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Sofia University St. Kliment Ohridski
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A Business-cycle-model with a Modified Cash-in-advance Feature, Government Sector and One-period Nominal Wage Contracts: The Case of Bulgaria

Aleksandar Vasilev

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A business-cycle model with a modified cash-in-advance feature, government sector and one-period nominal wage contracts: the case of Bulgaria

Aleksandar Vasilev*

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Abstract

We augment an otherwise standard business cycle model with a richer government sector, and add a modified cash in advance considerations, and one-period-ahead nominal wage contracts. In particular, the cash in advance constraint of Cooley and Hansen (1989) is extended to include private investment and government consumption. This specification, together with the nominal wage rigidity, when calibrated to Bulgarian data after the introduction of the currency board (1999-2016), gives a role to money in propagating economic fluctuations. In addition, the combinations of these ingredients allows the framework to reproduce better observed variability and correlations among model variables, and those characterizing the labor market in particular.

Keywords: business cycles, modified cash-in-advance constraint, one-period nominal wage contracts

JEL Classification: E32

*Independent Researcher, Bulgaria. E-mail: alvasilev@yahoo.com.

1 Introduction and Motivation

It is a well-known fact, *e.g.* Prescott (1986), that the perfectly-competitive (Walrasian) approach to modelling labor markets in real business cycles (RBC) does not fit data well, and thus creates a "puzzle" for neoclassical economists. More specifically, in the standard RBC model the fluctuations in employment are due to movements in labor supply. In other words, households increase hours in the face of a raise in the return on labor, the wage, driven by shocks to technology. Instead, if an RBC model is to fit data better along the labor market dimension, even for a small economy like Bulgaria, shocks that work on labor demand and shift it around would be much better candidates to explain the observed fluctuations in the wage rate, aggregate hours and employment.

In order to avoid running into the problem of "observational equivalence," an outcome in which two or more models of substantially different structure may explain equally well certain stylized facts, economists need to justify the inclusion of alternative propagation mechanisms. Therefore, in this paper we base our modeling approach on a particular empirical regularity in Bulgaria, namely a pre-contracted nominal wage of duration one year, which is the norm in the period following the introduction of the currency board arrangement (1999-2016). Indeed, annual nominal wage contracts are an important element of labor market reality in Bulgaria (and other continental European countries), as a large proportion of the labor force in Bulgaria are wage-earners, and a large portion of the labor force in manufacturing in particular engages in long-term wage contracting. In addition, the justification for the existence of such wage contracts is provided by Danziger (1988), and Gomme and Greenwood (1994), who demonstrate that pre-setting the wage rate in nominal terms can be an optimal way for workers to achieve efficient risk sharing over the cycle, and that is why workers engage in such contracts instead of say, renting their services in a spot market for labor.[1]

Note that one-period ahead nominal wage contracts used in this paper work in a different way than spot market (Walrasian labor market) contracts. While in the latter, labor supply is driven by the intertemporal substitution hypothesis, i.e. the household will increase

[1]One year indexation is also very close to optimal contract duration of 4 quarters, established in Cho, Cooley and Planeuf (1997).

(decrease) their labor supply in response to an increase (decrease) in the market wage rate, in an economy with wage contracts, variability in hours and employment will be driven by fluctuations in labor demand. The pre-contracted wage rate is equal to the rational expectation of the wage that would prevail at the labor market equilibrium of an economy with spot labor markets. At any time period, the nominal wage is then fixed, and the real wage will depend on the price level. This level depends on the monetary shocks (but in this model we do not have such shocks). Equilibrium employment level is then determined by the firm which uses its labor demand curve. The households accept to work as much as the firm needs them to, as it is the firm that specified and posted the nominal wage contract. Therefore, the variability of employment and hours will effectively depend on the firm side.

We adopt the approach followed by Cho (1993) and Cho and Cooley (1995) to incorporate one-period nominal wage contracts in RBC models in order to investigate the quantitative effect on business cycle fluctuations in aggregate variables in Bulgaria, and labor markets in particular.[2] We then proceed to evaluate how the presence of contracting issues affects business cycle fluctuations, and whether this non-Walrasian setup in the labor market, that some transactions are conducted at non-clearing prices, is able to address the "labor market puzzle," and validate certain labor market facts, while at the same time retain technology as the only shock process.[3]

As much as possible, we would stay within the RBC framework. However, since the wage is set in nominal terms, we need to introduce money in the setup through the cash-in-advance (CIA) constraint. More specifically, we assume that money is valuable as agents need to possess cash in order to make purchases for consumption or investment purposes. This assumption has already been used in Cooley and Hansen (1989), where only consumption was constrained bu the CIA. In this model, we assume that all components of output -

[2]We also abtsract away from staggered and overlapping wage contracts, such as Gray (1976) and Fischer (1977), and Talor (1980), as their modelling approach which interferes with model tractability. We also do not explore the optimal indexation issue, ot the welfare cost of indexation.

