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Uncertainty: Definition and Classification for the Task of Economic Forecasting

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Abstract: The aim of this text is to establish a working definition and classification of uncertainty for the task of economic forecasting. This is necessary in order to arrive at a common understanding of the term, reduce semantic ambiguity and define a clear distinction when it comes to quantifying forecast uncertainty. Two fundamental sources on uncertainty by John Maynard Keynes and Frank H. Knight are reviewed from the perspective of the classification of uncertainty into aleatoric and epistemic, which is a separation of growing use in engineering and machine learning. The concepts of aleatoric and epistemic uncertainty are explored and the possible ambiguity and interaction between them are discussed. Finally, a working definition and classification of uncertainty is laid out and refined for practical use in the context of economic forecasting.

JEL: C53, D80, D81

Keywords: Frank H. Knight, John Maynard Keynes, uncertainty, economic forecasting

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1 Introduction

The concept of uncertainty has been a topic of great interest to scholars across both natural and social sciences. It has played an important role across a diverse set of fields and thus over time the term "uncertainty" has adopted various meanings. The first recorded attempts to define uncertainty, were by the Greek philosophers of the school of Athens (Bernstein, 1996). The ancient Greek word εικος (eikos), which could be translated as probable or plausible, was defined by Socrates as "likeness to truth". Aristotle also theorized on the topic of decision-making in his Nichomachaen Ethics, yet he did not delve deep into the concept of uncertainty, but rather accepted luck or chance as a given. On the other hand, games of chance seem to be as old as history with some early known examples from ancient Egypt dating back to 3500 BC, it was not until the Renaissance, that gambling was used as a foundation to study uncertainty (Bernstein, 1996).

In 1654, a French nobleman Chevalier de Méré challenged the famous mathematician Blaise Pascal to solve a puzzle, which has confused mathematicians for some two hundred years, when it was posed by the monk Luca Paccioli. The puzzle was how to divide the winnings of an unfinished game of chance between two players, while one of them is ahead. With the help of another brilliant mathematician Pierre de Fermat, Pascal laid the foundation of what we know today as theory of probability. In consequence, various great thinkers and scholars have continued to explore and revisit the concept of uncertainty like Bayes, Bernoulli and Galton to name a few. In the beginning of the 20th century, the concept of uncertainty already played a critical role in the analysis in various fields from physics to psychology and this is also when some of the most prominent economic thinkers laid out some fundamental groundwork on the concept of risk and uncertainty.

Research focused on modeling and forecasting uncertainty can suffer from ambiguity and lack of precision, without clearly defining the concept of uncertainty and by using the term as self-explanatory. This text aims to establish a working definition and classification of uncertainty for the task of economic forecasting. This is necessary in order to arrive at a common understanding of the term, reduce semantic ambiguity and define a clear distinction when it comes to quantifying forecast uncertainty. Recently, in the fields of engineering and machine learning a separation of uncertainty into aleatoric and epistemic has grown in popularity. Aleatoric uncertainty roughly refers to the inherent stochasticity in the environment or its measurement, while epistemic uncertainty refers to the limitations to the knowledge of the observer. The practicality and ambition for clarity and unambiguous separation makes this classification appealing and considered suitable in the context of economic forecasting. Therefore, two fundamental sources on uncertainty by John Maynard Keynes and Frank H. Knight, which define and explore the concept in the economics literature are reviewed from the perspective of the classification of uncertainty into aleatoric and epistemic. Consequently, the concepts of aleatoric and epistemic uncertainty are explored and the possible ambiguity and interaction between them are discussed. Finally, a working definition and classification of uncertainty is laid out and refined for practical use in the context of economic forecasting.

2 Uncertainty in Economics

For some time uncertainty had no place in economics (Davidson, 1999). Classical economics theory dealt with agents possessing perfect information about the outcomes of their decisions and uncertainty was simply ignored. In the year 1921, two seminal works were published by two up-and-coming economists, who eventually became founding fathers of two prominent schools of economics: John Maynard Keynes and Frank H. Knight. In that year, Keynes, who was a trained mathematician, published his dissertation on probability theory, which Bertrand Russel (1922) would praise as “the most important work on probability that has appeared in a long time”. Knight also published a revised version of his dissertation in 1921, which dealt with the economics of uncertainty and judgement. These two modern thinkers and their seminal works had laid the foundations of what later became two separate schools of economic thought – the Keynesian school of macroeconomics (Snowdon and Vane, 2015; Faccarello and Kurz, 2016) and the Chicago school of microeconomics (Emmett, 2009).

