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Abstract

Robots are introduced into a real-business-cycle setup augmented with a detailed government sector. Robots are modeled as an imperfect substitute for labor services. The model is calibrated to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2020). The quantitative importance of the presence of robots in the economy is investigated for business cycle fluctuations in Bulgaria. In the presence of robots, wages increase, but employment falls after a technology shock. However, for plausible parameter values, the effect is predicted to be quite small.

Keywords: business cycles, robots, Bulgaria

JEL Classification Codes: E24, E32

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1 Introduction and Motivation

Recent developments in digitalization, machine learning and artificial intelligence (AI), collectively referred to as Industry 4.0, have raised new and interesting questions such as: "What is going to be the effect on the economy?" or "Are robots going to replace workers in the workplace?". Many have made the prediction that a substantial number of standard tasks can be mechanized, which will have an adverse effect on wages and employment.

These ideas are all taken seriously, and this paper incorporates robots in an otherwise standard real-business-cycle (RBC) model with a detailed government sector. Robots enter the model as a separate form of capital, which is an imperfect substitute for labor services.[1] The model is calibrated for Bulgaria in the period 1999-2020, as Bulgaria provides a good testing case for the theory. The paper then proceeds to quantitatively evaluate the effect of robots for business cycles. For plausible parameter values, the effect is predicted to be quite small. This is the first study on the issue using modern macroeconomic modelling techniques, and thus an important contribution to the field.

The rest of the paper is organized as follows: Section 2 describes the model framework and describes the decentralized competitive equilibrium system, Section 3 discusses the calibration procedure, and Section 4 presents the steady-state model solution. Sections 5 proceeds with the out-of-steady-state dynamics of model variables. Section 6 concludes the paper.

2 Model Description

There is a representative households which derives utility out of consumption and leisure. The time available to households can be spent in productive use or as leisure. The government taxes consumption spending, levies a common proportional ("flat") tax on labor and capital income, in order to finance non-productive purchases of government consumption goods, and government transfers. On the production side, there is a representative firm, which hires labor and physical and robot capital to produce a homogeneous final good, which

[1]At the same time, labor is still a complement with physical capital. As robots are substitute for labor, robots and physical capital are also complements.

could be used for consumption, investment, or government purchases.

2.1 Households

There is a representative household, which maximizes its expected utility function

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \gamma \ln(1 - h_t) \right\} \tag{2.1}$$

whereE_0 denotes household's expectations as of period 0, c_t denotes household's private consumption in period t, h_t are hours worked in period t, $0 < \beta < 1$ is the discount factor, $0 < \gamma < 1$ is the relative weight that the household attaches to leisure.[2]

The household starts with an initial stock of physical capital $k_0 > 0$, and has to decide how much to add to it in the form of new investment. The law of motion for physical capital is

$$k_{t+1} = i_t^k + (1 - \delta^k) k_t \tag{2.2}$$

and $0 < \delta^k < 1$ is the depreciation rate on physical capital. Next, the real interest rate on physical capital is r_t^k, hence the before-tax physical capital income of the household in period t equals $r_t^k k_t$.

The household also starts with an initial stock of robot capital $d_0 > 0$, and also has to decide how much to add to in the form of new investment. The law of motion for robot capital is

$$d_{t+1} = i_t^d + (1 - \delta^d) d_t \tag{2.3}$$

and $0 < \delta^d < 1$ is the depreciation rate on robot capital. Next, the real interest rate on robot capital is r_t^d, hence the before-tax robot capital income of the household in period t

[2]This utility function is equivalent to a specification with a separable term containing government consumption, e.g. Baxter and King (1993). Since in this paper we focus on the exogenous (observed) policies, and the household takes government spending as given, the presence of such a term is irrelevant. For the sake of brevity, we skip this term in the utility representation above.

equals $r_t^d d_t$.[3]

In addition to capital income, the household can generate labor income. Hours supplied to the representative firm are rewarded at the hourly wage rate of w_t, so pre-tax labor income equals $w_t h_t$. Lastly, the household owns the firm in the economy and has a legal claim on all the firm's profit, π_t.

