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How Quantitatively Important are Shocks to Consumption and Income Tax Rates for Business Cycle Fluctuations? Lessons from Bulgaria (1999-2020)

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Abstract

This paper analyzes the macroeconomic effects of fluctuations in the marginal tax rates of consumption and income. To this end, stochastic tax rates are introduced as in Braun (1992), into a real-business-cycle setup augmented with a detailed government sector. The model is calibrated to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2020). The quantitative importance of the presence of stochastic taxation is investigated for the stabilization of cyclical fluctuations in Bulgaria. The quantitative effect of such shocks to the marginal tax rates is found to be very small, and thus not important for either business cycle stabilization, or public finance issues.

Keywords: business cycles, stochastic consumption and income taxes, Bulgaria

JEL Classification Codes: E24, E32

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1 Introduction and Motivation

Are movements in taxes important sources for business cycle fluctuations in Bulgaria after the introduction of the currency board arrangement? Braun (1992) says yes, at least for the case of the US.[1] Transient movements in income taxes affect labor supply and capital accumulation decisions, and trigger interesting intertemporal substitution effects. Similarly, changes in the consumption tax over time can affect the optimal consumption path. The proposal in Braun (1992) is thus taken seriously, and this paper incorporates stochastic tax rates[2] in an otherwise standard real-business-cycle (RBC) model with a detailed government sector. The model is calibrated for Bulgaria in the period 1999-2020, as Bulgaria provides a good testing case for the theory.[3] The paper then proceeds to quantitatively evaluate the effect of stochastic consumption and income taxes as a tool for business cycle stabilization, and the implications for public finances. This is the first study on the issue using modern macroeconomic modelling techniques, and thus an important contribution to the field. Unfortunately, the quantitative effects are tiny. Stochastic taxes are neither a good instrument for demand management over the cycle, nor for raising additional tax revenue.

The rest of the paper is organized as follows: Section 2 describes the model framework and describes the decentralized competitive equilibrium system, Section 3 discusses the calibration procedure, and Section 4 presents the steady-state model solution. Sections 5 proceeds with the out-of-steady-state dynamics of model variables, and compared the simulated second moments of theoretical variables against their empirical counterparts. Section 6 concludes the paper.

[1]For a survey of different tax reforms in Bulgaria and their effect on the business cycle, the reader is referred to Vasilev (2017a), Vasilev (2015b), Di Nola *et al* (2019), as well as the references therein.

[2]Braun's (1992) model setup does not include consumption taxation, given that the model is calibrated for the US. On the other hand, Braun (1992) distinguishes between a stochastic labor income tax and a stochastic capital income tax, which we will not do.

[3]Before the introduction of proportional income taxation of 10 percent in 2008, Bulgaria operated a progressive income taxation regime during the period 1993-2007 with the same effective rate. In addition, the corporate income tax rate has been reduced, in several steps, to a proportional rate of 10 percent in 2007 as well, to avoid incentives to move earnings across the income categories.

2 Model Description

There is a representative households which derives utility out of consumption and leisure. The time available to households can be spent in productive use or as leisure. The government taxes consumption spending, and levies a common proportional (”flat”) tax on labor and capital income, in order to finance wasteful purchases of government consumption goods, and government transfers. On the production side, there is a representative firm, which hires labor and capital to produce a homogenous final good, which could be used for consumption, investment, or government purchases.

2.1 Household's problem

There is a representative household, which maximizes its expected utility function

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \gamma \ln(1 - h_t) \right\} \tag{2.1}$$

where E_0 denotes household's expectations as of period 0, c_t denotes household's private consumption in period t, h_t are hours worked in period t, $0 < \beta < 1$ is the discount factor, $0 < \gamma < 1$ is the relative weight that the household attaches to leisure.

