R E V I E W

on the contest for the acquisition of the academic position Professor

in Professional Direction 4.5 Mathematics (Finite Geometries)

at Sofia University "St. Kliment Ohridski" (SU)

Faculty of Mathematics and Informatics (FMI)

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The review is written by Prof. Ph.D. Azniv Kirkor Kasparian, Section of Algebra, Faculty of Mathematics and Informatics, Sofia University "St. Kliment Ohridski", Professional direction 4.5 Mathematics, as a member of the scientific juri for the contest, according to the Order RD-38-576/15.10.2023 of the Rector of Sofia University.

The only applicant for the announced contest is Associate Professor, D. Sc. Assia Petrova Rousseva from the Section of Geometry, Faculty of Mathematics and Informatics, Sofia University "St. Kliment Ohridski".

1 General description of the presented materials

1.1 Data of the application

The documents, presented by the applicant for the contest, comply with the requirements of the Law on the Development of Academic Staff of Republic Bulgaria, the Rules on its Implementation and the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at Sofia University "St. Kliment Ohridski".

Ass. Prof. D.Sc. Assia Rousseva participates in the contest with eighteen articles and a book. Twelve of the aforementioned articles are published in specialized scientific journals and six - in proceedings of international conferences and workshops. A part of the book "Aspects of combinatorics" can serve as a textbook in combinatorics and finite geometries for the students from the New Bulgarian University and Sofia University "St. Kliment Ohridski". The final chapters comprise some recent research results, so that this book can be viewed as a monograph, as well. Seventeen of the aforementioned articles and the book are co-authored and one of the articles is standalone. All the co-authors of Ass. Prof. D.Sc. Assia Rousseva claim the equipollence of the contributions of the co-authors in the joint publications. The documents on the contest include evidences for the articles for the ranking and the citations of the articles.

The official transcripts, provided by Ass. Prof. D.Sc. Assia Rousseva for the contest include diplomas for higher education, Ph.D. degree, Doctor of Sciences degree, Associate Professorship appointment, as well as a transcript for more than 30 years work experience at the Section of Geometry, Department of Mathematics and Informatics, Sofia University "St. Kliment Ohridski" and a transcript for overall and classroom occupation during the last 10 academic years.

1.2 Brief biography of the applicant

Ass. Prof. D.Sc. Assia Rousseva works at the Section of Geometry, Department of Mathematics and Informatics, Sofia University ”St. Kliment Ohridski” for more than thirty years. After her graduation as a Master in Mathematics (Geometry) in 1988, she is a part-time assistant for five years. From 1993 Ass. Prof. D.Sc. Assia Rousseva is a full Assistant Professor. In 2001 she becomes a Senior Assistant Professor and in 2003 - a Chief Assistant Professor. From 2009 the applicant is an Associate Professor at the Section of Geometry. She has defended a Ph. D. Thesis in 2004 and a Doctor of Sciences Thesis in 2020.

The h-index of Ass. Prof. D.Sc. Assia Rousseva is 4. She is a member of the Geometric Association of the Union of the Bulgarian Mathematicians. Ass. Prof. D.Sc. Assia Rousseva has been a head of ten scientific contracts ”Finite geometries, incidence structures and applications” with the Science Foundation of Sofia University ”St. Kliment Ohridski” and a member of two scientific contracts with the Science Foundation of the Ministry of the Science and Education. She has written twenty nine peer reviews for the specialized scientific journal ”Designs, codes and cryptography”.

1.3 General characterization of the scientific works and the contributions of the applicant

The scientific works of Ass. Prof. D.Sc. Assia Rousseva are in the field of the finite geometries and their applications to coding theory and cryptography. Combining geometric and combinatorial arguments with polynomial techniques and computer implementations, she has attained remarkable results for linear codes, arcs and blocking sets, concerning their construction, extendability and numerical characterizations. The articles of Ass. Prof. D.Sc. Assia Rousseva introduce new methods for solving problems on information transmission by mixture of classical geometric approaches with contemporary combinatorial techniques. Her contributions enrich the knowledge in coding theory and cryptography, solve difficult contemporary problems and and raise interesting conjectures.

Ass. Prof. D.Sc. Assia Rousseva has the total of 55 publications, from which 30 are in specialized scientific journals and 25 are in proceedings of international conferences and workshops. Six of her publications are standalone and the other 49 are ao-authored. Nineteen publications of Ass. Prof. D.Sc. Assia Rousseva are in journal with IF, 2 are with SJR and 29 are refereed in Zentralblatt. Her scientific works have 54 citations, from which 42 are indexed in Web of Science or Scopus. Ass. Prof. D.Sc. Assia Rousseva has delivered 87 talks at international and national conferences in Germany, Italy, France, Russia, USA, Bulgaria, as well as during her visits at Gent University, Belgium or at Metropoliten College of Boston’s University, USA.