[3]Note that a model with zero-period nominal wage contracts and is de facto a spot market for labor. In addition, in the presence of a CIA constraint such a model will feature the real-nominal dichotomy, i.e. money will be neutral.

private consumption, private investment, and government consumption - are all featured in the CIA constraint.[4] This is one of the novelties in this paper. In addition, in the face of nominal wage rigidities, money is no longer neutral (even though households feature rational expectations). More specifically, money will have a real effect, and given one-period nominal wage contracts, the response will be concentrated in the first few periods after a technology shock. In the model in this paper, both the second moments and the correlations of the main aggregate variables with output are affected. Importantly, real wages become a lagging variable, which helps with the dynamic correlation between hours and wages. Given the particular focus on labor markets, we abstract away from nominal variables, whose cyclical properties are badly reproduced nonetheless. One reason is the short time series, the fact that Bulgaria is not fully developed. Last but not least, the model is primarily real in nature.

The rest of the paper is organized as follows: Section 2 describes the model framework and describes the decentralized competitive equilibrium system, Section 3 discusses the calibration procedure, and Section 4 presents the steady-state model solution. Sections 5 proceeds with the out-of-steady-state dynamics of model variables, and compared the simulated second moments of theoretical variables against their empirical counterparts. Section 6 concludes the paper.

2 Model Setup

There is a representative household, which derives utility out of consumption and leisure. The time available to households can be spent in productive use or as leisure. The household engages in nominal wage contracts, where the nominal wage rate is determined one period (year) in advance. The government taxes consumption spending and levies a common tax on all income, in order to finance wasteful purchases of government consumption goods, and government transfers. The monetary authority follows an exogenous money supply rule, and redistributes all seigniorage back to the household. On the production side, there is

[4]Hairault and Portier (1995) also include private investment in the CIA. However, their model does not incorporate a government sector. In contrast, Fairise (1995) includes both private investment and government consumption in the CIA constraint, but their setup also features investment and employment adjustment costs.

a representative firm, which hires labor and utilized capital to produce a homogenous final good, which could be used for consumption, investment, or government purchases.

2.1 Household problem

Each household maximizes expected discounted utility, which, as in Cho (1993) and Cho and Cooley (1995), is of the form

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - \frac{\theta_1}{1+\gamma} n_t^{1+\gamma} e_t - \frac{\theta_2}{1+\sigma} e_t^{1+\sigma} \right\}, \tag{1}$$

where E_0 is the expectation operation conditional on information available as of $t = 0$, $0 < \beta < 1$ is the discount factor, c_t is individual household consumption in period t, n_t are hours worked, and e_t is the employment probability. Parameters $\theta_1, \theta_2 > 0$ are the weights attached to disutility of work, while parameters γ and σ capture the curvature of the utility function in hours and employment. The particular form (integration of hours and employment) is based on Kydland and Prescott (1991) and Cho (1994), who use aggregation and lotteris as in Hansen (1985) and Rogerson (1988) to convexify a discrete labor supply decision at individual level - work either zero hours or a full-time - to derive the preferences of an aggregate household. In particular, in equilibrium, a households will be chosen for work every period with a probability e_t, which, form the law of large numbers, will also equal the employment rate.

The household starts with a positive endownment of physical capital, k_0, in period 0, which is rented to the firm at the nominal rental rate R_t, that is, before-tax capital income equals $R_t k_t$. Therefore, each household can decide to invest in capital to augment the capital stock, which evolves according to the following law of motion:

$$k_{t+1} = i_t + (1 - \delta) k_t, \tag{2}$$

where $0 < \delta < 1$ is the depreciation rate of physical capital.

In addition to the rental income, the household owns the firm, and thus has a legal claim to the firm's nominal profit, Π_t. It may also decide to buy government debt, which pays a nominal interest rate of i_t^b. Lastly, the household agrees to work a certain number of hours, to be determined by the firm, at a one-period ahead precontracted nominal wage rate W_t^c, producing a total nominal labor income of $W_t^c e_t n_t$ in period t.

The budget constraint of the aggregate household, expressed in real terms, is then

$$\begin{aligned}(1+\tau^c)c_t + k_{t+1} - (1-\delta)k_t + \frac{M_{t+1}}{P_t}\frac{P_{t+1}}{P_{t+1}} + \frac{B_{t+1}}{P_t}\frac{P_{t+1}}{P_{t+1}} - (1+i_t^b)\frac{B_t}{P_t} = \\ (1-\tau^y)[w_t^c e_t n_t + r_t k_t] + \frac{M_t}{P_t} + G_t^t + \frac{\Pi_t}{P_t},\end{aligned} \tag{3}$$

where τ^c is the tax rate on final consumption, τ^y is the proportional rate on labor and capital income, P_t is the aggregate price level, i_t^b is the nominal interest rate on bonds. M_t and B_t denote the nominal quantities of money stock and bond holdings in period t, respectively). Money stock is treated like a consumption good, it stores wealth over time. That is why real money balances in period t are $m_t = M_t/P_t$ in period $t+1$ only buy M_t/P_{t+1} (next period purchasing power). Also, real bond holdings, $b_t = B_t/P_t$. Similarly, $w_t^c = W_t^c/P_t$, and $r_t = W_t^c/P_t$ are the real wage and the real interest rate.