The household starts with an initial stock of physical capital $k_0 > 0$, and has to decide how much to add to it in the form of new investment. The law of motion for physical capital is

$$k_{t+1} = i_t + (1 - \delta)k_t \tag{2.2}$$

and $0 < \delta < 1$ is the depreciation rate. Next, the real interest rate is r_t, hence the before-tax capital income of the household in period t equals $r_t k_t$. In addition to capital income, the household can generate labor income. Hours supplied to the representative firm are rewarded at the hourly wage rate of w_t, so pre-tax labor income equals $w_t h_t$. Lastly, the household owns the firm in the economy and has a legal claim on all the firm's profit, π_t.

Next, the household's problem can be now simplified to

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \gamma \ln(1 - h_t) \right\} \tag{2.3}$$

s.t.

$$(1+\tau_t^c)c_t + k_{t+1} - (1-\delta)k_t = (1-\tau_t^y)[r_t k_t + \pi_t + w_t h_t] + g_t^t \tag{2.4}$$

where where τ_t^c is the time-varying tax on consumption, τ_t^y is the time-varying proportional income tax rate ($0 < \tau_t^c, \tau_t^y < 1, \ \forall t$), and g_t^t denotes government transfers. The household takes the tax schedules $\{\tau_t^c, \tau_t^y\}_{t=0}^{\infty}$, government spending categories, $\{g_t^c, g_t^t\}_{t=0}^{\infty}$, profit $\{\pi_t\}_{t=0}^{\infty}$, the realized technology process $\{A_t\}_{t=0}^{\infty}$, prices $\{w_t, r_t\}_{t=0}^{\infty}$, and chooses $\{c_t, h_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize its utility subject to the budget constraint.[4]

The first-order optimality conditions are as follows:[5]

$$c_t \ : \ \frac{1}{c_t} = \lambda_t(1+\tau_t^c) \tag{2.5}$$

$$h_t \ : \ \frac{\gamma}{1-h_t} = \lambda_t(1-\tau_t^y)w_t \tag{2.6}$$

$$k_{t+1} \ : \ \lambda_t = \beta E_t \lambda_{t+1}\Big[1 + [1-\tau_{t+1}^y]r_{t+1} - \delta\Big] \tag{2.7}$$

$$TVC \ : \ \lim_{t\to\infty} \beta^t \lambda_t k_{t+1} = 0 \tag{2.8}$$

where λ_t is the Lagrangean multiplier attached to household's budget constraint in period t. The interpretation of the first-order conditions above is as follows: the first one states that for each household, the marginal utility of consumption equals the marginal utility of wealth, corrected for the (now time-varying) consumption tax rate. The second equation states that when choosing labor supply optimally, at the margin, each hour spent by the household working for the firm should balance the benefit from doing so in terms of additional after-tax income generates, and the cost measured in terms of lower utility of leisure. The third equation is the so-called "Euler condition," which describes how the household chooses to allocate physical capital over time, taking into consideration the time-varying income tax rate. The last condition is called the "transversality condition" (TVC): it states that at the end of the horizon, the value of physical capital should be zero.

[4]Note that by choosing k_{t+1} the household is implicitly setting investment i_t optimally.

[5]We are using standard optimization methods, e.g. as in Todorova (2010).

2.2 Firm problem

There is a representative firm in the economy, which produces a homogeneous product. The price of output is normalized to unity. The production technology is Cobb-Douglas and uses both physical capital, k_t, and labor hours, h_t, to maximize static profit

$$\Pi_t = A_t k_t^{\alpha} h_t^{1-\alpha} - r_t k_t - w_t h_t, \tag{2.9}$$

where A_t denotes the level of technology in period t. Since the firm rents the capital from households, the problem of the firm is a sequence of static profit maximizing problems. In equilibrium, there are no profits, and each input is priced according to its marginal product, *i.e.*:

$$k_t \ : \ \alpha \frac{y_t}{k_t} = r_t, \tag{2.10}$$

$$h_t \ : \ (1-\alpha) \frac{y_t}{h_t} = w_t. \tag{2.11}$$

In equilibrium, given that the inputs of production are paid their marginal products, $\pi_t = 0$, $\forall t$.