The scientific contributions of Ass. Prof. D.Sc. Assia Rousseva comply with and exceed considerably the minimal national requirements of Decree 26/13.02.2019 on the Amendments of the Rules of Implementation of the Law on Development of Academic Staff of Republic Bulgaria, as well as the specific requirements of the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at Sofia University ”St. Kliment Ohridski”. More precisely, her contributions rate the total of 988 points versus the required 550 ones. From these, the sciento-metric properties of the presented 18 publications for the contest earn her 699, instead of the required 300 points. The 19 citations for the contest rate 124 points, versus the necessary 100 ones. The D.Sc. degree, the participation in two national scientific projects and "Aspects of Combinatorics" earn her 115 points, instead of the required 100.

The scientific works, presented for the competition do not include ones, used in previous procedures for acquisition of academic degrees and occupation of academic positions. I am strongly convinced that there is no plagiarism in the scientific works of Ass. Prof. D.Sc. Assia Rousseva.

1.4 Characterization of the teaching activity of the applicant

Ass. Prof. D.Sc. Assia Rousseva has taught "Analytic geometry", "Basics of geometry", "Projective geometry", "Descriptive geometry", "Finite geometries" and other geometric courses for Bachelors and Masters at the Department of Mathematics and Informatics, Sofia University "St. Kliment Ohridski". During her experience of more than 30 years, she developed skills for masterful explaining the material and keeping a permanent contact with the audience. Ass. Prof. D.Sc. Assia Rousseva has prepared several part-time assistants in geometry. At the moment, she is working with the part-time Ph. D. student Emilyan Rogachev. Ass. Prof. D.Sc. Assia Rousseva has written the book "Aspects of combinatorics", which is simultaneously a textbook and a monograph on the subject.

1.5 Content analysis of the scientific contributions of the materials, presented for the contest

The scientific works of Ass. Prof. D.Sc. Assia Rousseva develop new methods for studying problems on information transmission by combining classical geometric techniques with sophisticated combinatorial considerations and computer implementations. That allows her to answer a series of difficult open problems from coding theory and cryptography, which have potential for practical applications. In such a way, Ass. Prof. D.Sc. Assia Rousseva has enriched the existing knowledge in the field of information transmission and became one of the leading specialists in the field. While solving difficult contemporary problems from coding theory and cryptography, her results lead to interesting conjectures, shaping the development of the subject.