Real money balances are needed to purchase output, hence the households face the following cash-in-advance constraint

$$(1+\tau^c)c_t + k_{t+1} - (1-\delta)k_t + g_t^c \leq \frac{M_t}{P_t} = m_t \tag{4}$$

Next, we set up the Lagrangean of the household's problem:

$$\begin{aligned}L = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \ln c_t - \frac{\theta_1}{1+\gamma} n_t^{1+\gamma} e_t - \frac{\theta_2}{1+\sigma} e_t^{1+\sigma} - \lambda_t \Bigg[(1+\tau^c)c_t + k_{t+1} - (1-\delta)k_t + \frac{M_{t+1}}{P_t}\frac{P_{t+1}}{P_{t+1}} \\ + \frac{B_{t+1}}{P_t}\frac{P_{t+1}}{P_{t+1}} - (1+i_t^b)\frac{B_t}{P_t} - (1-\tau^y)[w_t^c e_t n_t + r_t k_t] - \frac{M_t}{P_t} - G_t^t - \frac{\Pi_t}{P_t} \Bigg] \\ -\mu_t[(1+\tau^c)c_t + \nu(k_{t+1} - (1-\delta)k_t + g_t^c) - \frac{M_t}{P_t}] \Bigg\}\end{aligned} \tag{5}$$

The first-order optimality conditions (FOCs) are as follows:

$$
\begin{aligned}
c_t &: \quad \frac{1}{c_t} = (1+\tau^c)(\lambda_t + \mu_t) && (6)\\
n_t &: \quad \theta_1 n_t^{\gamma} = \lambda_t (1-\tau^y) w_t^c, && (7)\\
e_t &: \quad \theta_2 e_t^{\sigma} + \theta_1 \frac{\theta_1}{1+\gamma} n_t^{1+\gamma} = \lambda_t (1-\tau^y) w_t^c n_t, && (8)\\
k_{t+1} &: \quad \lambda_t + \mu_t = \beta E_t \Big[\lambda_{t+1}[1-\delta + (1-\tau^y) r_{t+1}] + \mu_{t+1} \nu (1-\delta) \Big], && (9)\\
b_{t+1} &: \quad \lambda_t \frac{P_{t+1}}{P_t} = \beta E_t \lambda_{t+1} (1 + i_{t+1}^b), && (10)\\
m_{t+1} &: \quad \lambda_t = \beta E_t \left[\frac{P_t}{P_{t+1}} (\lambda_{t+1} + \mu_{t+1}) \right] && (11)
\end{aligned}
$$

and the boundary (transversality) conditions for capital, real money balances and real bond holdings:

$$
\begin{aligned}
TVC_k &: \quad \lim_{t\to\infty} \beta^t \lambda_t k_{t+1} = 0 && (12)\\
TVC_m &: \quad \lim_{t\to\infty} \beta^t \lambda_t m_{t+1} = 0 && (13)\\
TVC_b &: \quad \lim_{t\to\infty} \beta^t \lambda_t b_{t+1} = 0 && (14)
\end{aligned}
$$

The interpretation of the optimality conditions is standard. In the first, the household equates the marginal utility of consumption, to the VAT adjusted shadow price of wealth and the CIA constraint. The second and the third FOC determine optimal number of hours worked and probability of employment (or employment rate), by balancing at the margin the cost and benefit from working. The remaining equations from the original FOCs are standard: for example, the Euler equation for capital stock describes how capital is allocated across any adjacent periods in order to maximize household's utility. The transversality conditions (TVCs) for real cash holdings, real bonds, real holdings of deposits, and physical capital are imposed to rule out explosive solutions.

2.2 Firm's problem

There is a stand-in firm in the economy, which uses homogeneous capital and labor to produce a final good, which can be used for consumption, investment, or government purchases, through the following production function:

$$y_t = A_t k_t^{\alpha} Q_t^{1-\alpha}, \tag{15}$$

where A_t denotes the level of total factor productivity in period t, Q_t are total hours used, and α and $1-\alpha$ are the share of capital and labor, respectively.

