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# A Business-cycle-model with Monopolistically-competitive Firms and Calvo Wages: The Case of Bulgaria after the Introduction of the Currency Board (1999-2016)

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# A business-cycle-model with

monopolistically-competitive firms and Calvo wages: the case of Bulgaria after the introduction of the currency board (1999-2016)

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#### Abstract

We augment an otherwise standard business cycle model with a richer government sector, and add monopolistic competition in the product market, and rigid prices, as well as rigid wages a la Calvo (1983) in the labor market. This specification with the nominal wage rigidity, when calibrated to Bulgarian data after the introduction of the currency board (1999-2016), allows the framework to reproduce better observed variability and correlations among model variables, and those characterizing the labor market in particular. As nominal wage frictions are incorporated, the variables become more persistent, especially output, capital stock, investment and consumption, which helps the model match data better.

**Keywords:** business cycles, monopolistic competition, rigid (Calvo) prices, rigid (Calvo) nominal wages

JEL Classification: D43, D58, E32

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### 1 Introduction

As shown in Vasilev (2009), the standard real-business-cycle (RBC) model does not capture well model dynamics for Bulgaria. One explanation for this failute could be the assumption of perfect competition imposed everywhere in the model, which might be too restrictive for a transition economy such as Bulgaria. Instead, as demonstrated in Lozev et al. (2011) and Vladova (2016) imperfections in the product and factor markets are observed in Bulgaria. In light of this evidence, in this paper we take those phenomena seriously in our modelling strategy, and adopt a New Keynesian approach, which departs from perfect competition in goods and factors market, and as a result, the prices of the factors of production no longer will equal their marginal products. In addition, instead of a stand-in firm, there will be imperfect competition in the intermediate goods firms, and the differentiated intermediate goods are then combined into a final good, which is produced by a perfectly-competitive firm. However, the final stage is identical to the standard Dynamic Stochastic General Equilibrium (DSGE) model.

Another important difference from the RBC model is that the prices of production inputs depend on the elasticity of substitution between the differentiated intermediate goods, which in turn reflects the market power of monopolistically-competitive firms to set prices. The distortion driven by the industry structure pushes the prices of labor and capital below their marginal products. More specifically, as shown in Rotemberd and Woodford (1995), the higher the market power of monopolistic firms, the higher the mark-up, and as a result, the greater the difference between the marginal product of labor and capital and their prices.<sup>2</sup> In addition, with imperfect competition in the output market, a technology shock affecting the marginal product of capital and labor leads to lower reaction of both the real wage rate and the real interest rate (compared to the perfectly competitive environment). As a result, the owners of the two factors of production will perceive a smaller effect from the productivity shock, and that will drive down the use of labor and capital, and that would lead to lower

<sup>&</sup>lt;sup>1</sup>This modelling approach was initially developed by Rotemberg (1982), Blanchard and Kiyotaki (1987), and Rotemberg and Woodford (1992, 1995), among others.

<sup>&</sup>lt;sup>2</sup>As pointed out in Torres (2013), this is a direct consequence of the assumption that the elasticity of substitution between differentiated goods is strictly greater than unity.

output at intermediate- and final-good level. In turn, the model no longer generates efficient allocations in equilibrium.

Another important novelty in this paper, which distinguishes the model setup from the standard New Keynesian model, would be the presence of rigidities in the wage determination process a la Canzoneri et al. (2005) and Christiano et al. (2005). The stickiness in nominal wages is an important ingredient in the transmission of technology shocks, and a rigidity which could potentially generate employment and wage fluctuations similar to the ones exhibited in Bulgarian data. In order to accommodate those, it will be also assumed that labor services are differentiated among households. This assumption implies that households also possess certain market power in setting their nominal wages.<sup>3</sup>

The rest of the paper is organized as follows: Section 2 describes the model framework and defines the decentralized monopolistically-competitive equilibrium system. Section 3 discusses the calibration procedure, and Section 4 presents the steady-state model solution. Sections 5 proceeds with the out-of-steady-state dynamics of model variables, and compared the simulated second moments of theoretical variables against their empirical counterparts. Section 6 concludes the paper.