Let us start by discussing some extendability results of Ass. Prof. D.Sc. Assia Rousseva for linear codes and arcs. In 1998 Dodunekov and Simonis establish a correspondence between the linear $[n,k,d]_q$-codes and the $(n, n-d)$-arcs in $\mathbb{P}^{k-1}(\mathbb{F}_q)$. That allows to translate some optimization problems from coding theory in terms of incidence relations in projective spaces over finite fields. For instance, the extendability of the linear codes, subject to Griesmer's lower bound $g_q(k,d)$ on the length of an $\mathbb{F}_q$-linear code of dimension k and minimum distance d is equivalent to the extendability of the corresponding arc. In a series of papers, Ass. Prof. D.Sc. Assia Rousseva derives sufficient conditions for extendability of Griesmer t-quasidivisible mod$q$ arcs. These are formulated in terms of the parameters of the associated blocking sets, containing a hyperplane, or in terms of the coefficients of a pseudo q-adic expression of d, or in terms of the dual $(t \mod q)$-arcs, introduced for the purpose by her works. Let H be a projective hyperplane in $\mathbb{P}^r(\mathbb{F}_q)$ and $P \in \mathbb{P}^r(\mathbb{F}_q) \setminus H$. The projection of $\mathbb{P}^r(\mathbb{F}_q) \setminus \{P\}$ onto H with center P pulls back an arbitrary arc $\mathcal{K}_o$ in $H \simeq \mathbb{P}^{r-1}(\mathbb{F}_q)$ to an arc $\mathcal{K} \subset \mathbb{P}^r(\mathbb{F}_q)$, which is called a lifting of $\mathcal{K}_o$. The structure of a $(t \mod q)$-arc is compatible with the liftability of arcs. More precisely, a $(t \mod q)$-arc in $\mathbb{P}^r(\mathbb{F}_q)$ is shown to be lifted if its restrictions $\mathcal{K}|_H$ to all projective hyperplanes $H \subset \mathbb{P}^r(\mathbb{F}_q)$ are lifted. One of the works of Ass. Prof. D.Sc. Assia Rousseva classifies of the $(t \mod q)$-arcs in $\mathbb{P}^r(\mathbb{F}_q)$ for $t \in \{2,3\}$ and $r \in \{2,3\}$. For an odd $q \geq 5$, the $(2 \mod q)$-arcs in $\mathbb{P}^2(\mathbb{F}_q)$ are shown to constitute four classes. Except arcs, which are lifted from appropriate configurations of lines, the models include $2\mathbb{P}^2(\mathbb{F}_q)$ and the union of a closed $(q+1)$-arc in $\mathbb{P}^2(\mathbb{F}_q)$ with one of its tangents. The scientific contributions of Ass. Prof. D.Sc. Assia Rousseva include the classification of the $(3 \mod 5)$-arcs in $\mathbb{P}^2(\mathbb{F}_5)$, which is done by the means of an appropriate software package, generating the associated $\mathbb{F}_5$-linear codes. The classification of the $(3 \mod 5)$-arcs $\mathcal{K}$ in $\mathbb{P}^3(\mathbb{F}_5)$ is based on the description of the point-line configurations of $\mathcal{K}$, as well as on the knowledge of the restrictions $\mathcal{K}|_H$ of $\mathcal{K}$ on projective hyperplanes $H \subset \mathbb{P}^3(\mathbb{F}_5)$. A software implementation implies that there are three $(3 \mod 5)$-arcs $\mathcal{K}_i$, $1 \leq i \leq 3$ in $\mathbb{P}^3(\mathbb{F}_5)$, which are not lifted and do not contain a full projective hyperplane. The multiplicities, the line distributions and the hyperplane distributions of all the points of $\mathcal{K}_i$, $1 \leq i \leq 3$ are listed explicitly, as well as the cardinalities, the hyperplane spectra and the orders of the automorphism groups of $\mathcal{K}_i$, $1 \leq i \leq 3$. As an application of the classification of the $(3 \mod 5)$-arcs in $\mathbb{P}^3(\mathbb{F}_5)$ is shown the non-existence of a $(104, 22)$-arc in $\mathbb{P}^3(\mathbb{F}_5)$. Any $(3 \mod 5)$-arc in $\mathbb{P}^r(\mathbb{F}_5)$ with $r \geq 4$ is conjectured to be lifted. One of the works of Ass. Prof. D.Sc. Assia Rousseva provides geometric constructions of the $(3 \mod 5)$-arcs $\mathcal{K}_i \subset \mathbb{P}^3(\mathbb{F}_5)$, $1 \leq i \leq 3$, which are not lifted and do not contain a full projective hyperplane. The arc $\mathcal{K}_1 \subset \mathbb{P}^3(\mathbb{F}_5)$ of size 128 is shown to be one of the complete 128-caps, constructed by Abatangelo-Korchmáros-Larato in 1996. For the description of $\mathcal{K}_2$ and $\mathcal{K}_3$, let $q = p^h$ be an odd primary integer, $F(x_0, x_1, \ldots, x_r) \in \mathbb{F}_q[x_0, x_1, \ldots, x_r]^{(2)}$ be a homogeneous polynomial of degree 2, $\mathbb{V}(F) := \{[a_0 : a_1 : \ldots : a_r] \in \mathbb{P}^r(\mathbb{F}_q) \,|\, F(a_0, a_1, \ldots, a_r) = 0\}$ be the hypersurface, cut by F and $S_q^{\text{not}}(F)$ be the set of the points $[a_0 : a_1 : \ldots : a_r] \in \mathbb{P}^r(\mathbb{F}_q) \setminus \mathbb{V}(F)$, for which $F(a_0, a_1, \ldots, a_r)$ is not a square in $\mathbb{F}_q$. Then $\mathcal{K}_F = \left(\frac{q+1}{2}\right) \mathbb{V}(F) + S_q^{\text{not}}(F)$ is shown to be $\left(\frac{q+1}{2} \mod q\right)$-arc in $\mathbb{P}^r(\mathbb{F}_q)$, which is not lifted for a non-degenerate $\mathbb{V}(F)$. For a non-degenerate elliptic quadric $\mathbb{V}(F) \subset \mathbb{P}^3(\mathbb{F}_5)$, the arc $\mathcal{K}_F = \mathcal{K}_2$ is the non-lifted $(3 \mod 5)$-arc of size 143, while for a non-degenerate hyperbolic quadric $\mathbb{V}(F) \subset \mathbb{P}^3(\mathbb{F}_5)$ one obtains the non-lifted $(3 \mod 5)$-arc $\mathcal{K}_F = \mathcal{K}_3$ of size 168. In a similar vein, if $q = p^{2h}$, $h \in \mathbb{N}$ and $H(x_0, x_1, \ldots, x_r) \in \mathbb{F}_q[x_0, x_1, \ldots, x_r]$ is a non-degenerate Hermitian form then $\sqrt{q}\mathbb{V}(H)$ is proved to be a $(\sqrt{q} \mod q)$-arc in $\mathbb{P}^r(\mathbb{F}_q)$, which is not lifted.

