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Optimal Time to Attain a Targeted Profit Function Under Unitary Transformation

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Abstract: The problem of fastest descent is solved by the calculus of variations. Calculus of variations is a branch of mathematics dealing with the optimization problem of physical quantities. In this paper, profit maximization problems are judged by using this idea. Profit velocity and time are key factors to optimize policy. That is why we have investigated the path of the profit function and the minimum time to reach the final destination of a profit function by utilizing a unitary operator. Given two states, the starting profit function and the targeted profit function, there exist different paths belonging to the set. This investigation uses the unitary transformation, which transforms the starting profit function to the targeted profit function in the least possible time.

Keywords: profit velocity; targeted profit function; unitary operator

JEL Classifications: C61; D21

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1. Introduction

The application of unitary operators into economics was introduced by the study of the targeted profit function that arises in any mathematical theory of an economic system with profit velocity. Profit velocity is profit generated per minute of production and optimal timing. The purpose of this paper is to present a detailed discussion of different paths of the targeted profit functions.

We ask how quickly can we reach the maximum targeted profit function and by which transformation? We say that there is a unitary transformation which decides a profit function to attain its maximum value in time. It laid the analytical foundations for the study of allocation of time within the household (Becker 1965) and for a national economy (Caragea 2009). Boland (1978) established that neoclassical economics is not timeless but treats time explicitly. A necessary and sufficient conditions for time consistently policies without domination under Ramsey policies, which are equivalent to first order conditions, has been derived by Brendon and Ellison (2018). Huang, Leng and Parlar (Huang et al. 2013) provide a comprehensive survey of commonly used demand models, which depend on time, space, quality etc. It is applied on price sensitivity and the taste of differentiation in the monopoly and duopoly setting (Chambers et al. 2006). Its continuing relevance in institutional, territorial (Corpataux and Crevoisier 2007) and empirical economics is a testimony of power (Heckman 2015). The basic issue concerning to attain targeted profit in competitive world is whether a market solution will yield the socially optimum kinds and quantities of commodities.

Let π be a complete mathematical description of the profit function to be studied (i.e.,π might be comprised of the difference between a revenue function and a cost function). Assumptions made a priori about π (assumption of continuity of revenue function and cost function), define the space Π of profit functions to which the study is restricted. By a revenue function or a cost function we mean a specific values of all the relevant endogenous variables (i.e., for revenue function we mean income of consumers, quality of the good, price etc., and by cost function it means cost price, wealth, government policies, tax rate, labor, capital etc.).

As a first test of sufficiency of this economic model, it must be possible to prove that for every element π of an adequately extensive class Π, the set Π is not empty and that means it obeys the existence problem. The mathematical tools for the solution were provided by unitary transformations in the form of linear operator (we have used matrix operators.) or directly related results. The mathematical techniques applied here were those of functional analysis.

Having obtained a general solution to the targeted profit function one must investigate the structure of Π (π_T) of targeted profit function π_T. If we consider an economy that has a targeted profit function near π_T such that in any neighborhood of π_T there are infinitely many other profit functions related with different type of transformations. In this situation the targeted profit function is indeterminate nearπ_T. Moreover, the economic system π_T is unstable in this sense that arbitrary small perturbation from π_T to a neighboring profit functions induce no tendency for the state of the economy to return to π_T. It is therefore desirable to have a function around π_T is discrete; such that for every π_T in Π there is a neighborhood of π_T is the unique solution. But if all the conditions here prevail mathematically well behaved one may obtain a set Π that may not be discrete.

There is another type of problem to determine targeted profit function by using transformation. Actually there are different types of transformations which transform a profit function to different targeted profit functions. Specifically if π is close to π'then the application of transformations T_1 and T_2 on π and π' respectively leads to $T_1(\pi)$ and $T_2(\pi')$ and then one would find $T_1(\pi)$ to be close to $T_2(\pi')$. Otherwise an arbitrary small error in the determination of targeted profit function would yield entirely different values and thus destroy the explanatory power of the theory also. Therefore, it is also desirable to choose a special type of transformation to be such that the application of transformation on the starting profit function reaches to the desired targeted profit function only without disabling the system.

This difficulty is solved by functional analysis by using unitary transformations. As it is known that there are many paths to join the two points in the Real Space, there exist many transformations to obtain the desired result. However, our aim is to obtain the targeted profit without hampering the dynamical system in the space Π, so we have chosen unitary transformation as it is known that a unitary transformation transforms a linear operator into a linear operator and leaves invariant any algebraic equation between linear operators.