## 2 Model Setup

There is a continuum of ex ante identical one-member households distributed uniformly on the unit interval, and indexed by j. Final output is obtained through the aggregation of intermediate good outputs, in an environment of perfect competition, and can be used for household consumption, investment, or government purchases. In contrast, in the intermediate goods sector, there is monopolistic competition with free entry, which means that each intermediate good is produced by a single monopolistic firm, which has market power and sets the price of the particular good they produce at a mark-up above their marginal cost. Lastly, the government is levying taxes on consumption, labor and capital income in order

<sup>&</sup>lt;sup>3</sup>This assumption could be easily rationalized with the presence of labor unions, as well as certain provisions in the Bulgarian Labor code, which protect workers' interests in labor diputes with employers. Both explanations are empirically plausible in Bulgaria, as demonstrated in Paskaleva (2016).

to finance spending on government purchases and lump-sum government transfers.

#### 2.1 Households' problem

Household j maximizes the expected discounted utility, which is of the form<sup>4</sup>

$$U = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{h_t^{1+\varphi}}{1+\varphi} \right\},\tag{1}$$

where  $E_t$  is the expectations operator as of period t,  $0 < \beta < 1$  denotes the discount factor,  $c_t$  is consumption of household j in period t,  $h_t$  are the hours worked by household j in period t,  $\sigma$  is the relative risk aversion parameter (and the inverse of the intertemporal elasticity of substitution parameter), and parameter  $\varphi$  denotes the curvature of the function capturing the disutility of hours worked.

The household starts with a unit endowment of time in each time period, and a positive endowment of physical capital,  $k_0$ , in period 0, which is rented to the firm at the nominal rental rate  $R_t$ , that is, before-tax capital income equals  $R_t k_t$ . Therefore, each household can decide to invest in capital to augment the capital stock, which evolves according to the following law of motion:

$$k_{t+1} = i_t + (1 - \delta)k_t, \tag{2}$$

where  $0 < \delta < 1$  is the depreciation rate of physical capital.

In addition to the rental income, each household owns an equal share of the final-goodproducing firm, and thus has a legal claim to the firms' nominal profit,  $\Pi_t$ . Household j's period t budget constraint is then

$$P_t(c_t + k_{t+1} - (1 - \delta)k_t) = W_t h_t + R_t k_t + \Pi_t + P_t g_t^t, \tag{3}$$

where  $P_t$  is the aggregate price index, and  $g_t^t$  are per household government transfers.

The problem faced by each household is then to choose  $\{c_t, h_t, k_{t+1}, W_t\}_{t=0}^{\infty}$  to maximize

 $<sup>^4</sup>$ To simplify notation, we will suppress the j index and use smallcase letters to denote individual allocations, and capital letters for aggregate quantities.

utility subject to budget constaint.<sup>5</sup>

The first-order conditions for the allocations are as follows:

$$c_t : c_t^{-\sigma} = \lambda_t P_t \tag{4}$$

$$h_t : h_t^{\varphi} = \lambda_t W_t \tag{5}$$

$$k_{t+1}$$
:  $\lambda_t P_t = \beta E_t \lambda_{t+1} [(1 - \delta) E_t P_t + E_t R_{t+1}]$  (6)

$$TVC : \lim_{t \to \infty} \beta^t \lambda^t k_{t+1} + 0. \tag{7}$$

The interpretation of the optimality conditions above is standard: the first states that at the margin, optimal consumption is characterized by the balance between the benefit of extra consumption utility and the cost in terms of shadow price of wealth. The second equation balances the disutility of extra work and the benefit in terms of extra income, weighted by consumption utility. The third equation, the so-called Euler equation, describes how capital should be allocated in any two congruent periods. The last condition, the "Transversality condition," is a boundary constraint, in order to rule out explosive solution paths.

In addition, since labor can be a differentiated product among households, this implies that households have some market power when setting wages. As in Christiano et al. (2005), and Canzoneri et al. (2005), each household supplies differentiated labor services in a market structure of monopolistic competition. These labor services are rented to a representative firm that aggregates these different types of labor  $h_j$  into a single labor input H. As in Junior (2016), the labor-aggregating firm is assumed to use the following constant-elasticity-of-substitution (CES) technology:

$$H_{t} = \left( \int_{0}^{1} h_{j,t}^{\frac{\psi_{W}-1}{\psi_{W}}} dj \right)^{\frac{\psi_{W}}{\psi_{W}-1}}, \tag{8}$$

where  $\psi_W$  is the elasticity of substitution between differentiated labor services and  $h_{j,t}$  is the amount of differentiated labor hours supplied by household j. Each type of labor hours j is paid for with a nominal wage  $W_{j,t}$ . The problem of the labor-aggregating firm is then to maximize its static profit:

$$\max_{h_{j,t}} W_t H_t - \int_0^1 W_{j,t} h_{j,t} dj$$
 (9)

