STATEMENT REPORT

according to the procedure for the defense of a PhD Thesis entitled

„Subdifferential analysis of convex-like functions“

for the acquisition of PhD degree of the Sofia University

by the PhD student: **Matey Boyanov Konstantinov**
in the field of higher education: **4. Natural sciences, mathematics and informatics**
professional direction: **4.5. Mathematics**
doctoral program: **Operations Research of the Faculty of Mathematics and Informatics (FMI) of the SU "St. Kliment Ohridski"**

The referee report is prepared by Prof. DSc. Mihail Ivanov Krastanov as a member of the scientific jury, according to Order No. RD № РД 38-113/06.03.2023 of the Rector of SU "Saint Kliment Ohridski".

1. General description of the PhD Thesis and the presented materials

The presented materials have been prepared in accordance with the Law on the Development of the Academic Staff in the Republic of Bulgaria (LDASRB), the Regulations for the Application of the LDASRB, as well as the Regulations for the Terms and Procedures for Acquiring Scientific Degrees and Holding Academic Positions (RT LDASRB) in SU" St. Cl. Ohridski". These include: PhD Thesis in English; Abstract in Bulgarian; Autobiography in Bulgarian; List of scientific publications on the topic of the dissertation; Reference on the fulfillment of the minimum requirements under Article 2b, para. 2 and 3 of the LDAS of the Republic of Bulgaria; Application by the PhD student; Doctoral student's declaration of originality and absence of plagiarism; Report from the research supervisor; Similarity report; StrikePlagiarism.com software product report for no plagiarism; Protocol for verification of originality; Statement regarding the anti-plagiarism procedure signed by the supervisor and others.

The PhD Thesis of Matej Konstantinov has a volume of 79 pages and contains an introduction, three chapters in which the obtained results are presented, a conclusion, an appendix and a bibliography of 55 titles. It is written in English. In the Introduction, a brief motivation of the research is made, some of the main concepts used in the thesis are defined and the main obtained results are presented. In the first chapter, uniformly proximally regular subsets of Hilbert spaces are defined and investigated. In the second chapter, uniformly lower regular functions are considered. In the third chapter, a new proof of the classical Moreau-Rockafellar theorem for integrability of the

subdifferential of semicontinuous lower convex functions defined in a Banach space is presented. For the convenience of the reader, some well-known facts as well as proofs of known statements used in the PhD Thesis are collected in an appendix.

2. Data and personal impressions about the candidate

From the presented autobiography, it can be seen that Matei Konstantinov was born on September 5, 1994. He completed his higher education at the Faculty of Mathematics and Informatics of Sofia University, receiving a bachelor's degree in 2017 and a master's degree in 2019. Both are in "Applied Mathematics" specialty. Even as a student he won silver medals from the National Student Olympiad in Computer Mathematics in 2015, 2016, 2017 and 2018, as well as bronze medals from the National Student Olympiad in Mathematics in 2015. and 2017. From February 10, 2020 to February 10, 2023, he is a full-time doctoral student in the "Operations Research" doctoral program of FMI, SU with scientific supervisor Prof. Dr. Nadia Zlateva.

I have known Matey Konstantinov since he was a student. He made a strong impression on me with his serious attitude towards mathematics. Whatever he undertakes, he strives to understand it down to the smallest detail. Moreover, Matei Konstantinov can also explain complex mathematical questions in an understandable and accessible way. And that's why his students like and respect him.

3. Analysis of the candidate's scientific achievements, contained in the presented thesis and the corresponding publications, included in the procedure

In the introduction of the presented PhD Thesis, a brief historical overview of the results known up to now on the subject of the Thesis is made and the main results obtained by the author are presented.