On the surface, it appears like these foundational works have developed a similar theory of uncertainty, which have led some to call it “the Knight-Keynes uncertainty concept” (e.g. Davidson, 1972; Hodgson, 2011). However, Packard et al. (2021) who have performed a critical review of the historical records and the works of Keynes and Knight, argue that fundamentally the two thinkers differed in their political views, scientific epistemologies, their ontological beliefs and ultimately their views on uncertainty. The authors believe that historically Keynes’ and Knight’s theories of uncertainty and their political philosophies in general have been wrongfully homogenized, which is in stark contrast to the differences in the schools of thought each of them laid the foundations of. Although it is undeniable there are similarities between their concepts of uncertainty, there are also nuanced differences, which led them to different conclusions about the nature of uncertainty and how one should deal with it.

3 Keynesian Uncertainty

Lawson (1985) gives an excellent summary of Keynes’ theory of uncertainty as is found in *Treatise on Probability* (1921) and his most acclaimed work *The General Theory of Employment, Interest and Money* (1936), which has a separate chapter devoted to the concept of uncertainty. Keynes himself believed that his chapter on uncertainty in economics was one of the truly innovative parts of his General Theory (Patinkin and Leith, 1977).

When Keynes discusses uncertainty in his General Theory, he uses the term exclusively in respect to future outcomes (Lawson, 1985). Keynes (1921) emphasized the existence of a logical relationship between two sets of propositions called a probability relation. The probability relation is between a conclusion and a premise or evidence. Finding new evidence does not necessarily render the initial relation wrong, but can lead to a new or updated relation. In essence, the probability relation is not a property of the nature of reality, but rather an inductive framework one could use to theorize about the world.

Keynes discriminates between two cases where knowledge about the probability relation is absent. In the first the probability relation is completely unknown and the second is when the associated probabilities are numerically immeasurable or indeterminate. In the first case, he meant that a probability relation is unknown no matter how much evidence one obtains.

> ‘To say…that a probability is unknown ought to mean that it is unknown to us through our lack of skill in arguing from given evidence. The evidence itself justifies a certain degree of knowledge, but the weakness of our reasoning power prevents our knowing what this degree is’ (Keynes, 1921, p. 34).

About the second case, he argues that either not all probabilities are measurable and not all pairs of probabilities are comparable in an ordinal manner. Here Keynes implies that even given some evidence, there is no method of calculation, which is available.

> ‘Some cases, therefore, there certainly are in which no rational basis has been discovered for numerical comparison. It is not the case here that the method of calculation, prescribed by theory, is beyond our powers or too laborious for actual application. No method of calculation, however impracticable, has been suggested’ (Keynes, 1921, p. 32).

Furthermore, Keynes makes a distinction between probability and uncertainty, which is perhaps why many have found his view very similar to Knight’s distinction between risk and uncertainty.

> ‘By "uncertain" knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty; nor is the prospect of a Victory bond being drawn. Or, again, the expectation of life is only slightly uncertain. Even the weather is only moderately uncertain. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention, or the position of private wealth-owners in the social system in 1970. About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know.’ (Keynes, 1937, p. 213-214)

The distinction between the probable and the uncertain is indeed on the surface very similar to Knight’s dichotomy. However, Keynes defined certainty of a rational belief as the confidence in the belief in combination with the correctness of the belief. In this sense, certainty is achieved by obtaining knowledge about the objective reality. This same view is expressed in the last two sentences of the quote above, which point that only knowledge acquired through a scientific method could lead to a “calculable probability” and a reduction of uncertainty. This leads to the conclusion that Keynes’ view of uncertainty is epistemic – uncertainty as a result from limitations on the knowledge and information of an economic agent possesses, which Packard et al. (2021) also support.

Finally, Packard et al. (2021) argue that Keynes was an objectivist, positivist and a determinist, who believed in a objective deterministic reality, which should be studied via empiricism and rationalism. In this light, the problem of uncertainty arising from the limitations, ignorance and irrationality of an actor, can be continually mitigated through systematic scientific inquiry and the constant pursuit of new evidence. However, such a view would reject the notion that certain aspects of reality might be inherently uncertain even in the case of perfect knowledge.

4 Knightian Uncertainty

Knight’s laid out a more explicit and well-structured theory of uncertainty in his work *Risk, Uncertainty and Profit* (1921), which is probably why it is so widely cited and used across the economics field. Gerunov (2019) presents a great summary of Knight’s views on the possible situations with respect to uncertainty an economic agent would generally face. In the economic context, there are situations of absolute certainty, which however are rare and trivial and require no deep analysis or risk assessment. There are situations of risk in which economic agents possess the knowledge of well-defined outcomes and a set of probabilities relating to each outcome. In order to make a decision in this situation, one should measure the risk by taking into account the probability distributions of the possible outcomes. There are also two situations of uncertainty, characterized by lack of information about the outcomes and the probabilities. In the first there is knowledge about the specific outcomes, but no knowledge about the set of probabilities attached to these outcomes. The second situation of uncertainty is one where the economic agents possess no knowledge about the outcomes or the related probabilities. Thus, Knight distinguishes between situations of risk and uncertainty, on the basis of the available information to the economic agent.