<sup>&</sup>lt;sup>5</sup>We postpone the discussion of optimal wage-setting until later.

s.t. the aggregation constraint above, or

$$\max_{h_{j,t}} W_t \left( \int_0^1 h_{j,t}^{\frac{\psi_W - 1}{\psi_W}} dj \right)^{\frac{\psi_W}{\psi_W - 1}} - \int_0^1 W_{j,t} h_{j,t} dj. \tag{10}$$

The first-order condition for each type of differentiated labor services is

$$W_t \left(\frac{\psi_W}{\psi_W - 1}\right) \left(\int_0^1 h_{j,t}^{\frac{\psi_W - 1}{\psi_W}} dj\right)^{\frac{\psi_W}{\psi_W - 1} - 1} \left(\frac{\psi_W - 1}{\psi_W}\right) h_{j,t}^{\frac{\psi_W - 1}{\psi_W} - 1} - W_{j,t} = 0 \tag{11}$$

or,

$$W_t \left( \int_0^1 h_{j,t}^{\frac{\psi_W - 1}{\psi_W}} dj \right)^{\frac{1}{\psi_W - 1}} h_{j,t}^{-\frac{1}{\psi_W}} - W_{j,t} = 0$$
 (12)

Next, noting that we can express aggregate hours as follows:

$$H_t^{\frac{1}{\psi_W}} = \left( \int_0^1 h_{j,t}^{\frac{\psi_W - 1}{\psi_W}} dj \right)^{\frac{1}{\psi_W - 1}}, \tag{13}$$

so we can replace it in the following equation:

$$W_t H_t^{\frac{1}{t^{\psi_W}}} h_{j,t}^{-\frac{1}{t^{\psi_W}}} - W_{j,t} = 0.$$
 (14)

After some algebra, we can express the demand for  $h_j$  as follows:

$$h_{j,t} = H_t \left(\frac{W_t}{W_{j,t}}\right)^{\psi_W} \tag{15}$$

Substitute now this expression back into aggregate hours to solve for the aggregate wage rate as a function of household-specific wage rates

$$W_{t} = \left(\int_{0}^{1} W_{j,t}^{1-\psi_{W}} dj\right)^{\frac{1}{1-\psi_{W}}} \tag{16}$$

In terms of the wage rigidity, in each period,  $1 - \theta_W$  households, chosen independently and at random, optimally define/set their wages in nominal terms. The remaining households,  $\theta_W$ , follow a wage stickiness rule a la Calvo (1983), and keep the same wage level as the previous period, or,  $W_{j,t} = W_{j,t-1}$ . In particular, the  $1 - \theta_W$  fraction of households that can choose wage levels in period t knows that, even setting optimal nominal wage  $W_{j,t}^*$  for the period, it faces a  $\theta_W^N$  probability of these wages remaining fixed for N future period. When household j chooses  $W_{j,t}^*$  to solve the following problem<sup>6</sup>

$$\max_{W_{j,t}^*} E_t \sum_{t=0}^{\infty} (\beta \theta_W)^t \left[ \dots - \frac{h_t^{1+\varphi}}{1+\varphi} - \lambda_t (\dots - W_j, t^* h_t) \right]$$

$$\tag{17}$$

<sup>&</sup>lt;sup>6</sup>As in Junior (2016), we keep only the relevant terms in the problem below.

s.t.

$$h_t = H_t \left(\frac{W_t}{W_{j,t}}\right)^{\psi_W} \tag{18}$$

Substituting the expression into the objective function

$$\max_{W_{j,t}^*} E_t \sum_{t=0}^{\infty} (\beta \theta_W)^t \left[ \dots - \frac{1}{1+\varphi} \left[ H_t \left( \frac{W_t}{W_{j,t}} \right)^{\psi_W} \right]^{1+\varphi} - \lambda_t (\dots - W_{j,t}^* H_t \left( \frac{W_t}{W_{j,t}} \right)^{\psi_W} ) \right]$$
(19)

After some algebraic manipulations, and using that  $\lambda_t = c_t^{-\sigma}/P_t$ , we can derive the expression for the optimal wage equation set by household j:

$$W_{j,t}^* = \left(\frac{\psi_W}{\psi_W - 1}\right) E_t \sum_{t=0}^{\infty} (\beta \theta_W)^t c_t^{\sigma} h_t^{\varphi} P_t \tag{20}$$