2.3 Government

In the model setup, the government is levying taxes on labor and capital income, as well as consumption, in order to finance spending on wasteful government purchases, and government transfers. The government budget constraint is as follows:

$$g_t^c + g_t^t = \tau_t^c c_t + \tau_t^y [w_t h_t + r_t k_t + \pi_t] \tag{2.12}$$

Consumption- and income tax rate and government consumption-to-output ratio would be chosen to match the average share in data. Finally, government transfers would be determined residually in each period so that the government budget is always balanced.[6]

2.4 Dynamic Competitive Equilibrium (DCE)

For a given process followed by technology $\{A_t\}_{t=0}^{\infty}$ tax schedules $\{\tau_t^c, \tau_t^k\}_{t=0}^{\infty}$, and initial capital stock $\{k_0\}$, the decentralized dynamic competitive equilibrium is a list of sequences

[6]It should be evident that the stochasticity of taxation does not affect tax revenue in the steady state.

$\{c_t, i_t, k_t, h_t\}_{t=0}^{\infty}$ for the household, a sequence of government purchases and transfers $\{g_t^c, g_t^t\}_{t=0}^{\infty}$, and input prices $\{w_t, r_t\}_{t=0}^{\infty}$ such that (i) the household maximizes its utility function subject to its budget constraint; (ii) the representative firm maximizes profit; (iii) government budget is balanced in each period; (iv) all markets clear.

3 Data and Model Calibration

To characterize business cycle fluctuations in Bulgaria, we will focus on the period following the introduction of the currency board (1999-2020). Quarterly data on output, consumption and investment was collected from National Statistical Institute (2021), while the real interest rate is taken from Bulgarian National Bank Statistical Database (2021). The calibration strategy described in this section follows a long-established tradition in modern macroeconomics: first, as in Vasilev (2016), the discount factor, $\beta = 0.982$, is set to match the steady-state capital-to-output ratio in Bulgaria, $k/y = 13.964$, in the steady-state Euler equation. The labor share parameter, $1 - \alpha = 0.571$, is obtained as in Vasilev (2017d), and equals the average value of labor income in aggregate output over the period 1999-2018. This value is slightly higher as compared to other studies on developed economies, due to the overaccumulation of physical capital, which was part of the ideology of the totalitarian regime, which was in place until 1989. Next, the average labor and capital income tax rate was set to $\tau^y = 0.1$. Similarly, the average tax rate on consumption is set to its value over the period, $\tau^c = 0.2$.

Next, the relative weight attached to the utility out of leisure in the household's utility function, γ, is calibrated to match that in steady-state consumers would supply one-third of their time endowment to working. This is in line with the estimates for Bulgaria (Vasilev 2017a) as well over the period studied. Next, the steady-state depreciation rate of physical capital in Bulgaria, $\delta = 0.013$, was taken from Vasilev (2016). It was estimated as the average quarterly depreciation rate over the period 1999-2020. Finally, the process followed by the TFP process is estimated from the detrended series by running an AR(1) regression and saving the residuals. The moments of the stochastic processes for the two taxes are set to the same values. Table 1 below summarizes the values of all model parameters used in the paper.

Table 1: Model Parameters

Parameter	Value	Description	Method
β	0.982	Discount factor	Calibrated
α	0.429	Capital Share	Data average
$1-\alpha$	0.571	Labor Share	Calibrated
γ	0.873	Relative weight attached to leisure	Calibrated
δ	0.013	Depreciation rate on physical capital	Data average
τ^y	0.100	Average tax rate on income	Data average
τ^c	0.200	VAT/consumption tax rate	Data average
ρ_a	0.701	AR(1) persistence coefficient, TFP process	Estimated
ρ_c	0.701	AR(1) persistence coefficient, cons. tax	Set
ρ_y	0.701	AR(1) persistence coefficient, inc. tax	Set
σ_a	0.044	st. error, TFP process	Estimated
σ_c	0.044	st. error, cons. tax	Set
σ_y	0.044	st. error, inc. tax	Set