Let p be a prime integer and $\mathcal{K}$ be a $(t \mod p)$-arc in $\mathbb{P}^r(\mathbb{F}_p)$,, whose multiplicities of the points can exceed t and are bounded above by $p-1$. After proving that the $(0 \mod p)$-arcs in $\mathbb{P}^r(\mathbb{F}_p)$ form an $\mathbb{F}_p$-linear space of dimension $\binom{p+r-1}{r}$, which is generated by the complements $\mathbb{P}^r(\mathbb{F}_p) \setminus H$ of the projective hyperplanes $H \subset \mathbb{P}^r(\mathbb{F}_p)$, one of the works of Ass. Prof. D.Sc. Assia Rousseva establishes that any $(t \mod p)$-arc is a sum of lifted arcs. In the projective plane $\mathbb{P}^2(\mathbb{F}_p)$, the space of the $(0 \mod p)$-arcs is shown to be generated by p spaces V_i,$1 \leq i \leq p$ of arcs, lifted from points P_i of a conic in $\mathbb{P}^2(\mathbb{F}_p)$. As a consequence, an arbitrary $(t \mod p)$-arc in $\mathbb{P}^2(\mathbb{F}_p)$ can be decomposed into a sum of at most p lifted arcs. Moreover, if a (q^2+1-t)-cap $\mathcal{C}$ in $\mathbb{P}^3(\mathbb{F}_q)$ does not contain a plane of cardinality

$q+2$ and $t < t_0$ for the minimal integer t_0, for which there is a $(q+1+t_0)$-blocking set in $\mathbb{P}^3(\mathbb{F}_q)$ without a full line, then $\mathcal{C}$ is shown to be extendable.