Uniformly proximal regular sets with constant r of Hilbert spaces are considered in the first chapter. At the beginning, the author makes a brief historical overview of this concept. Then the used notations are introduced, the necessary definitions are given and some properties of these sets are formulated (Theorem 1.1.3). The rest of the first chapter is devoted to the proof of a new internal characterization of an r-proximal regular subset of a Hilbert space (Theorem 1.1.1), which is one of the main results in this PhD thesis. Three properties are formulated in this theorem. They have been shown to be equivalent. The corresponding proof involves lengthy calculations. The drawings made by Matei Konstantinov facilitate the reading and understanding of this proof.

In the second chapter, epi uniformly lower regular functions are studied. And here, at the beginning of this chapter, a brief historical review of the papers exploring this concept is made. Next, the used notations are introduced and the necessary concepts used in this chapter are defined. The main results in this chapter (Theorem 2.2.1 and Theorem 2.2.2) give a relation between epi uniformly lower regular functions and their epigraphs. More specifically, in Theorem 2.2.1 it is proved that if a function is epi ρ lower regular, then its epigraph is an epi ρ proximal regular set. In Theorem 2.2.2 it is proved that if the epigraph of a function is an epi ρ proximal regular set, then there exists ρ' such that the corresponding function is epi ρ' lower regular. Here we should note that if the epigraph of a function is an epi ρ proximally regular set, then the epigraph is ρ -

uniformly proximally regular, but the converse is not true. Using Theorem 1.1.1., some properties of epiproximal regular sets are proved (see Theorem 2.3.1. and Theorem 2.3.2 As a consequence of Theorem 2.2.1, Theorem 2.2.2, Theorem 2.3.1. and Theorem 2.3.2 is a characterization of the epiproximal regularity property of an eigenbottom semicontinuous function (Theorem 2.4.1). A corollary is also obtained for the points of the graph of this function.

In the third chapter, a variant of the ε subdifferential method is presented, which is applied to a proper convex lower semicontinuous function. At the beginning of this chapter, the used notations are introduced, two propositions proved by Brøndsted and Rockafellar are formulated, and Lemma 3.1.5 is proved. This lemma provides a useful estimate used to prove that the number of iterations of the presented ε subdifferential method is finite (an estimate is obtained from above for their number). As an application of this result, a new proof of the Moreau-Rockafellar theorem for integrability of the subdifferential of semicontinuous lower convex functions defined in a Banach space is obtained: if two lower semicontinuous convex functions are defined in a Banach space, the subdifferential of one of which is contained in the subdifferential of the other, then these two functions differ by a constant.

The References include the titles of 55 papers. They are arranged in ascending order of the surnames of the first author according to standard lexicographical arrangement. These sources demonstrate an excellent knowledge of the scientific field of the thesis by the doctoral student. Perhaps the fact that his scientific supervisor together with associate professor Milen Ivanov actively work in the field of variational analysis plays an important role here.

4. **Approbation of the results**

The results of the PhD thesis have been published in 3 articles, all in well-known journals with impact factor or impact rank:

M. Konstantinov, N. Zlateva, Epsilon subdidifferential method and integrability, Journal of Convex Analysis 29 (2021), 571-582.

M. Konstantinov, N. Zlateva, Direct proofs of intrinsic properties of prox-regular sets in Hilbert spaces, Journal of Applied Analysis (2023) (to appear).

M. Konstantinov, N. Zlateva, Epigraphical characterization of uniformly lower regular functions in Hilbert spaces, Journal of Convex Analysis (2023) (to appear).