As  $1 - \theta_W$  fraction of households chooses the same nominal wages,  $W_{j,t}^* = W_t^*$ , and the mass of remaining households,  $\theta_W$ , set their wage equal to the nominal wage observed in the previous period. Thus, the aggregate nominal wage can be expressed as

$$W_t^{1-\psi_W} = \int_0^{\theta_W} W_{t-1}^{1-\psi_W} dj + \int_{\theta_W}^1 W_t^{*1-\psi_W} dj$$
$$W_t^{1-\psi_W} = [jW_{t-1}^{1-\psi_W}]_0^{\theta_W} + [W_t^{*1-\psi_W}]_{\theta_W}^1$$
$$W_t^{1-\psi_W} = \theta_W W_{t-1}^{1-\psi_W} + (1-\theta_W) W_t^{*1-\psi_W},$$

hence the aggregate nominal wage rule is:

$$W_t = \left[\theta_W W_{t-1}^{1-\psi_W} + (1-\theta_W) W_t^{*1-\psi_W}\right]^{\frac{1}{1-\psi_W}} \tag{21}$$

#### **2.2** Firms

The modelling approach of the industry structure in the setup follows Dixit and Stiglitz (1977), with a continuum of differentiated goods. In turn, these differentiated goods are then aggregated into a single final goods, which is consumed by the households. Each firm produces a single intermediate good that the final producer then uses as an input in the production of the final good via a CES output aggregator function. The final good producer takes prices as given, while intermediate good producers have power over setting their own prices.

#### 2.2.1 Final goods production sector (retail)

The functional form chosen for the aggregation technology is

$$Y_{t} = \left(\int_{0}^{1} Y_{j,t}^{\frac{\psi-1}{\psi}} dj\right)^{\frac{\psi}{\psi-1}},\tag{22}$$

where  $Y_t$  is aggregate output (the product of the retailer) in period t, and  $Y_{j,t}$ ,  $j \in [0,1]$  is the output of intermediate (wholesale) good j, and  $\psi > 1$  denotes the elasticity of substitution between the differentiated wholesale goods.

Let  $P_{j,t}$  denote the nominal price of wholesale good j, the price of each wholesale good is taken as a given by retail firms. The problem faced by each retail firm is then to

$$\max_{Y_{j,t}} P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj \tag{23}$$

s.t. (22), or when plugging that expression back into the objective function, to

$$\max_{Y_{j,t}} P_t \left( \int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}} - \int_0^1 P_{j,t} Y_{j,t} dj \tag{24}$$

Taking the first-order condition, and after some algebra we obtain

$$Y_{j,t} = Y_t \left(\frac{P_t}{P_{i,t}}\right)^{\psi}. (25)$$

In other words, the individual demand is proportional to aggregate demand, and inversely proportional to relative price level. Now substitute this expression back into aggregate output to obtain the expression for the price of the final (retail) goods in term of the prices of the intermediate goods:

$$P_{t} = \left(\int_{0}^{1} P_{j,t}^{1-\psi} dj\right)^{\frac{1}{1-\psi}},\tag{26}$$

which is also the aggregate price index.

#### 2.2.2 Intermediate goods production sector (wholesale)

As pointed out earlier, each wholesale firms sell their differentiated goods to the stand-in retail (final-goods) firm. Intermediate-good producers will be assumed to possess some market

power and will have some power in setting the price of their product (facing a downward-sloping demand for their product). In addition, it will be assumed that fixed (entry or period) costs do not exist. Since the retailer has constant-returns-to-scale technology, its marginal cost is independent of quantity produced. Furthermote, the marginal cost function coincides with the average cost function, and total cost equals the product of marginal cost times quantity. Net, the retail firm's problem can be split in two parts. In the first, the prices of capital and labor are taken as given, and the firm minimizes total cost subject to the production function (the technology constraint), or

$$\min_{h_t, k_t} W_t h_t + R_t k_t \tag{27}$$

s.t

$$Y_t = A_t k_t^{\alpha} h_t^{1-\alpha} \tag{28}$$

The FOCs are

$$h_t: (1-\alpha)\mu_t A_t k_t^{\alpha} h_t^{-\alpha} = W_t \tag{29}$$

$$k_t: \alpha \mu_t A_t k_t^{\alpha - 1} h_t^{1 - \alpha} = R_t, \tag{30}$$

where  $\mu_t$  is the Lagrange multiplier attached to the constraint. With  $\mu_t = MC_{j,t}$  the equation above become