In contrast to the standard representation, with one-period ahead nominal contracts, the firm still maximizes profit but now under the constraint determined by the pre-contracted wage W_t^c. We follow Cho's (1990) approach of including wage contracts in a RBC model. More specifically, the contractual nominal wage W_t^c is determined one-period in advance, in period $t-1$. In period t, based on the information available (i.e., k_t, A_t), and taking the nominal wage as given, the firm hires labor on the labor demand curve. Importantly, the contract stipulates also that the household leaves to the firm the right to manage in order for the firm to maximize profit. In case of an agreement in period $t-1$ between the household and the firm redarding a W_t^c, in period t the amount of labor demanded by the firm will be supplied.

In other words, the firm problem is to

$$\max_{(k_t, Q_t) \geq 0} P_t A_t k_t^{\alpha} Q_t^{1-\alpha} - R_t k_t - W_t^c Q_t \tag{16}$$

s.t.

$$\ln W_t^c = E_{t-1} \ln W_t. \tag{17}$$

The first-order optimality conditions determining optimal capital, and labor use in the Walrasian setup are

$$k_t \quad : \quad P_t \alpha \frac{y_t}{k_t} = R_t, \tag{18}$$

$$Q_t \quad : \quad P_t (1-\alpha) \frac{y_t}{Q_t} = W_t. \tag{19}$$

Note that the expected and actual capital rental rate are the same. What differs is the price of labor. To solve for the contracted wage, we take natural logarithms from both sides of the Walrasian wage equation to obtain

$$\ln W_t = \ln P_t + \ln(1-\alpha) + \ln y_t - \ln Q_t. \tag{20}$$

Note that consistency requires that in equilibrium total labor demand equals total labor supply, or $Q_t = e_t n_t$.[5] Next, and imposing the wage contracting rule by taking expectations as of period $t-1$ yields

$$\ln W_t^c = E_{t-1} \ln W_t = E_{t-1} \ln P_t + \ln(1-\alpha) + E_{t-1} \ln y_t - E_{t-1} \ln Q_t = \\ E_{t-1} \ln P_t + \ln(1-\alpha) + E_{t-1} \ln y_t - E_{t-1} \ln e_t - E_{t-1} \ln n_t. \tag{21}$$

As in Cho and Cooley (1995), the contractual nominal wage is equal to the rational expectations of the household, taken as of date $t-1$ of the equilibrium wage at period t of the Walrasian (perfectly-competitive spot) model.

2.3 Monetary Authority

In this paper the monetary authority (central bank) follows an exogenous process for the growth rate g_t of the money aggregate, M_t. This is an adequate approximation for a central bank operating under a currency board, where money supply is determined by external factors (ECB and foreign owned banks borrowing from the mother banks).[6]

$$M_{t+1} = g_t M_t. \tag{22}$$

Note that g_t is assumed to be known at the beginning of period t. All money created (seigniorage) in period t is distributed to the government, and then to the households in a lump-sum fashion

$$(g_t - 1)M_t = T_t, \tag{23}$$

where T_t is the lump-sum transfer to the household.[7]

[5] Note that the equilibrium quantity of labor will be determined from the firm. The household satisfy the firm's labor demand, whatever the quantity. The firm is holding the right to manage, so it is free to set employment at the optimal level, as it observes the realization of the technology shock. More specifically, the firm will equate the marginal product of labor with the expected marginal product of labor, which is the pre-contracted wage.

[6] In particular, as shown in a later section, money supply growth rate follows an AR(1) process.

[7] Alternatively, we can assume that the central bank distributes the seigniorage to the Ministry of Finance, which in turn distributes it to the household as part of the overall government transfer.

2.4 Government

In the model setup, the government is levying taxes on labor and capital income, as well as consumption in order to finance spending on government purchases and government transfers, as well as roll over government debt. The government budget constraint is as follows:

$$\tau^c c_t + \tau^y (w_t^c n_t e_t + r_t k_t) + \frac{B_{t+1}}{P_t} + \frac{T_t}{P_t} = (1 + i_t^b)\frac{B_t}{P_t} + g_t^t + g_t^c \tag{24}$$

Tax rates and government consumption-to-output ratio would be chosen to match the average share in data, and government transfers would be determined residually.

2.5 Stochastic process

Total factor productivity, A_t, is assumed to follow AR(1) processes in logs, in particular

$$\ln A_{t+1} = (1 - \rho_a) \ln A_0 + \rho_a \ln A_t + \epsilon_{t+1}^a,$$

where $A_0 > 0$ is steady-state level of the total factor productivity process, $0 < \rho_a < 1$ is the first-order autoregressive persistence parameter and $\epsilon_t^a \sim iidN(0, \sigma_a^2)$ are random shocks to the total factor productivity progress. Hence, the innovations ϵ_t^a represent unexpected changes in the total factor productivity process.

Money growth rate, g_t, is also assumed to follow AR(1) processes in logs, in particular

$$\ln g_{t+1} = (1 - \rho_m) \ln g_0 + \rho_m \ln g_t + \epsilon_{t+1}^m,$$

where $g_0 > 0$ is steady-state money supply growth rate, $0 < \rho_m < 1$ is the first-order autoregressive persistence parameter and $\epsilon_t^m \sim iidN(0, \sigma_m^2)$ are random shocks to the money growth rate. Hence, the innovations ϵ_t^a represent unexpected changes in the growth rate of the money supply.

