

STATEMENT REPORT

on the procedure for defense of a dissertation on the topic:
„Toroidal compactifications of discrete quotients of the complex two-ball“
to acquire Ph. D. degree
from
Pancho Georgiev Beshkov,

Area of Higher Education: **4. Natural Sciences, Mathematics and Informatics,**

Professional Field: 4.5 Mathematics

Ph.D. program: “Algebra, topology and applications”, department Algebra, Faculty of Mathematics and Informatics (FMI), Sofia University „St. Kliment Ohridski“

The Statement report was prepared by: Assoc. Prof. Dr. Silvia Parvanova Bumova, FMI and IMI, in my capacity as a member of the scientific jury, according to Order № RD 38-141 of 04.03.2022. of the Rector of Sofia University.

1 General characteristics of the dissertation and the presented materials

The presented dissertation: contains 126 pages and consists of an introduction and 4 chapters. There are 50 titles in the bibliography.

The dissertation and the abstract include the scientific contributions of the doctoral student, approbation of the results and declaration of originality of the presented results.

The submitted materials (in electronic media) include all documents required by the Law on Development of the Academic Staff and the Regulations for its implementation, as well as those of the relevant Regulations of Sofia University for acquiring PhD degree of the Sofia University “St. Kliment Ohridski”..

According to the reference attached in accordance with the requirements of NACID, the applicant meets the minimum national requirements.

2 Data and personal impressions of the candidate.

I know Pancho Beshkov from the National Coding Seminar "Professor Stefan Dodunekov", 2018. His good performance at the seminars does not go unnoticed. My impressions of his teaching work are also excellent.

3 Analysis of the scientific and scientific-applied achievements of the candidate, contained in the presented dissertation and the publications to it, included in the procedure

The dissertation considers the smooth toroidal compactifications of non - compact factors of the complex two - dimensional sphere by a discrete subgroup Γ of the unitary group, ie. $\Gamma < U(1, 2)$, corresponding to the Hermitian form with signature (1,2) and having a finite $\Gamma(1, 2)$ -invariant volume. The corresponding subgroups Γ are called lattices of $U(1, 2)$.

The first two chapters are introductory. The necessary concepts - definitions and necessary theorems are introduced. In the first chapter, the holomorphic vector bundles over a complex manifold and their Chern classes and numbers are considered. The logarithmic equality of Bogomolov-Miyaoka-Yao for smooth toroidal compactification is also presented.

The second chapter describes the construction of a toroidal compactification of the complex two-dimensional sphere on a lattice Γ from holomorphic isometries of the complex two-dimensional sphere S^2 . If the parabolic points (which are the orbits of the rational lattice boundary points and are a finite number) are added to the quotient, a complex projective variety is obtained. This variety is not always smooth, even with a smooth quotient. Toroidal compactification is the resolution of singularities at these parabolic points.

In the third chapter, a bijection is constructed between the unramified coverings of smooth toroidal compactions (of a certain degree d) and the unramified coverings of degree d of the respective minimum models of the same degree. Non-trivial finite unramified coverings set a partial order in the set of smooth toroidal compactions, with the minimum elements being called primitive and the maximum elements saturated. Smooth toroidal compactation has been shown to dominate primitive, and necessary and sufficient conditions have been found to dominate over saturated. Saturated and primitive smooth toroidal compactions with a negative Kodaira dimension are characterized.

In the fourth chapter the contraction of smooth irreducible rational (-1) -curves on a smooth toroidal compactification to the linear surface with an elliptical base is considered. From the logarithmic

equality of Bogomolov-Miyaoka-Yao lower limits are obtained for the number of parabolic points and the Euler characteristic, as well as for the number of non totally geodesic punctured spheres.