Except the extendability and divisibility results for linear codes and arcs, the contributions of Ass. Prof. D.Sc. Assia Rousseva include construction and classification of optimal codes and arcs. Let $n_q(k,d)$ be the minimal length of an $\mathbb{F}_q$-linear code of dimension k and minimum distance d. An important problem from coding theory is the study of the maximum $t_q(k)$ of $n_q(k,d) - g_q(k,d)$ over all d. Several authors have established that $\lim_{d\to\infty}[n_q(k,d) - g_q(k,d)] = 0$. In 1974 Belov-Logachev-Sandimirov show that $n_q(k,d) = g_q(k,d)$ for all $d \geq (k-2)q^{k-1}+1$, so that $t_q(k) = \max_{1\leq d \leq (k-2)q^{k-1}}[n_q(k,d) - g_q(k,d)]$. After translating the study of $t_q(k)$ in terms of arcs and blocking sets, one of the articles of Ass. Prof. D.Sc. Assia Rousseva derives a sufficient condition for extendability of certain blocking sets in $\mathbb{P}^{k-2}(\mathbb{F}_q)$ to blocking sets in $\mathbb{P}^{k-1}(\mathbb{F}_q)$. As a consequence, for any $h \in \mathbb{N}$ and any odd prime p are obtained the upper bounds $t_{2^h}(3) \leq 2^{h-1} - 5$, $t_{p^{2h}}(3) \leq 2p^h - 1$, $t_{p^{2h-1}}(3) \leq \frac{p^{2h-1}-3}{2}$. If $\mathcal{C}_q$ is a (q^2+1)-cap in $\mathbb{P}^3(\mathbb{F}_q)$ then $\mathcal{K}_q = \mathcal{C}_q + \mathbb{P}^3(\mathbb{F}_q)$ is known to be a Griesmer arc in $\mathbb{P}^3(\mathbb{F}_q)$ of size $n(\mathcal{K}_q) = |\mathcal{C}_q| + \left|\mathbb{P}^3(\mathbb{F}_q)\right| = q^3 + 2q^2 + q + 2$ and maximal plane multiplicity $w(\mathcal{K}_q) = w(\mathcal{C}_q) + w(\mathbb{P}^3(\mathbb{F}_q) = q^2 + 2q + 2$. For projective planes $\pi_0, \pi_1 \subset \mathbb{P}^3(\mathbb{F}_3)$ and a projective line $l \subset \mathbb{P}^3(\mathbb{F}_3)$, which is skew to $\pi_0 \cap \pi_1$, the arc $\mathcal{K}_3' = 2\mathbb{P}^3(\mathbb{F}_3) - (\pi_0 + \pi_1 + l)$ has the same size and maximal plane multiplicity as $\mathcal{K}_3 = \mathcal{C}_3 + \mathbb{P}^3(\mathbb{F}_3)$. A work of Ass. Prof. D.Sc. Assia Rousseva conjectures that for a sufficiently large q any $(n(\mathcal{K}_q), w(\mathcal{K}_q))$-arc in $\mathbb{P}^3(\mathbb{F}_q)$ is of the form $\mathcal{K}_q = \mathcal{C}_q + \mathbb{P}^3(\mathbb{F}_q)$. This conjecture is proved for $q=5$ and $q=7$, making use of the reducibility of an $(a(p+1)+1, a)$-blocking set $\mathcal{B} \subset \mathbb{P}^2(\mathbb{F}_p)$ to an $(a(p+1), a)$-one for a prime p, $1 \leq a < p$ and $\mathcal{B}$ with more than $2p-2$ points of multiplicity 0. Another work of Ass. Prof. D.Sc. Assia Rousseva describes the $(113, 29)$-arcs in $\mathbb{P}^3(\mathbb{F}_4)$ and makes use of the available characterization of the $(100, 26)$-arcs in $\mathbb{P}^3(\mathbb{F}_4)$, in order to derive the non-existence of five-dimensional Griesmer codes over $\mathbb{F}_4$, which are of minimum distance $d_0 \in \{295, 296, 335, 336\}$. As a consequence are determined the exact values of the minimal lengths $n_4(5, d_0)$ of the five-dimensional $\mathbb{F}_4$-linear codes of minimum distance d_0. The sum $\mathcal{B}_{r,q} = \mathbb{P}^2(\mathbb{F}_q) + 2\mathbb{P}^1(\mathbb{F}_q) \subset \mathbb{P}^r(\mathbb{F}_q)$ of a projective plane and two projective lines is known to be a blocking set of size $n(\mathcal{B}_{r,q}) = \left|\mathbb{P}^2(\mathbb{F}_q)\right| + 2\left|\mathbb{P}^1(\mathbb{F}_q)\right| = q^2 + 3q + 3$ with minimal hyperplane multiplicity $w(\mathcal{B}_{r,q}) = \left|\mathbb{P}^1(\mathbb{F}_q)\right| + 2\left|\mathbb{P}^0(\mathbb{F}_q)\right| = q + 3$. For $r \geq 3$ and $q \geq 5$ an article of Ass. Prof. D.Sc. Assia Rousseva shows that any $(n(\mathcal{B}_{r,q}), w(\mathcal{B}_{r,q}))$-blocking set $\mathcal{B}$ in $\mathbb{P}^r(\mathbb{F}_q)$ is of the form $\mathcal{B} = \mathcal{B}_{r,q} = \mathbb{P}^2(\mathbb{F}_q) + 2\mathbb{P}^1(\mathbb{F}_q)$. The same article establishes that there are five isomorphism classes of blocking sets in $\mathbb{P}^3(\mathbb{F}_3)$ with parameters $(n(\mathcal{B}_{3,3}), w(\mathcal{B}_{3,3})) = (21, 6)$. As a result, an arbitrary $(22, 6)$-blocking set in $\mathbb{P}^3(\mathbb{F}_3)$ turns to be reducible to a $(21, 6)$-one and, in general, any $(n(\mathcal{B}_{3,q}) + 1, w(\mathcal{B}_{3,q}))$-blocking set in $\mathbb{P}^3(\mathbb{F}_q)$ with $q \geq 3$ is reducible to an $(n(\mathcal{B}_{3,q}), w(\mathcal{B}_{3,q}))$-one. Any blocking set in $\mathbb{P}^3(\mathbb{F}_4)$ with parameters $(n(\mathcal{B}_{34,}), w(\mathcal{B}_{3,4})) = (31, 7)$ is shown to be isomorphic to $\mathcal{B}_{3,4} = \mathbb{P}^2(\mathbb{F}_4) + 2\mathbb{P}^1(\mathbb{F}_4)$ or to a cone with vertex of multiplicity 3, whose base curve is a Baer subplane.