In the first model, we observe that least possible time is dependent on profit velocity and the cost function. This case indicates that a targeted profit function is achieved by studying only the costs function and profit velocity ignoring the revenue function and cost increasing parameter. In the second model, the time is dependent on profit velocity and revenue function. We have put a parameter here, the revenue increasing parameter, but it has no effect in the least possible time. In the case of model three, the least possible time is dependent on profit velocity, revenue increasing parameter and cost function by surpassing the cost increasing parameter.

Our paper explores the relation between the profit velocity and the time of the shortest path between the starting profit function and the targeted profit function by using unitary transformation. Unitary transformation gives impulses on the starting profit function of a firm to get the final profit function in a least possible time.

2. Model

As it is known that profit of a firm is dependent on demand and cost function, so it can be expressed in the following form: π= R – C, where π denotes profit of a firm, R is a revenue function and C is a cost function. Suppose a new firm is tried to enter in a competition with potential big bosses with a new idea that will reduce costs and give maximum profit in a short span of time. The investor will try to understand the dynamics of the economy by observing the conditions of more investment in the research and development area, the velocity of the profit function and time factor to attain the maximum profit in a short span of time without giving chances to his opponents to understand his economic policy. But the question arises whether the investor will engage in direct competition with his opponents. Then the rent dissipation effect and replacement effects come into action. These problems were solved by Arrow (1962) and are well discussed by Aghion and Griffith (2005). Our investor, however, is not interested in this type of direct conflict but he will try to win the race by giving importance to the time factor. Now the problem is how does this relate to attain the maximum profit in the shortest time? This is discussed in the following model.

Here, an empirical researcher may face some methodological difficulties that arise due to lack of data. First, it is important to control the own firm and to investigate characteristics of the least possible time that affects the profit function. The reason behind this are the characteristics of the cost-dependent parameter of the firm and the dynamics of profit velocity. For example, we know that the least possible time in order to reach the targeted profit function is related to the cost-dependent parameter and profit velocity. This parameter is positively related to the cost function. However, we cannot reach the targeted profit of the firm by enforcing the parameter only, because it could be the case that the least possible time to reach the targeted profit is genuinely related to the profit velocity and the cost function. Then the cost-dependent parameter plays no role. But we have to keep in mind that unless we manage for at least the main observable and unobservable characteristics, we cannot be sure to predict the relationship among time with profit velocity and the cost function with the application of unitary transformation.

Second, there is a problem of different causality. While profit velocity and the cost function are likely to affect the least possible time for the targeted function, it is also possible that the revenue function affects the least possible time. Firms that are engaged in innovation always try to reduce costs and/or to increase revenue by producing superior quality goods. However, when firms are unable to reduce the total cost significantly, then firms rely on the revenue function (i.e. on price, income etc.) to mobilize the profit function. It is important that there is variation in the profit function; for example, policy changes (here we take unitary operators as a symbol of policy change) that make firms attain targeted profit in least possible time.

Third, the relationship we are interested in is among time, profit velocity, revenue-increasing factors and the cost function. Earlier contributions in this field (Caragea 2009; Boland 1978) have mainly focused on price sensitivity, quality and demand functions. It can be difficult to obtain the time factor to attain the targeted profit function in an industry. Our work is mainly concentrated on the transformation of a system without hampering the other factors in a firm, particularly its geographic and product area. Unitary operators play an important role because we assume that firms operate in large markets, so that traditional ways of application to study the market behavior could be misleading.

Let us assume that π_S be an initial profit and π_T be the targeted profit of a firm. We have to recall that π_T is reached from the initial profit $\pi_S$through the shortest path at time‘t’. We will find out the time ‘t’ and the related conditions to attain the target point. It is possible to construct different paths to join π_S and $\pi_T$but we have assumed the shortest path between the two points. We have taken two postulates for our models to set up the space in which our economy takes place.

Postulate 1: Associated to any isolated economic system in our model is a real valued inner product space, which is a complete metric space with respect to a distance function induced by the inner product space.

Postulate 2: The evolution of our economic system is described by a unitary transformation. That is the position of the starting profit function π_S at time t_1 is related to the state π_T at time t_2 by a unitary operator U_t which depends only on the times t_1 andt_2, and t = t_2 - t_1.

We know from postulate 2 that the targeted profit is produced by motion of the dynamics of the firm policy acting on the starting profit and the operator acting on the starting profit is a unitary operator (see note 1). Mathematically speaking, π_T = U_t $\pi_S$that is unitary operator $U_t$is applied on π_S and after time t. It would reach π_T without hampering the system preserved in π_S.

