

## **Opinion**

**In connection with the procedure for the defense of  
a dissertation entitled:  
" Subdifferential analysis of convex-like functions"  
(dissertation submitted for acquiring the educational  
and scientific degree "Doctor")**

**Candidate:** Matey Boyanov Konstantinov

**Field of graduate studies:** 4. Natural sciences, mathematics, and informatics

**Professional direction:** 4.5. Mathematics

**Doctoral program:** „Operations Research“, department: „ Probabilities, Operations Research and Statistics “, Faculty of Mathematics and Informatics (FMI) of Sofia University “Kliment Ohridski” (SU)

Prepared by: **Professor DMS Petar Stoyanov Kenderov**,  
in my capacity as a member of the scientific jury, appointed by Order  
№ ПД-38-113/06.03.2023 of the Rector of Sofia University.

### **1. General characteristics of the dissertation work and the presented materials**

The dissertation is presented in English and is 79 pages long. It consists of an Introduction, three chapters, a Conclusion, an Appendix, and a list of cited articles (Bibliography). The latter contains 55 items, three of which are the candidate's articles. The earliest cited literary source is from 1959, and the titles from the last 10 years are 12 (not including the candidate's articles). In addition to the dissertation, two Abstracts of the dissertation (one in Bulgarian and one in English, each 28 pages long) are also presented. All the documents required by the formal procedure (21 in total) are provided with the necessary signatures and confirmations. The academic supervisor of the candidate is Prof. Nadia Zlateva.

### **2. Data and personal impressions about the candidate**

Candidate Matei Konstantinov was born in 1994. He graduated from secondary education at the famous Sofia Mathematical High School "Paisii Hilendarski" (in 2013). He received his bachelor degree in mathematics from the Faculty of Mathematics and Informatics of Sofia University (in 2017). He received master's degree in applied mathematics (Optimization) in July 2019. The topic of the diploma thesis was "Research on models for advertising" and it was prepared under the supervision of Prof. Mihail Krastanov.

As a student, M. Konstantinov participated very successfully in student olympiads in mathematics and computer mathematics (7 participations are listed). He won one first place, 4 silver medals and two bronze medals. Already as a student (since 2015), he was assigned to teach classes with students, as a part-time assistant, in Linear and Mathematical Optimization, Differential and Integral Calculus, Differential Equations and Applications, Probability and Statistics.

He has participated in four research projects, one of which was supported by the Fund "Scientific Research" of the Ministry of Education and Sciences, and the others – from the similar Fund of Sofia University.

I know the candidate from his participation in the Optimization seminar and from his presentations during the 2018 and 2021 editions of the Workshop on Well-Posedness of Optimization Problems and Related Topics.

### **3. Content analysis of the candidate's scientific and applied scientific achievements, contained in the submitted dissertation and the publications to it, included in the procedure**

In many practical optimization problems, classical approaches are not directly applicable because the function to be optimized is not smooth (it cannot be locally linearized at every point of its domain). The very set of admissible points among which the optimum is sought is very often also not smooth, and the direct application of techniques of the type "Lagrange Multipliers" is not applicable. This led to the emergence of a number of new mathematical fields, among which is Convex Analysis - the framework to which, if there is a good will, a significant part of the considerations in the dissertation can be referred. The most general idea is to replace the usual uniquely defined derivative at a point with a "set of tangents". A multi-valued mapping appears, which received the already well-established name "subdifferential". For convex functions, this concept is defined relatively unambiguously. With more general functions, more subdifferentials exist, and this has given rise to a bewildering variety of subdifferentials, some convenient in one situation, others in another. It is now possible to speak of "sub-differential calculus". This calculus naturally includes concepts and facts concerning both functions and sets. For example, "primal lower-nice functions" and the so-called "prox-regular sets", which are the focus of this dissertation. These functions and sets have properties similar to those of convex functions and convex sets, but constitute a significantly larger class. In the first chapter (Theorem 1.1.1) a new and direct proof of equivalence of different formulations of proximally regular sets is given. This reasoning shows that the candidate has a very good command of analysis techniques. The

techniques developed here help in the following Chapter 2 to obtain the main results. Here we can note Theorems 2.2.1 and 2.2.2, which establish the natural connection between the epi-lower regular functions and the epi prox-regularity of the epigraph of the function. Central here is Theorem 2.4.1, in which a characterization of the epi lower regular (and proper semi-continuous) function is given, without using subdifferentials.

The third chapter contains a new proof of the classical result that a proper lower semicontinuous convex function defined in a Banach space is determined by its subdifferential up to a constant. To prove this, the author uses an original version of the "epsilon subdifferential method."

### **Approbation of the results**

The dissertation provides links to the electronic pages of three forums in which the candidate has participated. At least two of them have international participation.

The publications related to the results of the dissertation are three. One of them appeared in the Journal of Convex Analysis in 2021, one was accepted for publication in the same journal, and the third was accepted for publication in the Journal of Applied Analysis.

From the documents provided to me in connection with the procedure, it can be seen that the minimum national requirements (according to Art. 2b, paras. 2 and 3 of the ZRASRB) and the additional requirements of SU "St. Kliment Ohridski" for the acquisition of an educational and scientific degree "doctor" in the scientific field and professional direction of the procedure are filled with excess.

The plagiarism check was negative.

Until now, the candidate has not participated in procedures for obtaining a scientific degree and/or holding an academic position, which excludes the possibility that these works of his have already been used in such a sense.

### **4. Qualities of the Abstract**

The abstract correctly and fully reflects the content of the dissertation work. The handling of the literature (both in the abstract and in the dissertation) makes a particularly good impression. The contributions of other authors are noted with the necessary detail and expertise.

### **5. Critical notes and recommendations**

I have no substantive criticisms.

## 6. Conclusion

Having familiarized myself with the dissertation work and the accompanying scientific works presented in the procedure and based on the analysis of their significance and the scientific contributions contained in them, I **confirm** that the presented dissertation work and the scientific publications to it, as well as the quality and originality of the results and achievements presented in them meet the requirements of the RSARB, the Rules for its application and the relevant Rules of the SU "St. Kliment Ohridski" for the candidate's acquisition of the educational and scientific degree "doctor" in the scientific field **4.**

**Natural sciences, mathematics and informatics** and professional direction **4.5. Mathematics.** In particular, the candidate satisfies the minimum national requirements in the professional direction and no plagiarism has been found in the scientific works submitted for the competition.

Based on the above, I recommend the scientific jury to award Matei Boyanov Konstantinov an educational and scientific degree "doctor" in the professional field of Mathematics.

31.05.2023

Prepared by: .....  
(Prof. Petar Kenderov)