> 'Uncertainty must be taken in a sense radically distinct from the familiar notion of Risk, from which it has never been properly separated.... The essential fact is that 'risk' means in some cases a quantity susceptible of measurement, while at other times it is something distinctly not of this character; and there are far-reaching and crucial differences in the bearings of the phenomena depending on which of the two is really present and operating.... It will appear that a measurable uncertainty, or 'risk' proper, as we shall use the term, is so far different from an unmeasurable one that it is not in effect an uncertainty at all.' (Knight, 1921, p. 19)

One could see a resemblance to Keynes' own ideas about uncertainty and the cases where the probability relation is unknown or the probabilities are immeasurable. However, Knight's view of uncertainty came from a different viewpoint of a liberal, who was skeptical of positivism and objectivism and rejected determinism (Knight, 1925; Packard et al. 2021). As Gordon (1974) interprets Knight's view on the relationship between determinism and uncertainty "man is a free and thinking being because of uncertainty, yet it is uncertainty which imposes limits upon his effective use of reason, a complexity that is compounded by the fact that we are uncertain also to the limits of uncertainty". This view is in stark contrast with Keynes' view that human actions are predictable in principle.

> 'It should now be clear that we cannot separate the discussion of reality from the discussion of the knowledge of reality, the nature and structure of thinking and the conditions of its validity, or the workings of "mind" (meaning minds)' (Knight, 1940, p. 11)

This excerpt from an article titled '*"What is truth" in Economics?'* and other thoughts shared by Knight in the same article portray him as one who is also very much a subjectivist.

> 'Perhaps the most interesting epistemological datum for economic theory is that we actually both know … that maximum efficiency is … achieved through ideal allocation of allocable resources … and also know that no individual achieves this maximum … This divergence arises because ignorance, error, and "prejudice" in innumerable forms affect real choices.' (Knight, 1940, p. 20)

This view is again in contrast with Keynes' view of the predictability of human actions, but also implies about the inherent stochasticity in the social realm. According to Packard et al. (2021) for Knight the social realm does not to permit simple scientific explanation and is characterized by complexity and paradoxes. Therefore, uncertainty is a fundamental principle of social scientific epistemology.

According to Packard et al. (2021), Knight's view of uncertainty, in light of his worldview and especially his opposition to positivism in economic analysis, is an aleatoric one – related to the inherent stochasticity and unpredictability of processes, which is also irreducible by the accumulation of evidence. Others, among which Friedman (2007), interpret Knightian uncertainty as epistemic and thus similar to Keynes' view, an interpretation which has dominated mainstream economics. However, Knight (1921) himself implies on numerous occasions that in economics you have "a larger proportion of factors … of the variable and fluctuating sort" and states that "it is a world of change in which we live, and a world of uncertainty."

It appears that Knight believed that uncertainty was inherent in reality, which points to an uncertainty in the aleatoric sense. However, he often referred to the subjective, the psychological and the knowledge of the individual, which means he also perceived uncertainty in the epistemic sense. In his own words, he seems to extend his view in the epistemic sense, not only to the future, but to the present as well, which seems to coincide with ideas from psychology.

> 'We do not perceive the present as it is and in its totality, nor do we infer the future from the present with any high degree of dependability, nor yet do we accurately know the consequences of our own actions.' (Knight, 1921, p. 202)

Perhaps, in his rejection of positivism and the treatment of social science and economics in particular as an exact science, Knight achieved a more broad and comprehensive view on uncertainty, compared to Keynes, who believed in the predictability of human behavior and perceived uncertainty as almost exclusively epistemic in nature.

Finally, it is important to mention that Knight was very critical of Keynes' magnum opus *The General Theory of Employment, Interest and Money.* In his review, Knight (1937) commented on the general theory developed by Keynes in an almost hostile manner. He did not refer specifically to the chapter on uncertainty, but rather focused on Keynes' idea of liquidity preference. Knight believed that the stock of capital is the main determinant of interest rates instead of the stock of money. Therefore, reading Knight's review it is not hard to conclude that he was very much an opponent of Keynes' ideas.

In light of these nuances and differences in views, which are evident in the theories of uncertainty of Knight and Keynes it is appropriate to turn to a more contemporary classification of uncertainty, which was already mentioned, but was not elaborated – the division of uncertainty into aleatoric and epistemic.