2.6 Dynamic Competitive Equilibrium (DCE)

Given the processes followed by the stochastic processes $\{A_t, g_t\}_{t=0}^{\infty}$, average tax rates $\{\tau^c, \tau^y\}$, endowments $k_0, m_0, b_0 \forall i$, the decentralized dynamic competitive equilibrium is a list of sequences $\{c_t, i_t, k_t, e_t, n_t, m_t\}_{t=0}^{\infty}$, a sequence of government purchases and transfers $\{g_t^c, g_t^t\}_{t=0}^{\infty}$, price level sequence $\{P_t\}_{t=0}^{\infty}$ and input prices $\{w_t^c, r_t\}_{t=0}^{\infty}$ such that (i) each household i maximizes its utility function subject to its budget constraint, the CIA constraint, and the nominal wage contract; (ii) the representative firm maximizes profit s.t. the nominal wage contract; (iii) government budget constraint evolves according to its law of motion; (iv) money supply evolves according to its law of motion; (v) all markets clear.

3 Data and Model Calibration

To calibrate the model to Bulgarian data, we will focus on the period after the introduction of the currency board (1999-2014). Annual data on output, consumption and investment was collected from National Statistical Institute (2016), while the real interest rate is taken from Bulgarian National Bank Statistical Database (2016). The calibration strategy described in this section follows a long-established tradition in modern macroeconomics: first, the discount factor, $\beta = 0.982$, as in Vasilev (2017a), is set to match the steady-state capital-to-output ratio in Bulgaria, $k/y = 3.491$. The labor share parameter, $\alpha = 0.429$, was obtained from Vasilev (2017b) as the average value of labor income in aggregate output over the period 1999-2014.

The relative weights attached to the utility out of leisure in the household's utility function, $\theta_1 = 12.652$, and $\theta_2 = 0.612$, are calibrated to match the fact that in steady-state consumers would supply one-third of their time endowment to working, and the average employment rate $e = 0.533$. As in Cho (1990), the curvature parameters are set to $\sigma = 2$ and $\gamma = 1.2$ in order to generate plausible value for aggregate labor supply elasticity.[8] Net, the average inflation rate in Bulgaria over the 1999-2016 is 4.6%, and the average annual growth rate of money supply is $g = 4.8\%$. Average debt-to-output ratio over the period is $b/y = 0.19$.

The depreciation rate of physical capital in Bulgaria, $\delta = 0.05$, was taken from Vasilev (2015). It was estimated as the average depreciation rate over the period 1999-2014. Finally, the average income tax rate was set to $\tau^y = 0.1$. Finally, the tax rate on consumption is

[8] As pointed out in Cho (1993), the responsiveness of the quantity of the labor hired to technology shocks is not affected by the labor supply elasticity but on the labor demand elasticity (i.e. the production function)

set to its value over the period, $\tau^c = 0.2$.[9] Lastly, as in Vasilev (2017c), processes followed by total factor productivity and money growth, are estimated from the detrended series by running an AR(1) regression and saving the residuals. Table 1 below summarizes the values of all model parameters used in the paper.

Table 1: Model Parameters

Parameter	Value	Description	Method
β	0.982	Discount factor	Calibrated
α	0.429	Capital Share	Data average
$1-\alpha$	0.571	Labor Share	Calibrated
δ	0.050	Depreciation rate on physical capital	Data average
θ_1	12.615	Utility weight	Calibrated
θ_2	0.612	Utility weight	Calibrated
γ	1.200	Curvature, disutility of work	Set
σ	2.000	Curvature, disutility of work	Set
e	0.533	Employment rate	Data average
n	0.333	Share of time spent working	Data average
τ^c	0.200	VAT/consumption tax rate	Data average
τ^y	0.100	Average tax rate on income	Data average
g	1.012	Gross growth rate of money supply	Data average
π	0.047	Average inflation rate	Data average
ρ_a	0.701	AR(1) parameter, total factor productivity	Estimated
σ_a	0.044	st.dev, total factor productivity	Estimated
ρ_m	0.625	AR(1) parameter, money supply (M2)	Estimated
σ_m	0.709	st.dev, money supply (M2)	Estimated

4 Steady-State

Once the values of model parameters were obtained, the steady-state equilibrium system solved, the "big ratios" can be compared to their averages in Bulgarian data. The results are

[9] Here we abstract away from excise taxes and import duties.

reported in Table 2 below. The model matches consumption-to-output ratio by construction; The investment and government purchases ratios are also closely approximated. The shares of income are also identical to those in data, which is an artifact of the assumptions imposed on functional form of the aggregate production function.