4 Steady-State

Once the values of model parameters were obtained, the steady-state equilibrium system solved, the "big ratios" can be compared to their averages in Bulgarian data. The results are reported in Table 2 below. The steady-state level of output was normalized to unity (hence the level of technology A differs from one, which is usually the normalization done in other studies), which greatly simplified the computations. Next, the model matches consumption-to-output and government purchases ratios by construction; The investment ratios are also closely approximated, despite the closed-economy assumption and the absence of foreign trade sector. The shares of income are also identical to those in data, which is an artifact of the assumptions imposed on functional form of the aggregate production function. The after-tax return, where $\bar{r} = (1-\tau^y)r - \delta$ is also relatively well-captured by the model. Lastly, given the absence of debt, and the fact that transfers were chosen residually to balance the government budget constraint, the result along this dimension is understandably not so close to the average ratio in data.

Table 2: Data Averages and Long-run Solution

Variable	Description	Data	Model
y	Steady-state output	N/A	1.000
c/y	Consumption-to-output ratio	0.648	0.674
i/y	Investment-to-output ratio	0.201	0.175
k/y	Capital-to-output ratio	13.96	13.96
g^c/y	Government consumption-to-output ratio	0.151	0.151
wh/y	Labor income-to-output ratio	0.571	0.571
rk/y	Capital income-to-output ratio	0.429	0.429
h	Share of time spent working	0.333	0.333
$\bar{r}$	After-tax net return on capital	0.014	0.016

5 Out of steady-state model dynamics

Since the model does not have an analytical solution for the equilibrium behavior of variables outside their steady-state values, we need to solve the model numerically. This is done by log-linearizing the original equilibrium (non-linear) system of equations around the steady-state. This transformation produces a first-order system of stochastic difference equations. First, we study the dynamic behavior of model variables to an isolated shock to the total factor productivity process, a shock to the consumption tax rate, a shock to the income tax rate, and then we fully simulate the model to compare how the second moments of the model perform when compared against their empirical counterparts.

5.1 Impulse Response Analysis

This subsection documents the impulse responses of model variables to a 1% surprise innovation to technology. The impulse response functions (IRFs) are presented in Fig. 1. As a result of the one-time unexpected positive shock to total factor productivity, output increases upon impact. This expands the availability of resources in the economy, so uses of output - consumption, investment, and government consumption also increase contemporaneously.

At the same time, the increase in productivity increases the after-tax return on the two factors of production, labor and capital. The representative households then respond to the incentives contained in prices and start accumulating capital, and supplies more hours worked. In turn, the increase in capital input feeds back in output through the production function and that further adds to the positive effect of the technology shock. In the labor market, the wage rate increases, and the household increases its hours worked. In turn, the increase in total hours further increases output, again indirectly.