5 Aleatoric and Epistemic Uncertainty

The division of uncertainty into aleatoric and epistemic seems to have originated from the field of engineering (Hora, 1996; Faber, 2005; Kiureghian and Ditlevsen, 2009), although it is hard to find a scientific study which mentions the origins of the terms. Due to its practicality it has been used in computer science and machine learning (Dutta, 2013; Shaker and Hüllermeier, 2020; Hüllermeier and Waegeman, 2021, Lai et al. 2022) and in economics as well, although rarely (Dequech, 2004; Packard and Clark, 2020; Curto, Acebes and González-Varona, 2022).

Probabilistic approaches have been traditionally perceived as the best way to represent and deal with uncertainty in fields like statistics, economics and machine learning. However, the necessity for classifying uncertainty arises from the fact that the sources of uncertainty might be inherently different, and thus capturing the knowledge about uncertainty in a single probability distribution might be inadequate (Hüllermeier and Waegeman, 2021).

5.1 Aleatoric Uncertainty

Aleatoric (or statistical) uncertainty refers to the uncertainty related to the inherent stochasticity or randomness in data generating processes or in the outcome of an experiment (Hora, 1996, Hüllermeier and Waegeman, 2021). The term comes from the Latin word *alea*, which means bone or dice, since bones were used as dices in gambling games (Lewis and Short, 1879). Coin flipping and the degree of unpredictability related to it is a good example. No two flip coins are fully equivalent in terms of all the disturbing factors, which can affect them like the initial force applied, the initial angle of the coin, air resistance, gravity and such. Similarly, if an archer's bow is used to make multiple arrows shots, which duplicate perfectly each launch in terms of acceleration, altitude, direction and velocity, the arrow will not hit the same point on the target due to random and complex dynamics of the vibrations of the arrow shaft. The knowledge of these dynamics cannot be determined sufficiently as to eliminate the resulting randomness. Therefore, a property of aleatoric uncertainty is its irreducibility. It is assumed that with the current knowledge about a data generating process there is a degree of uncertainty that cannot be reduced by accumulating more evidence or changing or refining the statistical model of the given process. However, this uncertainty can be identified and quantified. A subtle convenience of the concept of aleatoric uncertainty is that it can be justified despite the ontological view of its user. It fits and can be used in the contexts of both determinism and indeterminism.

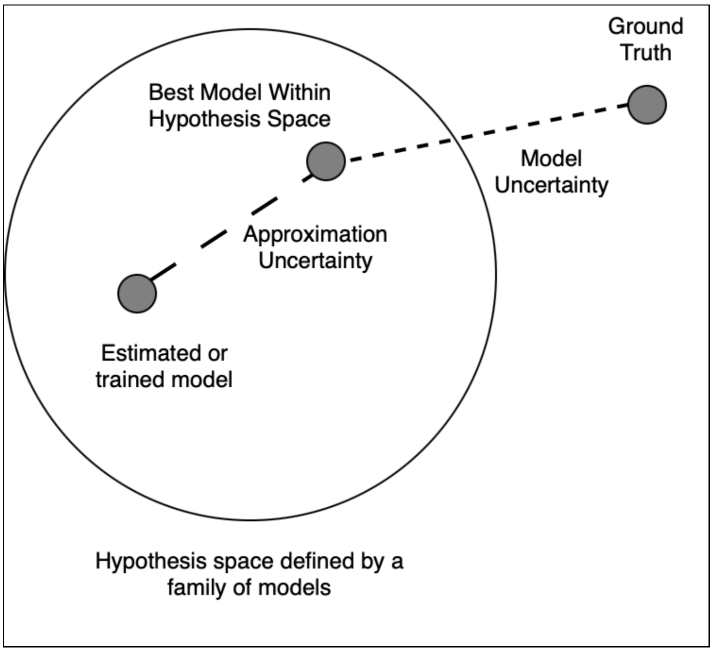
The concept of aleatoric uncertainty can be applied to the field of economics as well, since the economy being a complex system has been observed to exhibit stochasticity on various levels (Nelson and Plosser, 1982; King et al. 1987; Shiller, 1987; van Aarle and Kappler, 2012) and has been traditionally modelled as a stochastic dynamic system, subject of noise (Phelps, 1967; Friedman, 1968; Lucas and Sargent, 1979; Calvo, 1983; Diamond, 1965; Campbell and Mankiw, 1989). It could be argued that every economic indicator exhibits such inherent stochasticity, which can be a result of the nature of the measurement on the one hand, and the underlying factors which affect the data generating process on the other. For example, the measurement of GDP is fundamentally an approximation, an interaction of a multitude of factors and a reconciliation between two separate approaches – the production and expenditure approaches. The sheer complexity of the factors involved in the calculation can lead to a degree of stochasticity. On the other hand, as more data comes out for the underlying factors, GDP numbers get revised and thus change over time. Thus the GDP number for a specific quarter in most European countries gets updated three times after its flash release. After the flash release, there is a first release, second release and usually an annual revision as the data for the whole year comes out. The process for inflation and unemployment measurement although probably less complex involves also a significant number of dependencies to other factors and is a subject of revisions. This example demonstrates that measurement of core economic indicators is a subject of such aleatoric uncertainty even if the idea of an inherent stochasticity is put aside.