4 Approbation of the results

The original results of the doctoral student, obtained together with his supervisor Prof. Dr. Azniv Kasparyan are described in the third and fourth chapters of the dissertation. Two articles have been published. One is in the Annual of Sofia University (presented in Chapter 3 and indexed by MathSciNet and zbMATH), co-authored with its research supervisors Prof. Dr. Azniv Kasparyan and G. Sankaran.

The second article has an impact factor (IF = 0.378 for 2020) and was published in Proceedings of BAS (presented in Chapter 4) in co-authorship with its supervisor Prof. Dr. Azniv Kasparyan. From the attached declarations of the co-authors, I believe that the candidate's contribution to the joint publications is equivalent.

The results were reported at the National Seminar on Coding Theory "Professor Stefan Dodunekov", 2018, 2019 and the Spring Scientific Session of the Faculty of Mathematics and Informatics at Sofia University "St. Kliment Ohridski ", 2019 and 2021.

There are no reported citations. The articles were published in 2019 and 2021 and this is too short a period to reflect the results in articles by other scientists.

The plagiarism test (Applications 1 and 2) shows that the received results are original and there is no plagiarism.

The scientific works fully meet the minimum national requirements (under Art. 2b, par. 2 and 3 of ZRASRB) and respectively the additional requirements of Sofia University "St. Kliment Ohridski "for obtaining the educational and scientific degree "PhD Doctor "in the scientific field and professional field 4.5 Mathematics.

The results presented by the candidate in the dissertation and scientific works to him do not repeat those of previous procedures for acquiring a scientific title and academic position.

5 Qualities of the abstract

The abstract is written in Bulgarian (20 pages) and in English (19 pages) and it is in the required volume and format. The content of the dissertation is correctly reflected and it correctly reflects the contributions of the doctoral student, as well as the approbation of his results and meets all the requirements for its preparation.

6 Critical remarks and recommendations

In my opinion the presented dissertation is quite difficult for a non-specialist in algebraic geometry to read. In order to succeed in popularizing the field in which he works and his results in the future, he must pay attention to the way the text is arranged and take more care of the reader:

- to define terms (notions) before using them,
- to give more understandable and beautiful examples even of things already done, to make it easier to understand.

Although he has given motivation, I think that the reader is not left with a clear idea of where the inspiration for the task came from, what has been done so far and the remaining unsolved tasks.

At the very beginning of the introduction there is an inaccuracy in the definition of the unitary group - $U(1,2)$. It is written that the points z, z' are from the complex 2-dimensional sphere. In the second chapter (p. 39) it becomes clear that the points of the complex two-dimensional sphere are interpreted as lines of $M_{3 \times 1}(\mathbb{C}) \setminus \{O_{3 \times 1}\}$. But this causes some discomfort.

There are also a lot of spelling mistakes.

Nevertheless, I think that the doctoral student has worked quite thoroughly and knows the field of study well. This is evidenced by the numerous citations and literature used. In the presented dissertation there are original scientific results, which are an indisputable contribution in the field, which I appreciate.

7 Conclusion

After getting acquainted with the dissertation presented in the procedure and the accompanying scientific papers and based on the analysis of their significance and the scientific and applied contributions contained in them, **I confirm** that the presented dissertation and scientific publications to it, as well as the quality and originality of the results and achievements presented in them, meet the

requirements of the Law, the Regulations for its application and the respective Regulations of Sofia University “St. Kliment Ohridski ”for obtaining by the candidate the educational and scientific degree “PhD Doctor ” in the scientific field **4. Natural Sciences, Mathematics and Informatics and Professional Field 4.5. Mathematics.** In particular, the candidate satisfies the minimum national requirements in the professional field and no plagiarism has been established in the scientific papers submitted at the competition.

Based on the above, **I recommend** the scientific jury to award Pancho Georgiev Beshkov educational and scientific degree "PhD Doctor" in scientific field **4. Natural Sciences, Mathematics and Informatics**, professional field **4.5. Mathematics.**

29.04.2022 г.

Assoc. Prof. Dr. Silvia Boumova