Next, the household's problem can be now simplified to

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \ln c_t + \gamma \ln(1 - h_t) \Big\} \tag{2.4}$$

s.t.

$$(1+\tau^c)c_t + k_{t+1} - (1-\delta^k)k_t + d_{t+1} - (1-\delta^d)d_t = (1-\tau^y)[r_t^k k_t + r_t^d d_t + w_t h_t + \pi_t] + g_t^t \tag{2.5}$$

where where τ^c is the tax on consumption, τ^y is the proportional tax rate on labor and capital income ($0 < \tau^c, \tau^y < 1$), and g_t^t denotes government transfers.[4] The household takes the tax rates $\{\tau^c, \tau^y\}_{t=0}^{\infty}$, government consumption and transfers, $\{g_t^c, g_t^t\}_{t=0}^{\infty}$, profit $\{\pi_t\}_{t=0}^{\infty}$, the realized technology process $\{A_t\}_{t=0}^{\infty}$, prices $\{w_t, r_t^k, r_t^d\}_{t=0}^{\infty}$, and chooses $\{c_t, h_t, k_{t+1}, d_{t+1}\}_{t=0}^{\infty}$ to maximize its utility subject to the budget constraint.[5]

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[3]Alternatively, without a market for robot capital, the firm will be investing in robot capital itself. However, as long as the household is the claimant to all the firm's profit, the allocations from the two modeling setups will be equivalent.

[4]Again, in the absence of market for robot capital, profit earnings will be taxed as capital income. This means that capital income from robots should be taxed at the same rate as physical capital.

[5]Note that by choosing k_{t+1} the household is implicitly setting investment i_t^k optimally. Similarly, by choosing d_{t+1} the household is implicitly setting investment i_t^d optimally.

The first-order optimality conditions as as follows:[6]

$$c_t \quad : \quad \frac{1}{c_t} = \lambda_t(1+\tau^c) \tag{2.6}$$
$$h_t \quad : \quad \frac{\gamma}{1-h_t} = \lambda_t(1-\tau^y)w_t \tag{2.7}$$
$$k_{t+1} \quad : \quad \lambda_t = \beta E_t \lambda_{t+1}\Big[1+[1-\tau^y]r^k_{t+1} - \delta^k\Big] \tag{2.8}$$
$$d_{t+1} \quad : \quad \lambda_t = \beta E_t \lambda_{t+1}\Big[1+[1-\tau^y]r^d_{t+1} - \delta^d\Big] \tag{2.9}$$
$$TVC_k \quad : \quad \lim_{t\to\infty} \beta^t \lambda_t k_{t+1} = 0 \tag{2.10}$$
$$TVC_d \quad : \quad \lim_{t\to\infty} \beta^t \lambda_t d_{t+1} = 0 \tag{2.11}$$

where λ_t is the Lagrangian multiplier attached to household's budget constraint in period t. The interpretation of the first-order conditions above is as follows: the first one states that for each household, the marginal utility of consumption equals the marginal utility of wealth, corrected for the consumption tax rate. The second equation states that when choosing labor supply optimally, at the margin, each hour spent by the household working for the firm should balance the benefit from doing so in terms of additional income generates, and the cost measured in terms of lower utility of leisure. The third and forth equations are the so-called "Euler condition," which describe how the household chooses to allocate physical and robot capital over time. The last two conditions are called the "transversality condition" (TVC): they states that at the end of the horizon, the value of physical and robot capital should be zero.

2.2 Firm problem

There is a representative firm in the economy, which produces a homogeneous product. The price of output is normalized to unity. The production technology is Cobb-Douglas and uses physical capital, k_t, robot capital, d_t, and labor hours, n_t, to maximize static profit

$$\Pi_t = A_t\Big[\alpha k_t^\rho + (1-\alpha)[\theta d_t^\phi + (1-\theta)h_t^\phi]^{\rho/\phi}\Big]^{1/\rho} - r^k_t k_t - r^d_t d_t - w_t h_t, \tag{2.12}$$

where A_t denotes the level of technology in period t. The production function is constant elasticity of substitution in physical capital and the second input, where the second input is

[6] We are using standard optimization methods, e.g. as in Todorova (2010).

also CES, this time in robot capital and labor. In other words, robot capital and physical labor will be imperfect substitutes as factors of production. Next, since the firm rents both physical and robot capital from households, the problem of the firm is a sequence of static profit maximizing problems. In equilibrium, there are no profits, and each input is priced according to its marginal product, *i.e.*:

$$k_t \ : \ \alpha \frac{k_t^{\rho-1}}{\alpha k_t^\rho + (1-\alpha)[\theta d_t^\phi + (1-\theta)h_t^\phi]^{\rho/\phi}} = r_t^k, \tag{2.13}$$

$$d_t \ : \ (1-\alpha)\theta d_t^{\phi-1} \frac{[\theta d_t^\phi + (1-\theta)h_t^\phi]^{(\rho/\phi)-1}}{\alpha k_t^\rho + (1-\alpha)[\theta d_t^\phi + (1-\theta)h_t^\phi]^{\rho/\phi}} = r_t^d, \tag{2.14}$$

$$h_t \ : \ (1-\alpha)(1-\theta)h_t^{\phi-1} \frac{[\theta d_t^\phi + (1-\theta)h_t^\phi]^{(\rho/\phi)-1}}{\alpha k_t^\rho + (1-\alpha)[\theta d_t^\phi + (1-\theta)h_t^\phi]^{\rho/\phi}} = w_t. \tag{2.15}$$

In equilibrium, given that the inputs of production are paid their marginal products, $\pi_t = 0$, $\forall t$.