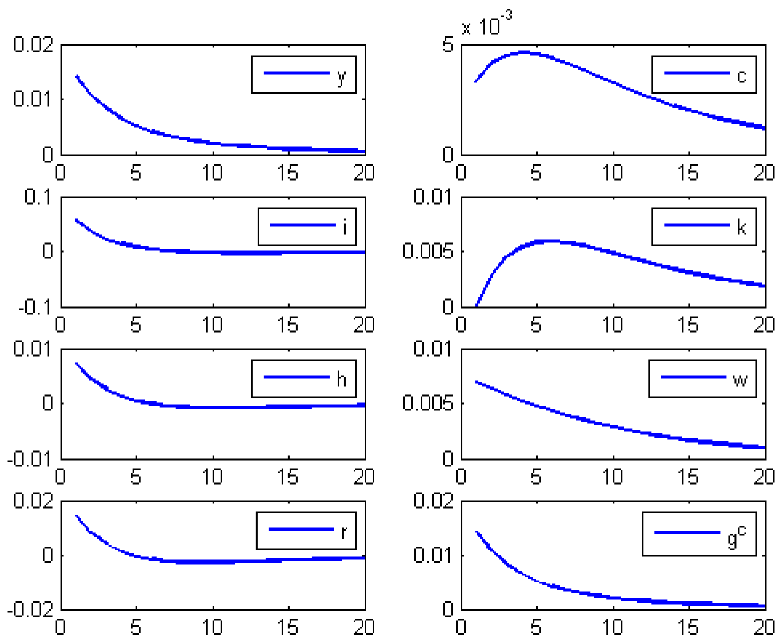

Figure 1: Impulse Responses to a 1% surprise innovation in technology

Over time, as capital is being accumulated, its after-tax marginal product starts to decrease, which lowers the households' incentives to save. As a result, physical capital stock eventually returns to its steady-state, and exhibits a hump-shaped dynamics over its transition path. The rest of the model variables return to their old steady-states in a monotone fashion as the effect of the one-time surprise innovation in technology dies out.

Next, as depicted in Fig.2 on the next page, in the case of consumption tax rate shock, despite being significant, the effect on the model economy is quite short-lived. A positive and unexpected increase in the consumption tax decreases consumption upon impact, and increases the marginal utility of wealth λ_t, thus decreasing λ_{t+1} and future consumption. Next, investment increases upon impact of the shock, and capital stock follows. Interest rate increases, as labor supply also increases (as the effective tax on labor has fallen). Wages fall from the marginal rate of substitution. Output thus increases indirectly as well, due to the higher input usage. As the consumption tax rate shock dies out, the variables return to their old steady-states in a monotone fashion, with the exception of consumption and capital, which follow hump-shaped dynamics.