Generally, as systems become more complex, they tend to be less predictable and characterized by a greater degree of uncertainty. An example from classical mechanics is the comparison of the two-body-problem and the three-body-problem. The two-body-problem is perfectly predictable. However, adding another body and excluding any constraints, we get a chaotic unpredictable system (Barrow-Green, 1996). It is the complex interactions of the elements of the system, which become the source of this uncertainty. This comparison also shows that a fully deterministic system can be perfectly unpredictable. Since, the economy is a complex system characterized by the interaction of a multitude of economic agents with unique pursuits and desires, it is only natural to accept its unpredictability and inherent uncertainty.

5.2 Epistemic Uncertainty

The word epistemic originates from the Ancient Greek word *επιστημη* (epistémē), which means knowledge. Epistemic uncertainty arises from the lack of knowledge of the observer, economic decision-maker or forecaster regarding the data generating process (Hüllermeier and Waegeman, 2021). Epistemic uncertainty coincides very much with Keynes' view of uncertainty, which could be reduced by the accumulation of a greater evidential weight or knowledge about the system in general. Thus, in contrast to aleatoric uncertainty, a main characteristic of epistemic uncertainty is that it can be reduced in principle (Hora, 1996). If this concept is applied to a forecasting task, epistemic uncertainty might relate to uncertainty related to the family of statistical or machine learning models chosen for a given task or the amount of data available. One could reduce this uncertainty, by picking a better model of the data generating process or by collecting more relevant data, in order to estimate the model parameters more precisely.

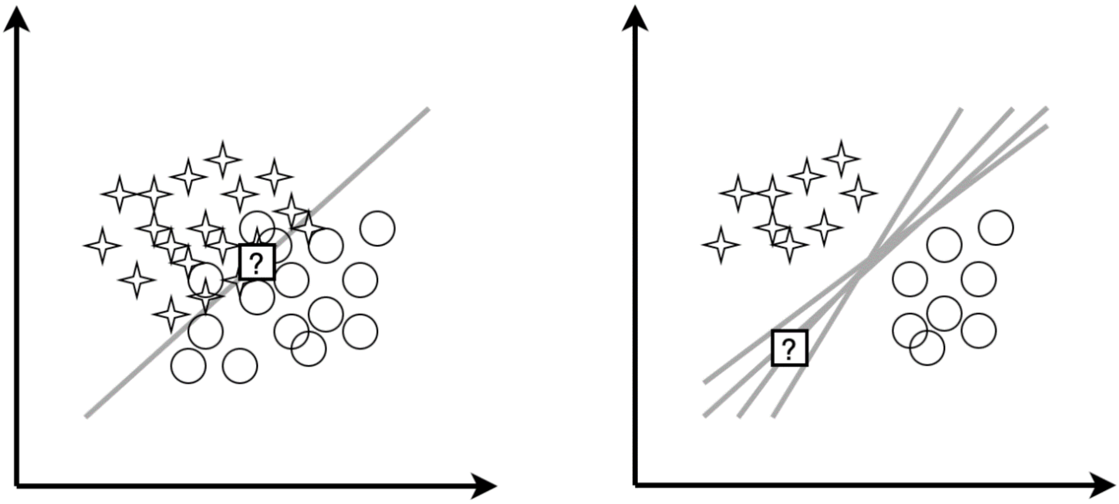
Figure 1: Types of Epistemic Uncertainty



Source: Author, Hüllermeier and Waegeman (2021)

Epistemic uncertainty can be further reduced into at least two more sub-categories: model uncertainty and approximation uncertainty (Hüllermeier and Waegeman, 2021) as shown on figure 1. Approximation uncertainty relates to the uncertainty surrounding the model parameters and can be expressed as the difference in chosen hypothesis or model and the optimal hypothesis within the chosen hypothesis space (family of models). Model uncertainty refers to the choice of the hypothesis space or family of models in general and can be expressed as the difference between the ground truth (or the population model) and the optimal hypothesis within the hypothesis space. If the task of economic modeling or forecasting is concerned, model uncertainty refers to both the choice of a family of models, but also the predictors used for modelling. For example, using a linear model, when modelling a quadratic relationship might be a source of epistemic uncertainty of the model sub-category. A missing variable bias, might again be a source of epistemic uncertainty of the same kind. An example of approximation uncertainty in a simple linear regression context, would be uncertainty surrounding a regression coefficient, which is usually expressed via a confidence interval. According to Hüllermeier and Waegeman (2021), given a consistent estimator, asymptotically one could eliminate approximation uncertainty by increasing the number of observations.