2.3 Government

In the model setup, the government is levying taxes on labor and capital income, as well as consumption, in order to finance spending on wasteful government purchases, and government transfers. The government budget constraint is as follows:

$$g_t^c + g_t^t = \tau^c c_t + \tau^y [w_t h_t + r_t^k k_t + r_t^d d_t + \pi_t] \tag{2.16}$$

consumption- and income tax rate and government consumption-to-output ratio would be chosen to match the average share in data. Finally, government transfers would be determined residually in each period so that the government budget is always balanced.

2.4 Dynamic Competitive Equilibrium (DCE)

For a given process followed by technology $\{A_t\}_{t=0}^\infty$ tax schedules $\{\tau^c, \tau^y\}_{t=0}^\infty$, and initial capital stocks $\{k_0, d_0\}$, the decentralized dynamic competitive equilibrium is a list of sequences $\{c_t, i_t^k, i_t^d, k_t, d_t, h_t\}_{t=0}^\infty$ for the household, a sequence of government purchases and transfers $\{g_t^c, g_t^t\}_{t=0}^\infty$, and input prices $\{w_t, r_t^k, r_t^d\}_{t=0}^\infty$ such that (i) the household maximizes its utility function subject to its budget constraint; (ii) the representative firm maximizes profit; (iii) government budget is balanced in each period; (iv) all markets clear.

3 Data and Model Calibration

To characterize business cycle fluctuations in Bulgaria, we will focus on the period following the introduction of the currency board (1999-2020). Quarterly data on output, consumption and investment was collected from National Statistical Institute (2021), while the real interest rate is taken from Bulgarian National Bank Statistical Database (2021). The calibration strategy described in this section follows a long-established tradition in modern macroeconomics: first, as in Vasilev (2018), the discount factor, $\beta = 0.982$, is set to match the steady-state capital-to-output ratio in Bulgaria, $k/y = 13.964$, in the steady-state Euler equation. The labor share parameter, $1 - \alpha = 0.571$, is obtained as in Vasilev (2017d), and equals the average value of labor income in aggregate output over the period 1999-2018. This value is slightly higher as compared to other studies on developed economies, due to the overaccumulation of physical capital, which was part of the ideology of the totalitarian regime, which was in place until 1989. Next, the average labor and capital income tax rate was set to $\tau^y = 0.1$. Similarly, the average tax rate on consumption is set to its value over the period, $\tau^c = 0.2$. We set $\rho = -1$ to increase the substitutability between capital and labor. We assume that robots are strong substitute for labor ($theta = 0.5$), but that their effect is initially weak ($\phi = -1$).

Next, the relative weight attached to the utility out of leisure in the household's utility function, γ, is calibrated to match that in steady-state consumers would supply one-third of their time endowment to working. This is in line with the estimates for Bulgaria (Vasilev 2017a) as well over the period studied. Next, the steady-state depreciation rate of physical capital in Bulgaria, $\delta = 0.013$, was taken from Vasilev (2016). It was estimated as the average quarterly depreciation rate over the period 1999-2014. Finally, the process followed by the TFP process is estimated from the detrended series by running an AR(1) regression and saving the residuals. Table 1 below summarizes the values of all model parameters used in the paper.

Table 1: Model Parameters

Parameter	Value	Description	Method
β	0.982	Discount factor	Calibrated
α	0.429	Capital Share	Data average
$1-\alpha$	0.571	Labor Share	Calibrated
γ	0.873	Relative weight attached to leisure	Calibrated
δ^k	0.013	Depreciation rate on physical capital	Data average
δ^d	0.013	Depreciation rate on physical capital	Data average
τ^y	0.100	Average tax rate on income	Data average
τ^c	0.200	VAT/consumption tax rate	Data average
ρ	-1.000	Substitutability capital and labor	Set
ϕ	-1.000	Substitutability robots and labor	Set
θ	0.500	Weight robots	Set
ρ_a	0.701	AR(1) persistence coefficient, TFP process	Estimated
σ_a	0.044	st. error, TFP process	Estimated

4 Out of steady-state model dynamics

Since the model does not have an analytical solution for the equilibrium behavior of variables outside their steady-state values, we need to solve the model numerically. This is done by log-linearizing the original equilibrium (non-linear) system of equations around the steady-state. This transformation produces a first-order system of stochastic difference equations. First, we study the dynamic behavior of model variables to an isolated shock to the total factor productivity process, and then we fully simulate the model to compare how the second moments of the model perform when compared against their empirical counterparts.

