diagonal principle" has been formulated and consistently applied for the composition and transformation of the respective matrices. For this purpose, methods of analytical geometry (vector-algebraic modeling, affine coordinate system, point coordinates, vector coordinates) and means of linear algebra (matrices and determinants) were used.

Г.8.3. *"Euler-Venn Diagrams or MZ-Cards in Primary School Mathematics"*

Summary

The subject of the article is the construction of mathematical models of problem situations caused by additive operations (unification, intersection and complementarity) with finite sets in primary school mathematics. Author's diagrams of situations caused by four basic sets are shown. A comparative analysis of two methodological approaches for modeling and solving problems of additive operations with sets and natural numbers is made. These are the already well-known Euler-Venn chart method and the innovative MZ map method. The idea is argued that the solution can be found more easily if the two approaches complement each other in the process of solving the problem.