Table 2: Data Averages and Long-run Solution

Variable	Description	Data	Model
y	Steady-state output	N/A	0.568
c/y	Consumption-to-output ratio	0.674	0.674
i/y	Investment-to-output ratio	0.201	0.175
g^c/y	Government cons-to-output ratio	0.159	0.151
b/y	Debt-to-output ratio	0.190	0.190
wen/y	Labor income-to-output ratio	0.571	0.571
rk/y	Capital income-to-output ratio	0.429	0.429
n	Share of time spent working	0.333	0.333
$\tilde{r}$	After-tax net return on capital	0.056	0.057

The after-tax return, net of depreciation, $\tilde{r} = (1 - \tau^y)r - \delta$, is also very closely captured by the model.

5 Out of steady-state model dynamics

Since the model does not have an analytical solution for the equilibrium behavior of variables outside their steady-state values, we need to solve the model numerically. This is done by log-linearizing the original equilibrium (non-linear) system of equations around the steady-state. This transformation produces a first-order system of stochastic difference equations. First, we study the dynamic behavior of model variables to an isolated shock to the total factor productivity process, and then we fully simulate the model to compare how the second moments of the model perform when compared against their empirical counterparts. Special focus is put on the cyclical behavior of labor market variables.

5.1 Impulse Response Analysis

This subsection documents the impulse responses of model variables to a 1% surprise innovation to technology. The impulse response function (IRFs) are presented in Fig. 1 below.

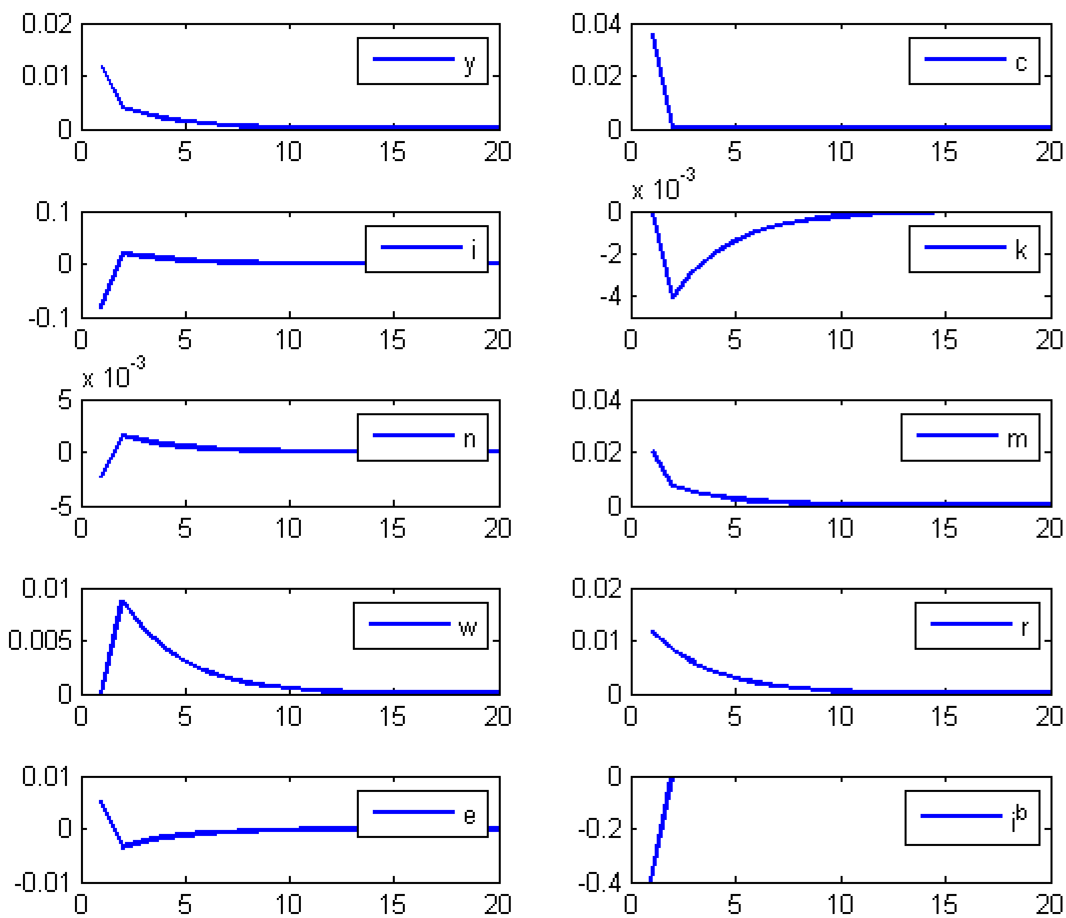

Figure 1: Impulse Responses to a 1% surprise innovation in technology

As a result of the one-time unexpected positive shock to total factor productivity, output increases. This expands the availability of resources in the economy, so consumption and government consumption also increase upon impact. Note that investment is countercyclical, due to the presence of private consumption, private investment, and government consumption in the CIA constraint, which means that those output components are to a certain degree substitutes. More specifically, government consumption moves perfectly with output, and consumption reacts more than output to the technology shock, so investment has to move down.

