Review

under the procedure for public defence of the PhD Thesis entitled: "Branching processes - optimization and applications" for acquisition of the educational and scientific degree Doctor by

Kaloyan Nikolaev Vitanov,

In the Scientific field: 4. Natural Sciences, Mathematics and Informatics, Professional field: 4.5. Mathematics, Doctoral program: Probability theory and Mathematical statistics, Department: Probability, Operations Research and Statistics (PORS), Faculty of Mathematics and Informatics (FMI), Sofia University "St. Kliment Ohridski" (SU).

This Review has been prepared by prof. Eugenia Stoimenova Dr.Sci., Institute of Mathematics and Informatics, BAS, in my capacity as a member of the scientific jury, according to Order for the defence of this PhD thesis according to Order RD-38-308/01.07.2022 of the Rector of the Sofia University "St. Kliment Ohridski".

1. General characteristics of the dissertation thesis and the presented materials

This thesis is dedicated to probabilistic models for branching processes called Multi-type Sevastyanov Branching Processes through Probabilities of Mutation between types (MSBPM). A new MSBPM model, which finds application in modeling populations avoiding degeneracy, is investigated. Optimization problems for sequential decision making involving branching processes are studied.

The thesis consists of three main chapters, a conclusion and an appendix, it is written in English with an abstract in Bulgarian and English. The total volume is 196 pages and includes a bibliography of 213 titles. The thesis is based on 4 scientific articles.

2. Short CV and personal impressions of the candidate

I know Kaloyan Vitanov briefly, only as a listener of his talks at the National Seminar on Stochastics and at international conferences. I have excellent impressions of his talks, where he has shown a very good knowledge of the topics presented. From the submitted documents, it can be seen that Kaloyan Vitanov obtained the Bachelor degree at FMI-SU and the Master degree again at FMI-SU, Master program "Probability, Actuarial Science and Statistics". He was enrolled as a full-time doctoral student at the PORS department at FMI-SU in 2018. He was dismissed with the right of defence as of 07/05/2021.

3. Content analysis of the scientific and applied achievements of the candidate, contained in the presented PhD thesis and the publications to it, included in the procedure

In the introductory chapter, the main concepts of the theory of branching processes are presented, an in-depth literature review of classical and modern results in the field of dissertation research is made.

Chapter 2 is devoted to a novel continuous time branching process model namely - the Multi-type Sevastyanov Branching Process through probabilities of Mutation between types (MSBPM). The novel characteristic of the MSBPM, with respect to the classical formulation, is the use of decomposition of the classical probabilities for a particle of type i of age u to transform into a number particles at the end of its lifespan into two components: 1) Probability $p_{ik}(u)$ for the total number of offspring, k, regardless of offspring type, of a type i particle of age u; and 2) Probability u_{ij} for mutation of an offspring particle of a type i towards a particle of type j. The main goal is towards obtaining results for the MSBPM regarding quantities that are of interest in the context of populations escaping extinction. Within the chapter, novel results for the MSBPM are obtained that are valid for the Decomposable Multi-type Sevastyanov Branching Process through probabilities of Mutation between types (DMSBPM) and some additional results that stem from the enforced decomposability are also explored.

In Chapter 3 several branching processes have been successfully incorporated into Sequential Decision Problems (SDPs) within the "Universal Modeling Framework" developed by Powell. These include Multi-type Bienayme-Galton-Watson branching process (BGW), multitype Bellman-Harris branching process with exponential lifespan distributions, multi-type Sevastyanov branching process, as well the MSBPM and its variants.

A new result concerning an efficient algorithm for finding the solution of SDPs with multitype BGW branching processes dynamics that uses Bellman's optimality equation has been obtained. Further, a novel result has been obtained for the case of the multi-type Bellman-Harris branching process with exponential lifespan distributions and as well as for the case of the Multi-type Bellman-Harris Branching Process through probabilities of Mutation between types with exponential lifespan distributions.

4. Approbation of the results

This thesis dissertation is based on 4 publications. Two of them are published in the renowned journal Stochastic Models with an impact factor, one is published in Proceedings of the BAS (also with an impact factor) and one is published in the Proceedings of the 21st European Young Statisticians meeting. All the articles are co-authored by the supervisor, and for me there is no doubt about Kaloyan Vitanov's significant contribution in them. The PhD student has reported his research at several international scientific forums and several national seminars. The quantity and quality of the publications show that the candidate exceeds the minimum national requirements (according to Art. 2b, paras. 2 and 3 of ZRASRB) and, accordingly, the additional requirements of SU "St. Kliment Ohridski" for the acquisition of an educational and scientific degree "Doctor" in the scientific field and professional direction of the procedure. There is no proven plagiarism in the submitted dissertation and scientific works under this procedure.

5. Qualities of the abstract

The 50-page abstract (in Bulgarian and English) correctly reflects the results presented in the dissertation and is prepared according to the requirements

6. Critical notes and recommendations

I have no significant criticisms regarding the results and the theorem proofs.

7. Conclusion

This thesis dissertation work presented by Kaloyan Nikolaev Vitanov contains theoretical results that represent an original contribution in the field of probability theory and mathematical statistics. I confirm that the presented thesis dissertation and the scientific publications attached to it, as well as the quality and originality of the results and achievements presented in them, meet the requirements of ZRASRB, the Regulations for its application and the relevant Regulations of SU "St. Kliment Ohridski" for the candidate's acquisition of the educational and scientific degree "Doctor" in the scientific field "Theory of Probability and Mathematical Statistics" and the professional field of

Mathematics. The candidate has in-depth theoretical knowledge and professional skills for independent research.

Based on the above, I evaluate **positively** the results and contributions in the thesis dissertation and recommend the scientific jury to award the educational and scientific degree "Doctor" to Kaloyan Nikolaev Vitanov in the field of higher education: 4. Natural sciences, mathematics and informatics; professional direction: 4.5. Mathematics ("Probability Theory and Mathematical Statistics").

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