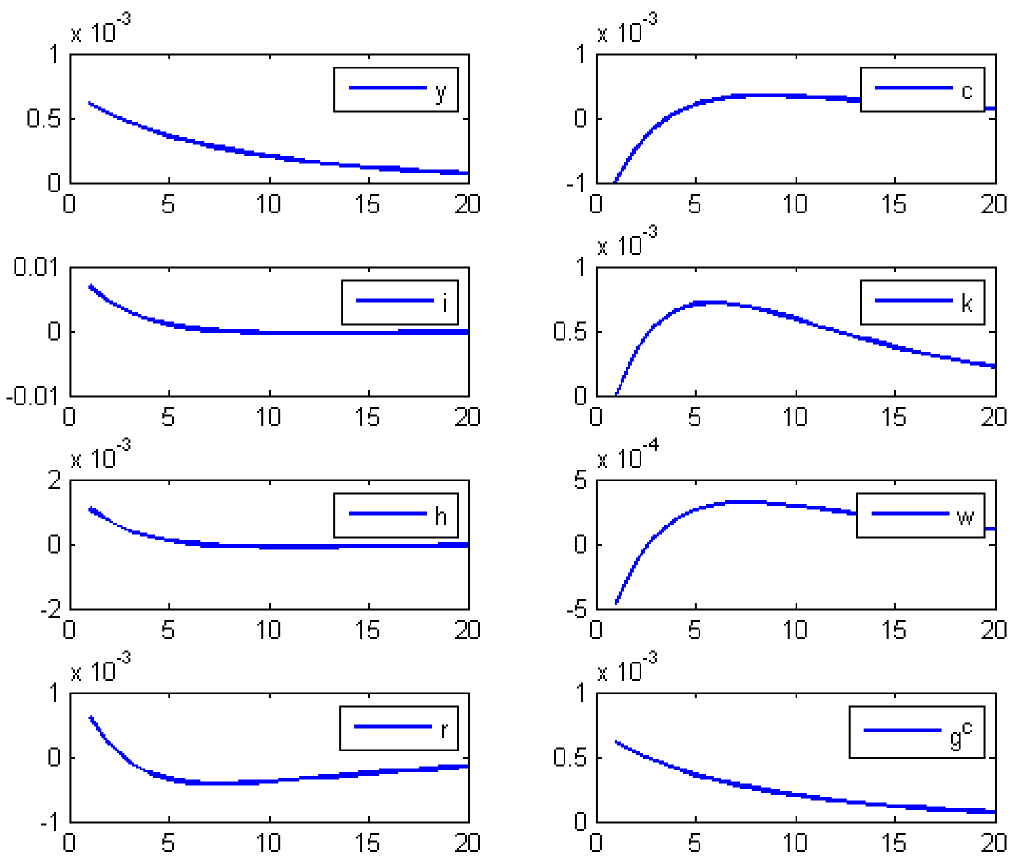

Figure 2: Impulse Responses to a 1% surprise innovation in consumption tax rate

Lastly, Fig.3 depicts the responses to model variables to a one-time income tax shock. In contrast to the consumption tax rate, the effect here is stronger, as the innovation represents a shock to labor income tax rate, and a shock to capital income tax rate at the same time. This would have a negative effect on both labor and capital labor supply. However, despite being significant, the effect is quite short-lived. As expected, a positive and unexpected increase in the income tax lowers both capital, investment and labor supply. In turn, that decreases the real interest rate, while wages go up. Output falls due to the lower input usage. As the income tax rate shock dies out, the variables return to their old steady-states in a monotone fashion, with the exception of consumption and capital, which follow hump-shaped dynamics.