Model 1: To evaluate the targeted profit time when only cost-dependent parameterα is considered:

Consider an economy with a starting profit function π_S, revenue function R, and cost function C with cost- dependent parameter α acting on the cost function only. Here there acts no parameter in the revenue function R because in the system the revenue function is unaltered. We can express the starting profit function in the following way:

(1) $$\pi_S = R - Ce^{\alpha}.$$

Here we have considered the targeted profit π_T as a combination of the revenue function, which is same as the starting profit, and cost function C. The cost-dependent parameter αacts negatively on the cost function. Here a new firm faces a severe challenge with the old system and is unable to increase the revenue remarkably. Consider the new revenue function R_1. R_1 = R + dR. As dR is a negligible quantity it follows that $R_1 \approx$ R. A firm manager has nothing to do only to reduce the cost function. However, since it is known that cost function and the revenue functions are correlated, he will try to reduce the factors depending on the cost dependent parameter α. For that reason, we have taken α as a negative variablethat acts on cost function. Then we write the targeted profit as,

(2) $$\pi_T = \text{R} - \text{C}\, e^{-\alpha} .$$

Equations 1 and 2 give,

$$\text{R} = \frac{\pi_T e^{\,\alpha} - \pi_I e^{-\,\alpha}}{e^{\,\alpha} - e^{-\,\alpha}}$$

(3) $$\text{R} = \frac{\pi_T e^{\,\alpha} - \pi_I e^{-\,\alpha}}{2\ \sin h\ \alpha}$$

and

(4) $$\text{C} = \frac{(\pi_T - \pi_S)}{2\ \sin \text{h}\, \alpha} .$$

[We get the results by using the properties, sin h $\alpha = \frac{e^{\,\alpha} - e^{-\,\alpha}}{2}$.]

Equations (1) and (2) may be expressed in a matrix form as

(5) $$\pi_S = \begin{pmatrix} R & C \\ -\,e^{\alpha} & 1 \end{pmatrix}$$

and

(6) $$\pi_T = \begin{pmatrix} R & C \\ -\,e^{-\alpha} & 1 \end{pmatrix}.$$

Equations (3) and (4) may be written as

(7) $$\text{C} = \frac{1}{2\ \sin \text{h}\, \alpha} \begin{pmatrix} \pi_T & -\,\pi_S \\ 1 & 1 \end{pmatrix})$$

and

(8) $$\text{R} = \frac{1}{2\ \sin \text{h}\, \alpha} \begin{pmatrix} \pi_T & \pi_S \\ -\,e^{-\,\alpha} & e^{\,\alpha} \end{pmatrix}.$$

Now set $U_t = \begin{pmatrix} \cos\omega\, t & \sin\omega\, t \\ -\sin\omega\, t & \cos\omega\, t \end{pmatrix}$, where ω is profit velocity.

Then, $U_t\, \pi_s = \begin{pmatrix} \cos\omega\, t & \sin\omega\, t \\ -\sin\omega\, t & \cos\omega\, t \end{pmatrix} \begin{pmatrix} R & C \\ -e^{\alpha} & 1 \end{pmatrix}$

(9) $$= \begin{pmatrix} R\cos\omega\, t - e^{\alpha}\sin\omega\, t & C\cos\omega\, t + \sin\omega\, t \\ -R\sin\omega\, t - e^{\alpha}\cos\omega\, t & -C\sin\omega\, t + \cos\omega\, t \end{pmatrix}.$$

As we know $U_t\, \pi_s = \pi_T$ from the role of unitary operator of postulate 2. That is, equating the equations (6) and (9) we get,

(10) $$\begin{pmatrix} R\cos\omega\, t - e^{\alpha}\sin\omega\, t & C\cos\omega\, t + \sin\omega\, t \\ -R\sin\omega\, t - e^{\alpha}\cos\omega\, t & -C\sin\omega\, t + \cos\omega\, t \end{pmatrix} = \begin{pmatrix} R & C \\ -e^{-\alpha} & 1 \end{pmatrix}.$$

Equating the both sides of equation (10) we get the only valid equation

$C\cos\omega\, t + \sin\omega\, t = C$, where other equations give negative time or improperequation of time. Now from $C\cos\omega\, t + \sin\omega\, t = C$, we get

(11) $$t = \frac{2}{\omega}\tan^{-1}\frac{1}{C}.$$

Claim 1: An increase in profit velocity should have a more positive effect on the time factor to achieve a maximum profit.