Several articles of Ass. Prof. D.Sc. Assia Rousseva study arcs and codes of almost constant Hamming distance. For any integer $0 \leq \alpha < q$ and any arc $\mathcal{K}_\alpha$ in $\mathbb{P}^r(\mathbb{F}_q)$, $r \geq 2$, whose hyperplane multiplicities are $w, w+1, \ldots, w+\alpha$, the projective subspaces $U \simeq \mathbb{P}^s(\mathbb{F}_q)$ of dimension $0 \leq s \leq r$ are shown to be of multiplicity $\mathcal{K}_\alpha(U) \in \{u_s, u_s + 1, \ldots, u_s + \alpha\}$ for some $u_s \in \mathbb{Z}^{\geq 0}$. As a result, any such arc is of the form $\mathcal{K}_\alpha = m\mathbb{P}^r(\mathbb{F}_q) + \mathcal{K}_\alpha'$ for some $m \in \mathbb{Z}^{\geq 0}$ and an arc $\mathcal{K}_\alpha' \subset \mathbb{P}^r(\mathbb{F}_q)$, whose points are of multiplicity $0, 1, \ldots, \alpha$.

Let $L_r \simeq \mathbb{P}^1(\mathbb{F}_q) \subset \mathbb{P}^r(\mathbb{F}_q)$ be a line of multiplicity $q, q+1$ or $q+2$, which consists of points of multiplicity $0, 1$ or 2. Denote by $\mathcal{O} \subset \mathbb{P}^2(\mathbb{F}_q)$ a hyperoval, consisting of $q+1$ points of multiplicity 0 and one point of multiplicity 2. If a plane arc $\mathcal{K}_2 \subset \mathbb{P}^2(\mathbb{F}_q)$, whose hyperplane multiplicities are $w, w+1, w+2$, contains a point of multiplicity 0 and a point of multiplicity 2 then $\mathcal{K}_2$ is shown to be of the form $\mathcal{K}_{2,1} = 2P$ for some $P \in \mathbb{P}^2(\mathbb{F}_q)$, $\mathcal{K}_{2,2} = L_2 + [\mathbb{P}^2(\mathbb{F}_q) - L_2]$, $\mathcal{K}_{2,3} = \mathcal{O} + [\mathbb{P}^2(\mathbb{F}_q) - \mathcal{O}]$ or a complement of $\mathcal{K}_{2,j}$ for some $1 \leq j \leq 3$. Let $\mathcal{K}'_{2,1} = P_1 + P_2$ for some $P_1, P_2 \in \mathbb{P}^r(\mathbb{F}_q)$, $r \geq 3$, $\mathcal{K}'_{2,2} = \mathbb{P}^r(\mathbb{F}_q) - \mathcal{K}'_{2,1}$ or $\mathcal{K}'_{2,3} = L_r + [\mathbb{P}^r(\mathbb{F}_q) - \mathrm{Supp}(L_r)]$. If $\mathcal{K}_2$ is an arc in $\mathbb{P}^r(\mathbb{F}_q)$ with $r \geq 3$, whose hyperplane multiplicities belong to $[w, w+2]$ and attain the ends of this segment then $\mathcal{K}_2$ is proved to be of the form $\mathcal{K}_2 = \mathcal{K}'_{2,j} + \gamma \mathbb{P}^r(\mathbb{F}_q)$ for some $1 \leq j \leq 3$ and $\gamma \in \mathbb{Z}^{\geq 0}$. Let $A_2(n, \{d_1, d_2\})$ be the maximal cardinality of a binary code of length n with Hamming distances $d_1 \neq d_2$. A work of Ass. Prof. D.Sc. Assia Rousseva shows that if $d_2 > 2d_1$ then $A_2(n, \{d_1, d_2\}) \leq n+1$. That allows to derive the bounds $A_2(n, \{2, 4\}) = \binom{n}{2} + 1$ for $\forall n \geq 6$, $A_2(n, \{2, d\}) = n$ for $\forall 5 \leq d \leq n-2$ and $A_2(n, \{2, n-1\}) = n+1$ for $\forall n \geq 6$, which are conjectured in 2020 by Boyvalenkov-Delchev-Zinoviev-Zinoviev. Moreover, if a binary code $C \subset \mathbb{F}_2^n$ of cardinality $|C| > n+1$ contains the origin $0^n \in C$, has k_j words of weight d_j for $1 \leq j \leq 2$ and $d_1 + d_2 \in 2\mathbb{N} + 1$ is an odd integer then the parameters of C satisfy the equality $2(k_1+1)k_2(d_2 - d_1) + (k_1 + k_2 + 1)d_1 = 0$. For any $p, q \in \mathbb{N}$ there follow $A_2(n, \{2p, 2p+2q-1\}) = n+1$ and $A_2(n, \{2p-1, 2p+2q-2\}) = n+2$. Besides, for arbitrary $n, d_1, d_2 \in \mathbb{N}$ is derived that $A_2(n, \{d_1, d_2\}) \leq \frac{(n+1)(n+2)}{2}$. There is a proposed construction of binary codes $C \subset \mathbb{F}_2^n$ with distances $2k-2, 2k$ of cardinality $|C| = \frac{n(n-1)}{k(k-1)}$, which makes use of $2-(n,k,1)$-designs.