Figure 2: Aleatoric and Epistemic Uncertainty



Source: Author, Hüllermeier and Waegeman (2021)

Figure 2 demonstrates how epistemic uncertainty differs from aleatoric uncertainty in context of a simple classification model. On the left hand side, at the point denoted by a question mark, the prediction is aleatorically uncertain, due to the overlap of the two classes in this region around the decision boundary. On the right hand side, the point denoted by a question mark is a case of epistemic uncertainty, due to the lack of knowledge about the model parameter, in turn caused by lack of enough data.

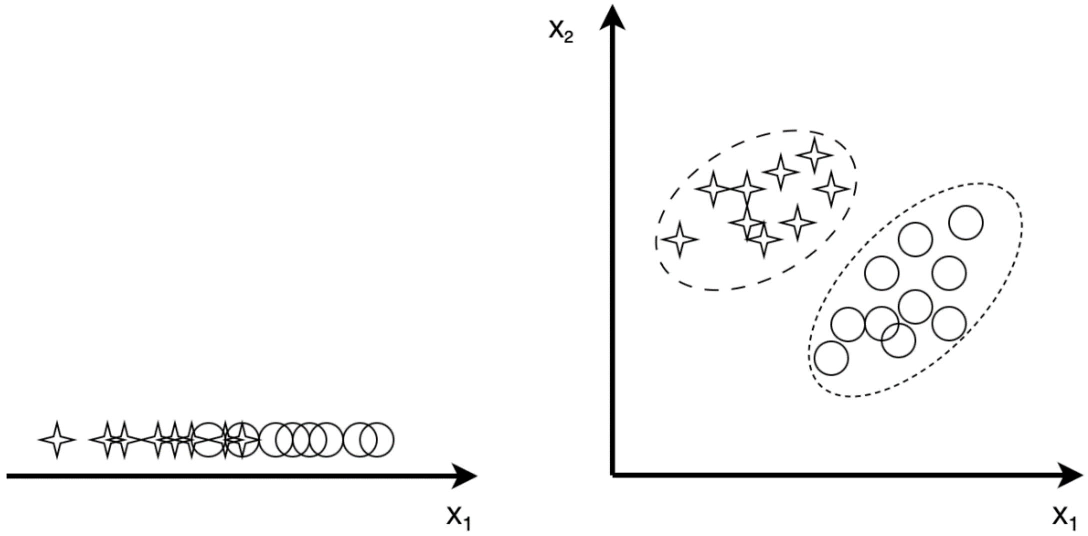
5.3 Interaction Between Aleatoric and Epistemic Uncertainty

The distinction between aleatoric and epistemic uncertainty should not be perceived as clear-cut and definitive. According to Kiureghian and Ditlevsen (2009) and Hüllermeier and Waegeman (2021) they are very much context-dependent and thus changing the context might change the nature of the uncertainty.

In figure 3, it is demonstrated that given a univariate dataset, we have overlapping classes in a certain interval of the independent variable. This would be perceived as aleatoric uncertainty. However, by including a second variable in the dataset and thus effectively changing the context, in a higher-dimension space, there are two clearly separable classes and the uncertainty is resolved.

The example in figure 3 demonstrates that aleatoric uncertainty can be reduced and even eliminated by changing the context defined by the inputs and outputs in the dataset, the choice of hypothesis and the probability measure. As Hüllermeier and Waegeman (2021) point out, a toss of a coin can be generally perceived as aleatorically uncertain, unless all physical conditions are taken into account (Laplace's demon). In this case, the known laws of physics can be applied to predict the outcome of the coin toss, but in this case the uncertainty would be epistemic, since it would be the limitations of the known physical model, which would be the source of the uncertainty. This points out to the fact that the exact definition and classification depends very much on the perspective of the observer or forecaster and her/his way of approaching the modeling task. This example, might lead one to believe that in essence all uncertainty is epistemic and as Einstein, Podolski and Rosen thought it is all about finding the "hidden variables". However, this goes to the debate between determinism and indeterminism, which is ongoing and unresolved in many branches of science.

Figure 3: Resolving Aleatoric Uncertainty



Source: Author, Hüllermeier and Waegeman (2021)

6 Definition and Classification of Uncertainty for Economic Forecasting

Based on the discussion so far, uncertainty could be defined as the lack of certain knowledge or understanding about the realizations of a given situation or event. Uncertainty can be considered a fundamental characteristic of the future, but as Knight (1921) points out it could extend to the present as well, especially when complex interactions are concerned. Uncertainty stems both from the stochasticity inherent in the environment as whole or in a given data-generating process, as well as from the limited knowledge of the observer or forecaster, who attempts to model the said data-generating process. Gneiting et al. (2007) claims "forecasts characterize and reduce but generally do not eliminate uncertainty." Based on the fact that forecasts provide additional knowledge about the realization of future events assuming they are accurate and unbiased, this claim could be accepted as true. In light of this, it is most appropriate if forecasts are probabilistic in nature in order to provide the most information about future realizations, which can be expressed as a probability distribution over future events (Dawid, 1984).