Claim 2: Enhancement of the cost function increases the time factor to a larger extent in achieving the optimum profit function. That is firms with higher costs will not attain its maximum value in a short span of time.

Model 2: To find out the time for the targeted profit function when a revenue increasing parameter is considered:

Now if we express the starting and the targeted profit functions in the following way,

(12) $$\pi_S = \mathrm{R}\, e^{-\beta} - \mathrm{C}$$

and

(13) $$\pi_T = \mathrm{R}\, e^{\beta} - \mathrm{C},$$ where β is the revenue increasing factor.

As we assume that firm size, innovative output, market structure and other characteristics are co-related. Furthermore, all factors are dependent on revenue function. Firms that are successful in obtaining the maximum profits have to focus on the revenue increasing factor. This is also important when firms operate in new markets. Then traditional market policy might not be effective but could be misleading. Hence, we consider equations (12) and (13).

We also arrange the values of the revenue and the cost function as

(14) $$\mathrm{R} = \frac{\pi_T - \pi_S}{e^{\beta} - e^{-\beta}} = \frac{\pi_T - \pi_S}{2\ \sin h\ \beta}$$

and

(15) $$\mathrm{C} = \frac{\pi_T\ e^{-\beta} - \pi_S\ e^{\beta}}{e^{\beta} - e^{-\beta}} = \frac{\pi_T\ e^{-\beta} - \pi_S\ e^{\beta}}{2\ \sin h\ \beta}.$$

Now π_S and π_T can be written in the matrix form as,

(16) $$\pi_S = \begin{pmatrix} R & -C \\ 1 & e^{-\beta} \end{pmatrix}$$

and

(17) $$\pi_T = \begin{pmatrix} R & -C \\ 1 & e^{\beta} \end{pmatrix}$$

Again from postulate 2 we get,

(18 $$U_t\,\pi_s = \begin{pmatrix} \cos\omega\,t & \sin\omega\,t \\ -\sin\omega\,t & \cos\omega\,t \end{pmatrix}\begin{pmatrix} R & -C \\ 1 & e^{-\beta} \end{pmatrix}$$

$$= \begin{pmatrix} R\cos\omega\,t + \sin\omega\,t & -C\cos\omega\,t + e^{-\beta}\sin\omega\,t \\ -R\sin\omega\,t + \cos\omega\,t & -C\sin\omega\,t + e^{-\beta}\cos\omega\,t \end{pmatrix}.$$

Now from the relation $U_t\,\pi_s = \pi_T$ we obtain,

$$\begin{pmatrix} R\cos\omega\,t + \sin\omega\,t & -C\cos\omega\,t + e^{-\beta}\sin\omega\,t \\ -R\sin\omega\,t + \cos\omega\,t & -C\sin\omega\,t + e^{-\beta}\cos\omega\,t \end{pmatrix} = \begin{pmatrix} R & -C \\ 1 & e^{\beta} \end{pmatrix}.$$

Equating corresponding elements of both matrices, we get the legitimate equation $R\cos\omega\,t + \sin\omega\,t = \mathrm{R}$ leads to

(19) $$\mathrm{t} = \frac{2}{\omega}\tan^{-1}\frac{1}{R}.$$

Claim 1: As in the claim1of the first model, when profit velocity tends to gain its momentum, then time factor decreases significantly.

Claim 2: If a firm always tries to earn a revenue function without an interruption, then it is impossible to attain the maximum profit in a stipulated time.

Model 3: To find out the time for the targeted profit function when both, revenue increasing parameter and cost-dependent parameter are considered

Here we are interested in the extent to which the effect of the cost-dependent parameter and the revenue-increasing function on the cost function and the revenue function influences the time factor. In this case the cost-dependent parameter acts positively on the starting profit function and acts negatively on the targeted or final profit function whereas the revenue-increasing parameter acts negatively on the starting profit function and acts positively on the targeted profit function.

We set the starting and the targeted profit in the following way,

(20) $$\pi_s = \mathrm{R}\,e^{-\beta} - \mathrm{C}\,e^{\alpha}$$

and

(21) $$\pi_T = \mathrm{R}\, e^{\beta} - \mathrm{C}\, e^{-\alpha}.$$

Here we get revenue and cost function as,

(22) $$\mathrm{R} = \frac{\pi_T e^{\alpha} - \pi_s e^{-\alpha}}{2 \sin h\, (\alpha + \beta)}$$

and

(23) $$\mathrm{C} = \frac{\pi_T e^{-\beta} - \pi_s e^{\beta}}{2 \sin h\, (\alpha + \beta)}.$$