$$h_t = (1 - \alpha)MC_{j,t} \frac{Y_{j,t}}{W_t}$$
 (31)

$$k_t = \alpha M C_{j,t} \frac{Y_{j,t}}{R_t} \tag{32}$$

The expressions above are the optimal demand for the two inputs (capital and labor) by each wholesale firm. Deriving the total and marginal cost function can be done from the dual problem - the profit maximization one:

$$\max_{h_t, k_t} \pi_t = A_t k_t^{\alpha} h_t^{1-\alpha} P_{j,t} - W_t h_t - R_t k_t$$
(33)

<sup>&</sup>lt;sup>7</sup>Actually, that assumption is not that relevant - we can have an equivalent representation with fixed "period costs" and free entry, which leads the firm to set price above mc to cover the amount of the fixed costs and have 0 profit. This is the representation utilized in Torres (2013).

FOCs:

$$k_t : R_t = \alpha \frac{Y_{j,t}}{k_t} \tag{34}$$

$$h_t : W_t = (1 - \alpha) \frac{Y_{j,t}}{h_t}$$
 (35)

After some algebra, we arrive at the following expression:<sup>8</sup>

$$MC_{j,t} = \frac{1}{A_t} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{R_t}{\alpha}\right)^{\alpha} \tag{37}$$

Next, the second stage of the problem of the wholesale firm j is optimally-setting the price of its product. This firm decides how much to produce in each period a la Calvo (Calvo 1983). More specifically, in each period, each wholesale firm has a  $\theta$  probability of keeping the price of its good unchanged in the next period  $(P_{j,t} = P_{j,t-1})$  and a  $1 - \theta$  probability of optimally setting its price. Therefore, the problem of a wholesale firm j that is able to reset the price of its good is

$$\max_{P_{j,t}^*} E_t \sum_{t=0}^{\infty} (\beta \theta)^t (P_{j,t}^* Y_{j,t} - T C_{j,t+i})$$
(38)

s.t. demand constraint, or

$$\max_{P_{j,t}^*} E_t \sum_{t=0}^{\infty} (\beta \theta)^t (P_{j,t}^* = Y_t \left(\frac{P_t}{P_{j,t}}\right)^{\psi} - = Y_t \left(\frac{P_t}{P_{j,t}}\right)^{\psi} MC_{j,t+i})$$
(39)

The first-order condition (after some algebra)

$$P_{j,t}^* = \left(\frac{\psi}{\psi - 1}\right) E_t \sum_{t=0}^{\infty} (\beta \theta)^t M C_{j,t}$$
(40)

Note that all wholesale firms that fix their price have the same mark-up on the same marginal cost. Thus, in all periods,  $P_{j,t}^*$  is the same price for all the  $1 - \theta$  firms that set their prices.

$$MC_{i,t} = TC_{i,t}/Y_{i,t}. (36)$$

<sup>&</sup>lt;sup>8</sup>Note that the expression below is consistent with the requirement that

Following the argument in Junior (2016), the aggregate price level is

$$P_{t}^{1-\psi} = \int_{0}^{\theta} P_{t-1}^{1-\psi} dj + \int_{\theta}^{1} P_{t}^{*1-\psi} dj$$

$$P_{t}^{1-\psi} = [jP_{t-1}^{1-\psi}]_{0}^{\theta} + [jP_{t}^{*1-\psi}]_{\theta}^{1}$$

$$P_{t}^{1-\psi} = \theta P_{t-1}^{1-\psi} + (1-\theta)P_{t}^{*1-\psi}$$

$$P_{t} = [\theta P_{t-1}^{1-\psi} + (1-\theta)P_{t}^{*1-\psi}]_{1-\psi}^{\frac{1}{1-\psi}}$$
(41)

Note that since there is a continuum of intermediate good producers, and the share that can reset its price (and the group that cannot) is chosen randomly, regardless of when each firm last altered its price. As a result, the distribution of prices among firms does not change between periods.

#### 2.3 Government

In the model setup, the government is levying taxes on labor and capital income, as well as consumption in order to finance spending on government purchases and government transfers. The government budget constraint is as follows:<sup>9</sup>

$$\tau^{c}c_{t} + \tau^{y}(w_{t}h_{t} + r_{t}k_{t}) = g_{t}^{t} + g_{t}^{c}$$
(42)

Tax rates and government consumption-to-output ratio would be chosen to match the average share in data, and government transfers would be determined residually.