The scientific metrics of these articles, compared with the minimum requirements for the educational and scientific degree "doctor", according to Resolution No. 26 of February 13, 2019 on the amendment and addition of the Regulations for the implementation of the Law on the development of the academic staff in the Republic of Bulgaria, adopted with Decree No. 202 of the Council of Ministers of 2010 (promulgated, SG No. 75 of 2010; amended and supplemented, No. 19 of 2011, No. 9 of 2012, No. 62 of 2013, No. 60 of 2014, No. 57 of 2015 and No. 56 of 2018) are as follows: All publications collect a **total of 102 points,** with a minimum requirement of **30 points** for the acquisition of the educational and scientific degree "doctor" in the scientific field **4. Natural sciences, mathematics and informatics**, professional direction **4.5 Mathematics**. The first publication is in Q4 of WEB of Sciences and is valued at

36 points, the second is in Q4 in SCOPUS and is valued at 30 points and the third is of WEB of Sciences and is valued at 36 points. The obtained **102 points** significantly (**more than 3 times**) exceed the minimum requirements for obtaining the educational and scientific degree "doctor" in the scientific field and professional direction of the procedure.

Based on the submitted materials, the reviewer assumes that there is no proven plagiarism in the submitted thesis and research papers under this procedure.

The results of the dissertation have been presented by the author so far in the following reports:

M. Konstantinov, N. Zlateva, Epsilon Subdifferential Method And Integrability, 15-th International Workshop on Well-Posedness of Optimization Problems and Related Topics, June 28-July 2, 2021, Borovets, Bulgaria, http://www.math.bas.bg/~bio/WP21/;

M. Konstantinov, N. Zlateva, Direct proofs of intrinsic properties of prox-regular sets in Hilbert spaces, Spring Scientific Session, Faculty of Mathematics and Informatics, Sofiaa University "St. Kliment Ohridski", 26 March, 2022, Sofiaa, Bulgaria, https://www.fmi.uni-sofia.bg/bg/proletna-nauchna-sesiya-na-fmi-2022/;

M. Konstantinov, N. Zlateva, Epsilon Subdifferential Method and Integrability, 10-th International Conference on Numerical Methods and Applications, August 22-26, 2022, Borovets, Bulgaria, http://www.math.bas.bg/~nummeth/nma22/index.html.

5. Qualities of the Resume

The Resume is written in Bulgarian in a volume of 28 pages. It accurately reflects the content of the dissertation work and corresponds to the requirements of LDASRB and the Regulations on the terms and conditions for acquiring scientific degrees and holding academic positions at SU "Kliment Ohridski". Fortunately, the numbering of the assertions, definitions, references and others in the auto-reference matches perfectly to their numbering in the thesis. All this makes it easy to read. I was impressed by the drawing that visualizes the version of the ε subdifferential method that he considered.

6. Critical notes and recommendations

It seems to me that the thesis would have gained if the Matey Konstantinov had devoted a little more space at the beginning of each chapter to explain the ideas of the proofs, to emphasize the difficulties he has overcome. All this would make this thesis more readable.

7. Conclusion

Having read this PhD thesis and the accompanying publications presented in the procedure and based on the analysis of their significance and the scientific results contained in them, I confirm that the **presented PhD thesis and the scientific publications to it**, as well as the quality and originality of the presented in them, results and achievements, **meet the requirements of LDASRB**, the Regulations for the application of LDASRB and the Regulations for the conditions and procedures for acquiring scientific degrees and holding academic positions in the SU for the candidate's acquisition of the educational and scientific degree "doctor" in the

scientific field: 4. Natural sciences, mathematics and informatics, professional direction: 4.5. Mathematics, doctoral program **Operations Research** of the Faculty of Mathematics and Informatics (FMI) of the SU "St. Kliment Ohridski". I want to emphasize that the qualities of the thesis **significantly exceed the minimum national requirements** in the professional field. Here I must note that no plagiarism was found in the scientific papers submitted to the competition.

Based on the above, I **strongly recommend that the scientific jury award** Matey Boyanov Konstantinov the educational and scientific degree "doctor" in a scientific field: **4.Natural sciences, mathematics and informatics**, professional direction: **4.5. Mathematics,** doctoral program **Operations Research** of the Faculty of Mathematics and Informatics (FMI) of the SU "St. Kliment Ohridski".

29.05. 2023 Prepared the referee report:

/prof. DSc. Mikhail Ivanov Krastanov/