With respect to this definition, the separation of uncertainty into aleatoric and epistemic is appropriate for the purpose of economic forecasting, due to its practicality, relative concreteness and comprehensiveness. While Knight's description is widely used in economics, its practicality is limited when it comes to the task of forecasting specifically. To a great extent the limited notation used here follows from Hüllermeier and Waegeman (2021) who define it for the case of supervised learning and predictive uncertainty. However, it is adjusted as to be more relevant to a more traditional representation for the economics and econometrics literature.

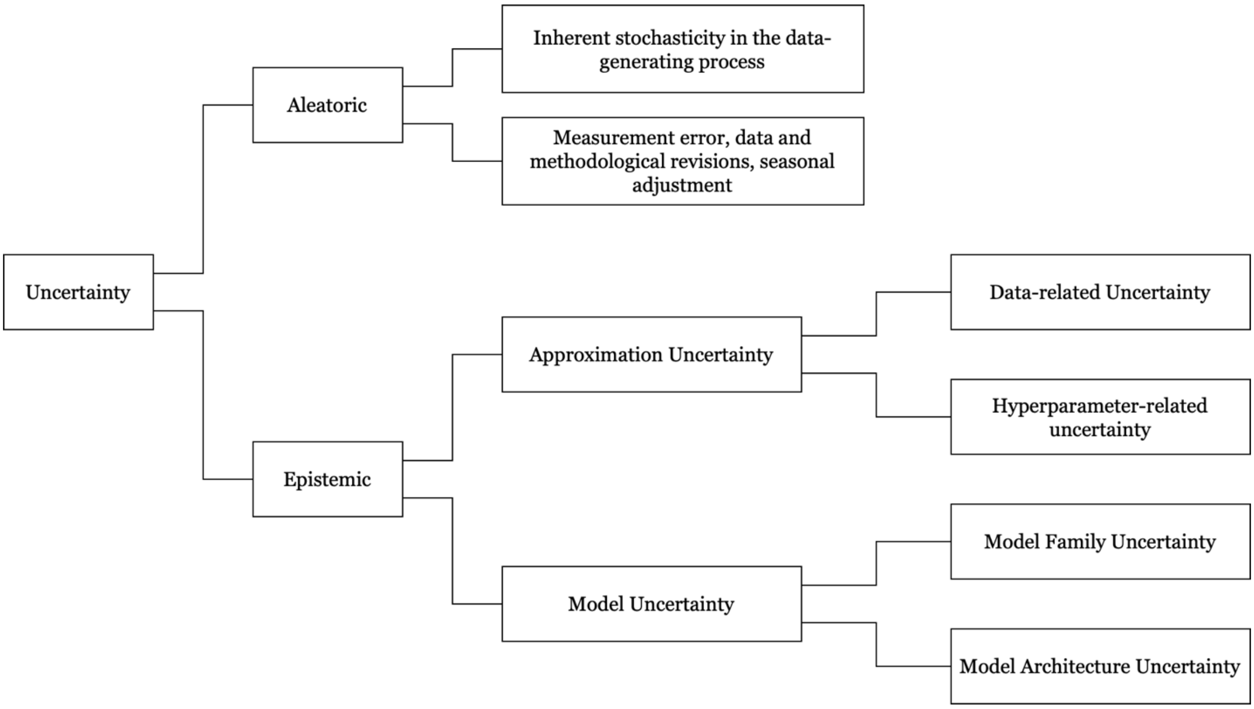
When we consider aleatoric uncertainty we can express it in the following way. First and foremost, $\mathcal{D}^T = (Y, X, X_{T-h})$ is the information set available to the forecaster. Y is vector of the target variable or dependent variable. X is a vector of the predictors or the independent variables up to time T and X_{T-h} is a vector of predictors available after time T in order to generate forecasts Y_{T-h} for a forecast horizon with length h. We can measure the aleatoric uncertainty in a simple way using the unconditional or empirical distribution of Y given by $f(Y)$ however this allows to make only naive forecasts. The predictive distribution $f(Y_{T+h}|\mathcal{D}^T)$ is a what is referred to as a density forecast since it describes the distribution of Y for future values and is conditional on the information set.