4.1 Impulse Response Analysis

This subsection documents the impulse responses of model variables to a 1% surprise innovation to technology. The impulse response functions (IRFs) are presented in Fig. 1. As a result of the one-time unexpected positive shock to total factor productivity, output increases upon impact. This expands the availability of resources in the economy, so uses of output - consumption, physical capital investment, investment in robots, and government consumption also increase contemporaneously.

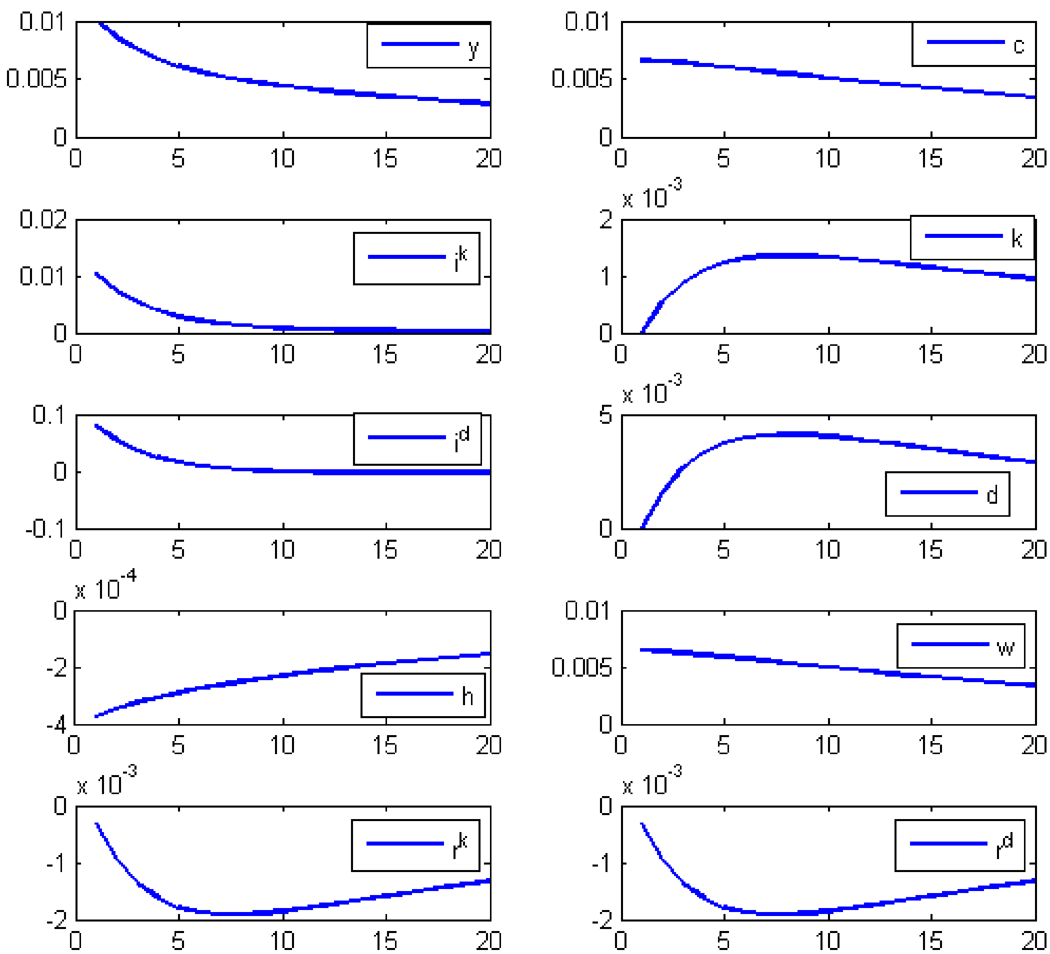

Figure 1: Impulse Responses to a 1% surprise innovation in technology

At the same time, the increase in productivity increases the after-tax return on the factors of production, labor and physical and robot capital. The representative households then respond to the incentives contained in prices and start accumulating both types of capital, and supplies less hours worked, as robots and labor services are substitutes. In turn, the increase in capital input feeds back in output through the production function and that further adds to the positive effect of the technology shock.

Over time, as both physical and robot capital is being accumulated, their after-tax marginal product starts to decrease, which lowers the households' incentives to save. As a result, physical and robot capital stock eventually return to their steady-state, and exhibits a hump-shaped dynamics over its transition path. The rest of the model variables return to their old steady-states in a monotone fashion as the effect of the one-time surprise innovation in technology dies out.

5 Conclusions

Robots are introduced into a real-business-cycle setup augmented with a detailed government sector. The model is calibrated to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2020). The quantitative importance of the presence of robots in the economy is investigated for business cycle fluctuations in Bulgaria. In the presence of robots, wages increase, but employment falls after a technology shock. However, for plausible parameter values, the effect is predicted to be quite small.

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