Now we express π_s and π_T in the matrix form as,

(24) $$\pi_s = \begin{pmatrix} R & C \\ -e^{\alpha} & e^{-\beta} \end{pmatrix}$$

and

(25) $$\pi_T = \begin{pmatrix} R & C \\ -e^{-\alpha} & e^{\beta} \end{pmatrix}.$$

By using postulate 2 we get,

(26) $$U_t\, \pi_s = \begin{pmatrix} \cos\omega\, t & \sin\omega\, t \\ -\sin\omega\, t & \cos\omega\, t \end{pmatrix} \begin{pmatrix} R & C \\ -e^{\alpha} & e^{-\beta} \end{pmatrix}$$

$$= \begin{pmatrix} R\cos\omega\, t - e^{\alpha}\sin\omega\, t & C\cos\omega\, t + e^{-\beta}\sin\omega\, t \\ -R\sin\omega\, t - e^{\alpha}\cos\omega\, t & -C\sin\omega\, t + e^{-\beta}\cos\omega\, t \end{pmatrix}$$

The relation $U_t\, \pi_s = \pi_T$ gives,

$$\begin{pmatrix} R\cos\omega\, t - e^{\alpha}\sin\omega\, t & C\cos\omega\, t + e^{-\beta}\sin\omega\, t \\ -R\sin\omega\, t - e^{\alpha}\cos\omega\, t & -C\sin\omega\, t + e^{-\beta}\cos\omega\, t \end{pmatrix}$$

$$= \begin{pmatrix} R & C \\ -e^{-\alpha} & e^{\beta} \end{pmatrix}$$

leads to $t = \frac{2}{\omega} \tan^{-1}(\frac{e-\beta}{C})$.

Claim 1: Profit velocity makes a positive impact on the time factor.

Claim 2: The decrease of the cost function has a negative impact on time and vice versa.

Claim 3: The revenue-increasing parameter should be regulated, because otherwise a firm cannot achieve its goal in the stipulated time.

3. Conclusion

We observe that with the application of different unitary operators a starting profit function of a firm may achieve its final destination in arbitrarily short time. Now a million dollar question arises whether such an operation is implemented in practice? If the answer is affirmative then it has an immense implication in the profit maximization policy of a firm and in other related fields.

If managers of a firm study the characteristics of the profit velocities and observes the conditions of the starting profit function and also study the trajectory path of the destination point then it is possible to find out the least time to reach the target through a well-developed computer.

Note 1: A matrix U is said to be unitary if U^TU = I, where I is a unit matrix. Similarly an operator U is unitary if $(U^T)^*$U = I, where $U^*$is adjoined of U and I is the identity operator. An operator is unitary if and only if each of its matrix representations is unitary. The property of a unitary operator is that if a vector is acted on by a unitary operator, the length of the vector remains unchanged. The corresponding physical meaning is that no new system would be created or the system in π_s would not collapse under unitary evolution. That is why we have chosen unitary operator.

References

Aghion, P.; Griffith, R. (2005), Competition and Growth Reconciling Theory and Evidence, Zeuthen Lectures, MIT Press, Cambridge.

Arrow, K. J. (1962), Economic Welfare and the Allocation of Resources for Invention, The Rate and Direction of Inventive Activity: Economic and Social Factors: Princeton University Press.

Becker, G. S. (1965), A theory of the allocation of time, *Economic Journal* 75(299), pp. 493-517.

Boland, L. A. (1978), Time in economics vs economics in time: The 'Hayek Problem', *Canadian Journal of Economics* 11(2), pp. 240-262.

Brendon, C.; Ellison, M. (2018), Time Consistently Undominated Policies, CEPR Discussion Paper No. DP12656. Available at SSRN: https://ssrn.com/abstract=3112280

Caragea, N. (2009), Time allocation in economics and the implications for economic development, *Romanian Journal of Economics* 29(2), pp. 122-141.

Corpataux, J.; Crevoisier, O. (2007),Economic theories and spatial transformations clarifying the space-time premises and outcomes of economic theories, *Journal of Economic Geography* 7(3), pp. 285-309.

Heckman, J. (2015), Introduction to A Theory of the Allocation of Time by Gary Becker, *Economic Journal* 125(583), pp. 403 -409.

Huang, J.; Leng, M.; Parlar, M. (2013), Demand functions in decision modelling: A comprehensive survey and research directions, *Decision Sciences* 44(3), pp. 557-609.

Quiggin, J.; Chambers, R. G. (2006), Capital market equilibrium with moral hazards and flexible technology, *Journal of Mathematical Economics* 42(3), pp. 358-363.