The scientific contributions of Ass. Prof. D.Sc. Assia Rousseva include construction of blocking sets of small size. Generalizing Ball's idea for obtaining a $(q^2, q-1)$-blocking set in $\mathbb{F}_q^3$, one of the articles constructs a $(q^2, q-n+2)$-blocking set in $\mathbb{F}_q^n$ for any $3 \leq n \leq q-1$. More precisely, let $T \simeq \mathbb{P}^{n-2}(\mathbb{F}_q)$ be a projective subspace of $\mathbb{P}^n(\mathbb{F}_q)$ of codimension 2 and $H_0, H_1, \dots, H_q$ be the projective hyperplanes in $\mathbb{P}^n(\mathbb{F}_q)$, containing T. If $\mathcal{C} = \{x_1, \dots, x_q\}$ is a q-arc in T and $L_i \simeq \mathbb{P}^1(\mathbb{F}_q)$, $1 \leq i \leq q$ are projective lines in $\mathbb{P}^n(\mathbb{F}_q)$ with $L_i \cap T = \{x_i\}$ then $\mathcal{B}_T = \cup_{i=1}^q (L_i \setminus \{x_i\})$ is a $(q^2, q-n+2)$-blocking set in $\mathbb{F}_q^n$, which attains Bruen's lower bound $\mathrm{Br}(t,n,q) := (n+t-1)(q-1)+1$ on the cardinality of a blocking set in $\mathbb{F}_q^n$. Removing $n-2+s$ points from each of the lines $L_1, \dots, L_s$, one obtains a $(q^2 - s(n-2+s), q-(n-2+s))$-blocking set $\mathcal{B}'_s$ in $\mathbb{F}_q^n$. In particular, $\mathcal{B}'_1$ is a $(q-n+1)$-blocking set in $\mathbb{F}_q^n$ of minimal cardinality $q^2 - n + 1$. If $M(t,n,q)$ is the minimal cardinality of a t-blocking set in $\mathbb{F}_q^n$ then for any $t \geq 3$ and any $c \in \mathbb{R}^{>0}$ the same article shows the existence of such $n_0 \in \mathbb{N}$ that $M(t,n,q) \geq \mathrm{Br}(t,n,q) + c$ for all $n \geq n_0$. The authors conjecture that for all $n \in \mathbb{N}$ and $t \geq q$, Bruen's lower bound $\mathrm{Br}(t,n,q)$ on the cardinality of a t-blocking set in $\mathbb{F}_q^n$ is not sharp. A similar construction yields a $(q^2+2q-1, q-n+3)$-blocking set in $\mathbb{F}_q^n$. The article provides also a table of lower and upper bounds on the cardinality of a t-blocking set in $\mathbb{F}_q^n$ for some $t \in \{3,4\}$, $3 \leq n \leq 5$ and $4 \leq q \leq 13$. More generally, let $S \simeq \mathbb{P}^r(\mathbb{F}_q)$, $2 \leq r \leq n-2$ and $T \simeq \mathbb{P}^{n-r-1}(\mathbb{F}_q)$ be disjoint complementary projective subspaces of $\mathbb{P}^n(\mathbb{F}_q)$, $\mathcal{K} = \{P_1, \dots, P_M\}$ be an (M, w)-arc in S, $H \simeq \mathbb{P}^{n-1}(\mathbb{F}_q)$ be a projective hyperplane in $\mathbb{P}^n(\mathbb{F}_q)$, containing S and $\mathcal{L} = \{Q_1, \dots, Q_N\}$ be an (N, u)-blocking set in $T \setminus (T \cap H) \simeq \mathbb{F}_q^{n-r-1}$. For any $1 \leq j \leq N$ consider the projective space $S_j \simeq \mathbb{P}^{r+1}(\mathbb{F}_q)$, generated by S and Q_j. Let $a = \left[\frac{M}{N}\right]$ and partition $\mathcal{K}$ into N subsets $C_1, \dots, C_N$ of size a or $a+1$. For any $P_i \in C_j$, let