This new dynamics is driven by the nominal wage contracts. Since wages are pre-determined one period in advance, and prices increase following the increase in output, real wages fall upon impact of the technology shock. As a result, hours and employment fall. In turn, real interest fall due to the fall in the marginal product of labor. This is because hours and capital are complements in the production function. The decrease in the return on capital drives down investment and capital accumulation. In the money market, the increase in output increases the transaction demand for money and decreases the demand for bonds. As a result, the price of bonds increases, and the nominal interest rate on bonds decreases.

In the period following the unexpected innovation in technology real wages adjust and increase. All households respond to the incentives contained in prices and start accumulating capital, and supplying more hours worked. In the labor market, the increase in the marginal product of labor also makes the value of marginal product of labor higher, so firms increase employment. In turn, the increase in employment further increases output. After this delayed effect, the model variables return to their old steady-states in a monotone fashion as the effect of the one-time surprise innovation in technology dies out.

5.2 Simulation and moment-matching

We will now simulate the model 10,000 times for the length of the data horizon. Both empirical and model simulated data is detrended using the Hodrick-Prescott (1980) filter. Table 3 on the next page summarizes the second moments of data (relative volatilities to output, and contemporaneous correlations with output) versus the same moments computed from the model-simulated data at annual frequency.[10] To minimize the sample error, the simulated moments are averaged out over the computer-generated draws. The model matches quite well the absolute volatility of output. However, the model substantially overestimates the variability in consumption, and investment. This shortcoming of the model could be

[10]The model-predicted 95 % confidence intervals are available upon request.

explained by structural factors in Bulgaria, such as privatization of state assets, and the short annual time series for Bulgaria. In addition, public investment in infrastructure has been also substantial in the last few years due to the EU accession funds. Still, the model is qualitatively consistent with the stylized fact that investment is more volatile than output. By construction, government spending in the model varies as much as in data.

Table 3: Business Cycle Moments

	Data	Model
σ_y	0.05	0.05
σ_c/σ_y	0.55	1.68
σ_i/σ_y	1.77	2.67
σ_g/σ_y	1.21	1.00
σ_e/σ_y	0.63	0.32
σ_w/σ_y	0.83	1.12
$\sigma_{y/n}/\sigma_y$	0.86	1.12
σ_u/σ_y	3.22	0.32
$corr(c, y)$	0.85	0.83
$corr(i, y)$	0.61	-0.27
$corr(g, y)$	0.31	1.00
$corr(n, y)$	0.49	0.42
$corr(w, y)$	-0.01	0.82
$corr(u, y)$	-0.47	-0.41
$corr(n, y/n)$	-0.14	0.68

With respect to the labor market variables, the variability of employment predicted by the model is less than in data, but the variability of wages in the model is higher than that in data.The model fails in matching unemployment volatility. In the model it varies as much as the employment rate. The reason behind this mismatch could be driven by several possible explanatory factors: the fact that the model misses the "out-of the-labor-force" segment, as well as the significant emigration to EU member states.

Next, in terms of contemporaneous correlations, the model slightly over-predicts the procyclicality of the main aggregate variables - consumption and government consumption. This, however, is a common limitation of this class of models. In addition, investment is counter-cyclical due to the presence of a modified CIA constraint that incorporates also investment and government consumption. Still, along the labor market dimension, the contemporaneous correlation of employment with output, and unemployment with output, is relatively well-matched. With wages, the model predicts strong cyclicality, while wages in data are acyclical.

In the next subsection, we investigate the dynamic correlation between labor market variables at different leads and lags, thus evaluating how well the model matches the phase dynamics among variables. In addition, the autocorrelation functions (ACFs) of empirical data, obtained from an unrestricted VAR(1) are put under scrutiny and compared and contrasted to the simulated counterparts generated from the model.

5.3 Auto- and cross-correlation

This subsection discusses the auto-(ACFs) and cross-correlation functions (CCFs) of the major model variables. The coefficients empirical ACFs and CCFs at different leads and lags are presented in Table 4 against the simulated AFCs and CCFs. Following Canova (2007), this comparison is used as a goodness-of-fit measure. As seen from Table 4 on the next page, the model compares well vis-a-vis data. Empirical ACFs for output and investment are slightly outside the confidence band predicted by the model, while the ACFs for total factor productivity and household consumption are well-approximated by the model.