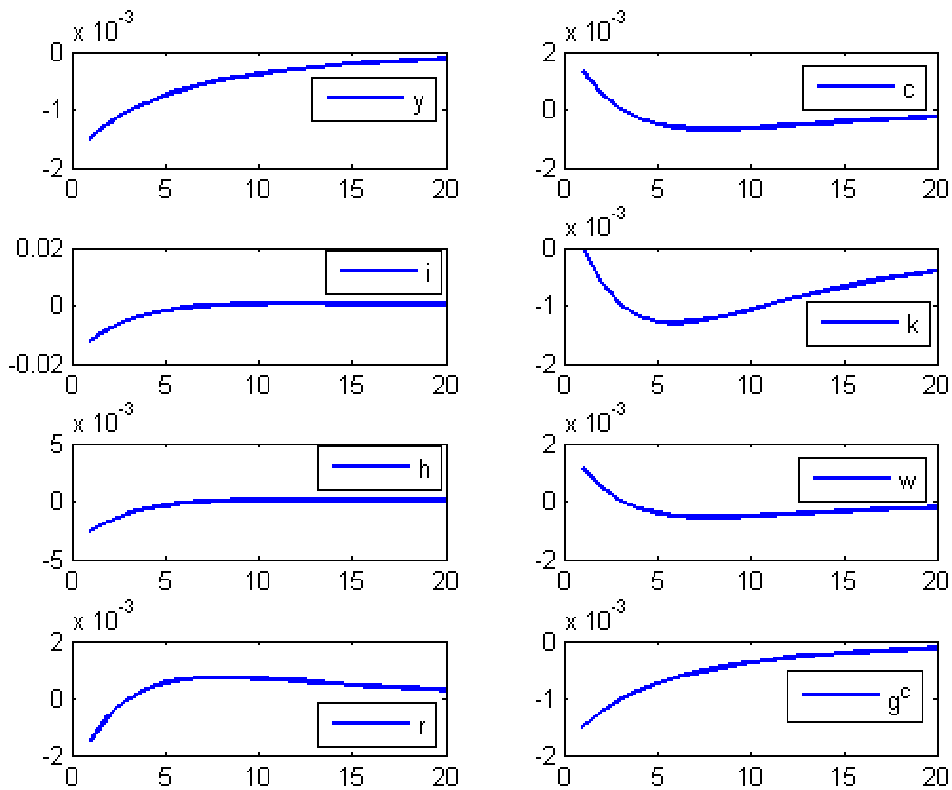

Figure 3: Impulse Responses to a 1% surprise innovation in income tax rate

5.2 Simulation and moment-matching

As in Vasilev (2017b), we will now simulate the model 10,000 times for the length of the data horizon. Both empirical and model simulated data is detrended using the Hodrick-Prescott (1980) filter. Table 3 on the next page summarizes the second moments of data (relative volatilities to output, and contemporaneous correlations with output) versus the same moments computed from the model-simulated data at quarterly frequency.The "Model" denotes the case with technology and consumption tax shocks, with technology and income tax shock, and with technology, consumption and income tax shocks, while the "Benchmark RBC" is a setup with technology shocks only and constant consumption and income tax rate. In addition, to minimize the sample error, the simulated moments are averaged out over the computer-generated draws. As in Vasilev (2016, 2017b, 2017c), all models match quite well the absolute volatility of output. By construction, government consumption in the model varies as much as output. In addition, the predicted consumption and investment volatilies are too high across all models. Still, all models are qualitatively consistent with the stylized fact that consumption generally varies less than output, while investment is more volatile than output. The model with consumption tax shocks is indistinguishable from the model without stochastic tax rate. This is because a tax on final consumption is a tax on demand, and thus has little quantitative effect. In contrast, with income tax shocks, consumption is a bit smoother and investment and hours are more volatile; this is because a varying income tax rate makes labor supply more responsive. However, the quantitative effect is again small. Overall, the models are almost indistinguishable from one another.

With respect to the labor market variables, the variability of employment predicted by the models is lower than that in data, but the variability of wages in the models is very close to that in data. This is yet another confirmation that the perfectly-competitive assumption, e.g. Vasilev (2009), as well as the benchmark calibration here, does not describe very well the dynamics of labor market variables. Next, in terms of contemporaneous correlations, the models systematically over-predicts the pro-cyclicality of the main aggregate variables - consumption, investment, and government consumption. This, however, is a common limitation of this class of models. Along the labor market dimension, the contemporaneous correlation of employment with output is too low. With respect to wages, the models predicts

Table 3: Business Cycle Moments

	Data	Model (technology and cons. tax shocks)	Model (technology and income tax shocks)	Model (all shocks)	Benchmark RBC
σ_y	0.05	0.05	0.05	0.05	0.05
σ_c/σ_y	0.55	0.82	0.81	0.81	0.82
σ_i/σ_y	1.77	2.35	2.42	2.41	2.35
σ_g/σ_y	1.21	1.00	1.00	1.00	1.00
σ_h/σ_y	0.63	0.28	0.30	0.30	0.28
σ_w/σ_y	0.83	0.86	0.85	0.85	0.86
$\sigma_{y/h}/\sigma_y$	0.86	0.86	0.85	0.85	0.86
$corr(c, y)$	0.85	0.89	0.88	0.89	0.90
$corr(i, y)$	0.61	0.82	0.82	0.83	0.83
$corr(g, y)$	0.31	1.00	1.00	1.00	1.00
$corr(h, y)$	0.49	0.58	0.59	0.59	0.59
$corr(w, y)$	-0.01	0.96	0.95	0.95	0.96

strong cyclicality, while wages in data are acyclical. This shortcoming is well-known in the literature and an artifact of the wage being equal to the labor productivity in the model.

6 Conclusions

Stochastic consumption and income taxes are introduced into a real-business-cycle setup augmented with a detailed government sector. The model is calibrated to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2020). The quantitative importance of the presence of stochastic taxation is investigated for the stabilization of cyclical fluctuations in Bulgaria. The quantitative effect of such shocks to the marginal tax rates is found to be very small, and thus not important for either business cycle stabilization, or public finance issues.

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