## 2.4 Stochastic process

Total factor productivity,  $A_t$ , is assumed to follow AR(1) processes in logs, in particular

$$\ln A_{t+1} = (1 - \rho_a) \ln A_0 + \rho_a \ln A_t + \epsilon_{t+1}^a,$$

where  $A_0 > 0$  is steady-state level of the total factor productivity process,  $0 < \rho_a < 1$  is the first-order autoregressive persistence parameter and  $\epsilon_t^a \sim iidN(0, \sigma_a^2)$  are random shocks to the total factor productivity progress. Hence, the innovations  $\epsilon_t^a$  represent unexpected changes in the total factor productivity process.

<sup>&</sup>lt;sup>9</sup>Given that there is a unit mass of households, individual and total allocations are identical.

#### 2.5 Dynamic Monopolistically-Competitive Equilibrium (DMCE)

Given the processes followed by the stochastic process  $\{A_t\}_{t=0}^{\infty}$ , average tax rates  $\{\tau^c, \tau^y\}$ , endowments  $k_0 \forall j$ , the decentralized dynamic competitive equilibrium is a list of sequences  $\{c_t, i_t, h_t\}_{t=0}^{\infty}$ , a sequence of government purchases and transfers  $\{g_t^c, g_t^t\}_{t=0}^{\infty}$ , price level sequence  $\{P_t\}_{t=0}^{\infty}$  and prices  $\{w_t, r_t\}_{t=0}^{\infty}$  such that (i) each household j maximizes its utility function subject to its budget constraint, (ii) the representative final-good firm maximizes profit; (iii) intermediate-good firms maximize profit; (iv) government budget is balanced in each period; (v) all markets clear.

### 3 Data and Model Calibration

To calibrate the model to Bulgarian data, we will focus on the period after the introduction of the currency board (1999-2016). Annual data on output, consumption and investment was collected from National Statistical Institute (2016), while the real interest rate is taken from Bulgarian National Bank Statistical Database (2016). The calibration strategy described in this section follows a long-established tradition in modern macroeconomics: first, the discount factor,  $\beta=0.982$ , as in Vasilev (2017a), is set to match the steady-state capital-to-output ratio in Bulgaria, k/y=3.491. The labor share parameter,  $\alpha=0.429$ , was obtained from Vasilev (2017b) as the average value of labor income in aggregate output over the period 1999-2016. The depreciation rate of physical capital in Bulgaria,  $\delta=0.05$ , was taken from Vasilev (2015). It was estimated as the average depreciation rate over the period 1999-2016. The curvature parameters of consumption and leisure components in the household's utility function are set to  $\sigma=2$  and  $\varphi=1.5$  in order to generate plausible value for aggregate labor supply elasticity. The average income tax rate was set to  $\tau^y=0.1$ , and the tax rate on consumption,  $\tau^c=0.2$ , are set to their values over the period.

Next, the elasticity of substitution between differentiated intermediate goods was set to  $\psi = 5$  to generate the average mark-up of 25 percent estimated by Korosi *et al.* (2004) for Bulgaria. Similarly, the elasticity of substitution between differentiated labor services was set to  $\psi_W = 7$  to generate a wedge between marginal utility of consumption and the marginal disutility of leisure of 17 percent, which is the average inter-industry wage difference. The

price stickiness parameter was set to  $\theta = 0.74$  following Paskaleva (2016)'s estimate on the share of firms that do not change prices. Similarly, as in Lozev et al. (2011) the wage stickiness parameter,  $\theta_W = 0.68$ , was set to the share of firms that set wages to the previous period wages. Al those values are consistent with the values of those parameters in the literature for the US and other EU countries. Lastly, as in Vasilev (2017c), processes followed by total factor productivity and money growth, are estimated from the detrended series by running an AR(1) regression and saving the residuals. Table 1 below summarizes the values of all model parameters used in the paper.