The persistence of labor market variables are also well-described by the model dynamics: the ACFs unemployment and wages are close to the simulated ones until the third lag. Same holds true for output and investment. The ACF for consumption and employment is well-captured only until the first lag. Overall, the model with one-period nominal wage contracts generates the right persistence in model variables, and is able to respond to the criticism in Nelson and Plosser (1992), Cogley and Nason (1995) and Rotemberg and Woodford (1996), who argue that the RBC class of models do not have a strong internal propagation mech-

Table 4: Autocorrelations for Bulgarian data and the model economy

		k			
Method	Statistic	0	1	2	3
Data	$corr(u_t, u_{t-k})$	1.000	0.765	0.552	0.553
Model	$corr(u_t, u_{t-k})$	1.000	0.818	0.629	0.442
	(s.e.)	(0.000)	(0.035)	(0.063)	(0.084)
Data	$corr(e_t, e_{t-k})$	1.000	0.484	0.009	0.352
Model	$corr(e_t, e_{t-k})$	1.000	0.818	0.629	0.442
	(s.e.)	(0.000)	(0.035)	(0.063)	(0.084)
Data	$corr(y_t, y_{t-k})$	1.000	0.810	0.663	0.479
Model	$corr(y_t, y_{t-k})$	1.000	0.915	0.625	0.438
	(s.e.)	(0.000)	(0.039)	(0.070)	(0.094)
Data	$corr(a_t, a_{t-k})$	1.000	0.702	0.449	0.277
Model	$corr(a_t, a_{t-k})$	1.000	0.814	0.624	0.437
	(s.e.)	(0.000)	(0.038)	(0.072)	(0.096)
Data	$corr(c_t, c_{t-k})$	1.000	0.971	0.952	0.913
Model	$corr(c_t, c_{t-k})$	1.000	0.816	0.626	0.439
	(s.e.)	(0.000)	(0.036)	(0.065)	(0.089)
Data	$corr(i_t, i_{t-k})$	1.000	0.810	0.722	0.594
Model	$corr(i_t, i_{t-k})$	1.000	0.816	0.629	0.442
	(s.e.)	(0.000)	(0.038)	(0.063)	(0.084)
Data	$corr(w_t, w_{t-k})$	1.000	0.760	0.783	0.554
Model	$corr(w_t, w_{t-k})$	1.000	0.816	0.628	0.442
	(s.e.)	(0.000)	(0.035)	(0.063)	(0.084)

anism besides the strong persistence in the TFP process. Furthermore, the nominal wage mechanism dominates other non-Walrasian models such as Vasilev (2016, 2017b,d).

Next, as seen from Table 5 on the next page, over the business cycle, in data labor productivity leads employment. The model with nominal wage contracts, however, cannot account for this fact. In this model, as well as in the standard RBC model a technology shock can be regarded as a factor shifting the labor demand curve, while holding the labor supply curve constant. Therefore, the effect between employment and labor productivity is only a contemporaneous one. Still, the model with nominal wage contracts is a clear improvement over the perfectly-competitive labor market paradigm used in Vasilev (2009).

Table 5: Dynamic correlations for Bulgarian data and the model economy

		k						
Method	Statistic	-3	-2	-1	0	1	2	3
Data	$corr(n_t, (y/n)_{t-k})$	-0.342	-0.363	-0.187	-0.144	0.475	0.470	0.346
Model	$corr(n_t, (y/n)_{t-k})$	-0.023	-0.017	-0.040	0.708	0.022	0.016	0.012
	(s.e.)	(0.738)	(0.648)	(0.532)	(0.572)	(0.516)	(0.635)	(0.724)
Data	$corr(n_t, w_{t-k})$	0.355	0.452	0.447	0.328	-0.040	-0.390	-0.57
Model	$corr(n_t, w_{t-k})$	-0.023	-0.017	-0.040	0.708	0.022	0.016	0.012
	(s.e.)	(0.738)	(0.648)	(0.532)	(0.572)	(0.516)	(0.635)	(0.724)

6 Conclusions

We augment an otherwise standard business cycle model with a richer government sector, and add a modified cash in advance considerations, and one-period-ahead nominal wage contracts. In particular, the cash in advance constraint of Cooley and Hansen (1989) is extended to include private investment and government consumption. This specification, together with the nominal wage rigidity, when calibrated to Bulgarian data after the introduction of the currency board (1999-2016), gives a role to money in propagating economic fluctuations. In addition, the combinations of these ingredients allows the framework to reproduce better observed variability and correlations among model variables, and those characterizing the labor market in particular. These results suggest that technology shocks seem to be the dominant source of economic fluctuations, but nominal wage contracting might be an important aspect of the labor markets in Bulgaria, which should be incorporated in any model that studies cyclical movements in employment and wages. Still, the model suffers from some of the usual shortcomings inherent in this class of RBC models. As a suggestion for future research, the model might be extended to accommodate other important (and real) frictions in the labor market, possibly along the lines of Vasilev (2016, 2017b, 2017d), and study the interplay between nominal and real rigidities.

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