If we assume that $f(Y_{T+h}|\mathcal{D}^T)$ is generated using a hypothesis or model $H(\theta)$ with a vector of parameters θ. The epistemic approximation uncertainty can be defined as the uncertainty around the parameters $f(\theta|\mathcal{D}^T)$. As was mentioned asymptotically increasing the size of the information set $N \to \infty$ would in principle eliminated this approximation uncertainty. However, another important aspect of approximation uncertainty is hyperparameter optimization. Given a fixed hypothesis space, optimizing the hyperparameters of the learning algorithm can reduce the distance towards the optimal model within the hypothesis space and thus reduce epistemic approximation uncertainty. Therefore, two subtypes of approximation uncertainty are the data-related uncertainty and the hyperparameter-related uncertainty.

With respect to model uncertainty, two more subtypes of uncertainty can be identified. One related to the family of statistical or machine learning models used and one associated with the model architecture. The latter is relevant to the growing use of artificial neural networks, where different architectures or modifications of an architecture can be thought of as defining a new hypothesis space within the same family of models. Since the ground truth or the population distribution $f^*(Y_{T+h}|\mathcal{D}^T)$ is unknown, reducing model uncertainty is not straightforward. One possible way to reduce epistemic model uncertainty would be to compare different families of models and pick the one with the best evaluation performance, assuming that a better performance given some evaluation metric means closer to the ground truth. Such evaluation is usually done in an out-of-sample or pseudo out-of-sample setting. In economics generally, the statistical definition of ground truth is used which refers to the population. The population parameter or distribution is the ground truth. In the machine learning field, a slightly different view is taken as to what is ground truth compared to economics. An assumption is made about an optimal hypothesis sometimes called a Bayes predictor, which is the optimal predictor given the information set. This may or may not refer in any way to the population parameter and represents a practical framework of thinking, in the context of prediction.

In the context of using artificial neural networks one could optimize the architecture of the model via experimentation or using a search algorithm similarly to the hyperparameter optimization process. Thus, in principle one would reduce epistemic model uncertainty. Given the simple definition given above figure 4 presents the classification of uncertainty in the context of an economic forecasting task.

Figure 4: Classification of Uncertainty for Economic Forecasting



Source: Author

7 Conclusion

In Christopher Sims' Nobel Prize lecture in 2011, one of the main themes was the necessity of a probability approach to inference and forecasting, which was realized and outlined by Haavelmo in two seminal papers back in 1943 and 1944. According to Sims (2011), current methods in macroeconomics still lack in this regard and especially with hindsight to the global financial crisis of 2008-2009.

Others have also pointed out the advantage and necessity of using density forecasts instead of point forecasts (Anscombe, 1968; Granger and Pesaran, 2000; Tay and Wallis, 2000; Clements, 2005; Timmermann, 2006; Gneitling, 2011; Gaglianone and Lima, 2011). According to Tay and Wallis (2000) a density forecast "provides a complete description of the uncertainty associated with a prediction and stands in contrast to a point forecast which by itself contains no description of the associated uncertainty". Clements (2005) points out that an assessment of the uncertainty is important especially for policy-making decisions, which is based on forecasting. Moreover, according to Diebold et al. (1998) historically most economic forecasts were provided in the form of point forecasts and there was little interest in density forecasts. This seems to be true not only for economics, but also for the more specialized time-series forecasting field as well as is seen in the top entries in the M3 and M4 competitions (see Makridakis, 2000, 2018).

While it seems that the advantage of density forecasting over point-forecasts is established, it is important to analyze uncertainty according to its source. Hüllermeier and Waegeman (2021) argue that uncertainty analysis "for individual instances, is arguably important and practically more relevant than a kind of average accuracy or confidence, which is often reported in machine learning". They give the example of medical diagnosis, where a patient would be interested in the reliability of the prediction in her/his case rather than some average reliability measure of the model. Kull and Flach (2014) also argue that assigning reliability scores to each instance is much more powerful compared to assigning an aggregate reliability score. The additional information provided by a quantification of uncertainty in this transductive way, would affect and perhaps improve decision-making due to a greater transparency and information value. One could see how this view can be applied to economic forecasting as well in order to provide greater informational value when communicating forecasts.

In this critical review, the theories on uncertainty of Keynes and Knight were reviewed from the point of view of a classification of uncertainty, which originates from engineering and is becoming predominant in machine learning. Its main premise is separating uncertainty in one inherent in the environment or related to its measurement (aleatoric) and one arising from the limitations of the forecaster and her/his knowledge (epistemic). Due to the practical convenience and transparency of this approach to uncertainty it is adopted to the task of economic forecasting. This classification of uncertainty is enriched and adjusted for the context of economic forecasting and outlines the sub-classes of aleatoric and epistemic uncertainty. Conclusively, it is established that a forecaster should not avoid an assessment of uncertainty and should attempt to dissect the uncertainty in order to increase the informational value of her/his forecasts.

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