Table 1: Model Parameters						
Parameter	Value	Description	Method			
β	0.982	Discount factor	Calibrated			
$\alpha$	0.429	Capital Share	Data average			
$1 - \alpha$	0.571	Labor Share	Calibrated			
$\delta$	0.050	Depreciation rate on physical capital	Data average			
arphi	1.500	Curvature, disutility of work	Set			
$\sigma$	2.000	Curvature, utility of consumption	Set			
h	0.333	Share of time spent working	Data average			
$ au^c$	0.200	VAT/consumption tax rate	Data average			
$ au^y$	0.100	Average tax rate on income	Data average			
$\theta$	0.740	Price stickiness parameter	Data average			
$ heta_W$	0.680	Wage stickiness parameter	Data average			
$\psi$	5.000	Elasticity of substitution, intermediate goods	Calibrated			
$\psi_W$	7.000	Elasticity of substitution, differentiated labor	Calibrated			
$ ho_a$	0.701	AR(1) parameter, total factor productivity	Estimated			
$\sigma_a$	0.044	st.dev, total factor productivity	Estimated			

## 4 Steady-State

Once the values of model parameters were obtained, the steady-state equilibrium system solved, the "big ratios" can be compared to their averages in Bulgarian data. The results

are reported in Table 2 below. The model matches consumption-to-output ratio by construction; The investment and government purchases ratios are also closely approximated. The shares of income are also identical to those in data, which is an artifact of the assumptions imposed on functional form of the aggregate production function. The after-tax return, net of depreciation,  $\tilde{r} = (1 - \tau^y)r - \delta$ , is also very closely captured by the model.

Table 2: Data Averages and Long-run Solution

Table 2. Data riverages and Long run polition					
Variable	Description	Data	Model		
c/y	Consumption-to-output ratio	0.674	0.674		
i/y	Investment-to-output ratio	0.201	0.175		
$g^c/y$	Government cons-to-output ratio	0.159	0.151		
wh/y	Labor income-to-output ratio	0.571	0.571		
rk/y	Capital income-to-output ratio	0.429	0.429		
$ ilde{r}$	After-tax net return on capital	0.056	0.057		

## 5 Out of steady-state model dynamics

Since the model does not have an analytical solution for the equilibrium behavior of variables outside their steady-state values, we need to solve the model numerically. This is done by log-linearizing the original equilibrium (non-linear) system of equations around the steady-state. This transformation produces a first-order system of stochastic difference equations. First, we study the dynamic behavior of model variables to an isolated shock to the total factor productivity process, and then we fully simulate the model to compare how the second moments of the model perform when compared against their empirical counterparts. Special focus is put on the cyclical behavior of labor market variables.

## 5.1 Impulse Response Analysis

This subsection documents the impulse responses of model variables to a 1% surprise innovation to technology. The impulse response function (IRFs) are presented in Fig. 1 on the next page. 10 As a result of the one-time unexpected positive shock to total factor productivity, output increases. This expands the availability of resources in the economy, so consumption, investment and government consumption also increase upon impact. As a result of the increase in productivity, the real interest rate increases as well, and households increase their capital accumulation. Wages also increase but the effect of technology is somehow muted because of the "Calvo" effect, which creates a rigidity in the wage rate, and a delay in the response in hours. Note that while capital becomes immediately productive, the slower response in the wage, and the resulting increase in hours arrives with a delay of one period, which mutes the feedback effect of hours on final output. This new dynamics is driven by the nominal wage rigidity as in Calvo (1983). Importantly, compared to the perfectly competitive case, the effects over the model variables are smaller upon impact, as demonstrated in Junior (2016), and Torres (2013), since the inefficiencies produced by imperfect competition at intermediate level itself reduce the effect of the technology shock. In addition, with nominal wage frictions, model variables become more persistent, especially output, capital stock and consumption. The "adjustment stickiness" of wages also causes households' labor supply to initially increase more than in other models, e.g. the standard RBC, and New Keynesian models without wage frictions, as shown in Junior (2016).

Over time, as the effect of the shock wanes, the return on capital decreases, which drives down investment and capital accumulation back to their old steady-state values. The other model variables return to their old values in a monotone manner as well as the effect of the one-time surprise innovation in technology dies out.

## 5.2 Simulation and moment-matching

We will now simulate the model 10,000 times for the length of the data horizon. Both empirical and model simulated data is detrended using the Hodrick-Prescott (1980) filter. Table 3 on the next page summarizes the second moments of data (relative volatilities to output, and contemporaneous correlations with output) versus the same moments computed from the model-simulated data at annual frequency.<sup>11</sup> To minimize the sample error, the simu-

<sup>&</sup>lt;sup>10</sup>The results are insensitive to the degree of wage persistence, as captured by parameter  $\psi_W$ .

 $<sup>^{11}</sup>$ The model-predicted 95 % confidence intervals are available upon request.