$L_i \simeq \mathbb{P}^1(\mathbb{F}_q)$ be a projective line, which is contained in S_j but not in S. Assume that the lines L_i, contained in S_j are skew to each other. Then $\mathcal{B} = \cup_{i=1}^{M}(L_i \setminus \{P_i\})$ is shown to be a blocking set in $\mathbb{P}^n(\mathbb{F}_q) \setminus H \simeq \mathbb{F}_q^n$ of cardinality qM with hyperplane multiplicities $\geq t = \min(M - w, aqu)$. The application of the aforementioned general construction to various specific cases provides a table of lower and upper bounds on the cardinality of a t-blocking set in $\mathbb{F}_4^n$ for $4 \leq t \leq 7$ and $3 \leq n \leq 9$.

One of the articles of Ass. Prof. D.Sc. Assia Rousseva studies the p-rank $\mathrm{rk}_p(A)$ of the points-by-lines incidence matrix A of a projective Hjelmslev plane $\mathbb{P}^2(R)$ over a chain ring R with $|R| = q^2$, $R/\mathrm{Rad}(R) \simeq \mathbb{F}_q$. It establishes that over the residue ring $R = \mathbb{Z}_4$ the 2-rank of A is $\mathrm{rk}_2(A) = 12$, while for $R = \mathbb{F}_2[u]/\langle u^2 \rangle$ there holds $\mathrm{rk}_2(A) = 13$. In the case of an odd prime p, the article derives that $\mathrm{rk}_p(A) \geq \binom{p+1}{2}^h (q+1) + 3q^2 - 2q$, making use of $(0 \mod p)$-arcs in $\mathbb{P}^2(R)$. Specifically, for $|R| = 9$ there holds $45 \leq \mathrm{rk}_3(A) \leq 76$.

Ass. Prof. D.Sc. Assia Rousseva participates in the contest with six articles with IF from the second quartile, one article with IF from the third quartile, four articles with IF from the fourth quartile, two articles with SJR and five articles, published in refereed journals. Among the nineteen citations, provided for the contest, twelve are indexed in Web of Science or Scopus. All co-authors of joint works have declared the equipollence of the contributions in the joint articles. The aforementioned articles and citations are not used in previous procedures for acquisition of academic degrees and occupation of academic positions. There is no doubt of plagiarism. The above circumstances have convinced me that Ass. Prof. D.Sc. Assia Rousseva complies and even exceeds considerably all the requirements for occupation of the academic position Professor at the Section of Geometry, Department of Mathematics and Informatics, Sofia University "St. Kliment Ohridski".

1.6 Critical remarks and suggestions

I have no critical remarks and suggestions.

1.7 Personal impressions

Ass. Prof. D.Sc. Assia Rousseva is dedicated to the excellence in research and teaching. She is a distinguished specialist in finite geometries and a skillful teacher. Ass. Prof. D.Sc. Assia Rousseva is highly respected for her qualification, responsiveness, honesty and precision in any job. She has developed excellent teamwork skills, which lead to remarkable results in her research and earn her the reputation of a profound profesionalist.

1.8 Conclusion on the application

After getting acquainted with the materials and the scientific works, presented for the competition, and based upon the aforementioned analysis of their scientific significance and applicability, I confirm that the scientific contributions comply with the Law on Development of Academic Staff of Republic Bulgaria, the Rules on its implementation and the Rules on the terms and conditions for acquisition of academic degrees and occupation of academic positions at Sofia University "St. Kliment Ohridski" for occupation of the academic position ”Professor” in professional direction ”4.5 Mathematics” (Finite geometries). In particular, the applicant satisfies the minimal national requirements in the professional direction and no plagiarism was found in the presented scientific works. That is why,

I evaluate positively the application.

2 General conclusion

Based upon the aforementioned, **I strongly recommend** the scientific Juri to propose the appropriate election authority of the Faculty of Mathematics and Informatics of Sofia University ”St. Kliment Ohridski” to elect

Ass. Prof. D. Sc. Assia Petrova Rousseva
to take up the academic position ”Professor”

in Professional Direction 4.5 Mathematics (Finite geometries).

November 15, 2023

Review written by:
Prof. Ph.D. Azniv Kasparian