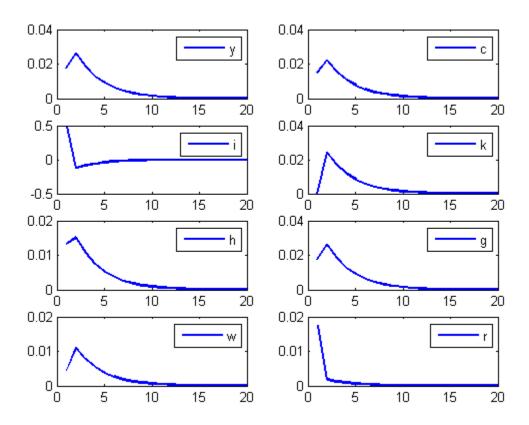


Figure 1: Impulse Responses to a 1% surprise innovation in technology

lated moments are averaged out over the computer-generated draws. The model matches quite well the absolute volatility of output. However, the model slightly overestimates the variability in consumption, and more substantially that of investment. This shortcoming of the model could be explained by structural factors in Bulgaria, such as privatization of state assets, and the short annual time series for Bulgaria. In addition, public investment in infrastructure has been also substantial in the last few years due to the EU accession funds. Still, the model is qualitatively consistent with the finding the consumption varies less than output, and investment varies more than output. By construction, government spending in the model varies as much as in data.

With respect to the labor market variables, the variability of employment predicted by the model is very close to that in data, but the variability of wages in the model is much lower

Table 3: Business Cycle Moments

	Data	Model
$\sigma_y$	0.05	0.05
$\sigma_c/\sigma_y$	0.55	0.85
$\sigma_i/\sigma_y$	1.77	3.81
$\sigma_g/\sigma_y$	1.21	1.00
$\sigma_h/\sigma_y$	0.63	0.61
$\sigma_w/\sigma_y$	0.83	0.39
$\sigma_{y/h}/\sigma_y$	0.86	0.39
$\sigma_u/\sigma_y$	3.22	0.32
$\sigma_w/\sigma_h$	1.32	0.67
corr(c,y)	0.85	1.00
corr(i,y)	0.61	0.08
corr(g,y)	0.31	1.00
corr(h,y)	0.49	0.99
corr(w,y)	-0.01	0.99
corr(u, y)	-0.47	-0.91
corr(h, y/h)	-0.14	0.99

than that in data. The model fails in matching unemployment volatility. In the model it varies as much as the employment rate. The reason behind this mismatch could be driven by several possible explanatory factors: the fact that the model misses the "out-of the-labor-force" segment, as well as the significant emigration to EU member states. Next, in terms of contemporaneous correlations, the model slightly over-predicts the pro-cyclicality of the main aggregate variables - consumption and government consumption. This, however, is a common limitation of this class of models. In addition, investment is very weakly procyclical due to the depressing effect of the Calvo-wage effect on capital and investment. With rigid wages labor becomes more important, and the substitutability between the two factors of production increases. Along the labor market dimension, the contemporaneous correlation of employment with output, and unemployment with output, is of the right sign, but the model predicts it to be quite strong, while in data the linear relationship is more

moderate. With wages, the model predicts strong cyclicality, while wages in data are acyclical. The same is true with the contemporaneous correlation between productivity and hours.

In the next subsection, we investigate the dynamic correlation between labor market variables at different leads and lags, thus evaluating how well the model matches the phase dynamics among variables. In addition, the autocorrelation functions (ACFs) of empirical data, obtained from an unrestricted VAR(1) are put under scrutiny and compared and contrasted to the simulated counterparts generated from the model.

#### 5.3 Auto- and cross-correlation

This subsection discusses the auto-(ACFs) and cross-correlation functions (CCFs) of the major model variables. The coefficients empirical ACFs and CCFs at different leads and lags are presented in Table 4 against the simulated AFCs and CCFs. Following Canova (2007), this comparison is used as a goodness-of-fit measure. As seen from Table 4 on the next page, the model compares well vis-a-vis data. Empirical ACFs for output and investment are slightly outside the confidence band predicted by the model, while the ACFs for total factor productivity and household consumption are well-approximated by the model.

The persistence of labor market variables are also well-described by the model dynamics: the ACFs unemployment and wages are close to the simulated ones until the third lag. Same holds true for output and investment. The ACF for consumption is well-captured only until the first lag. Overall, the model with persistence a la Calvo (1983) in nominal wages generates the right persistence in model variables, and is able to respond to the criticism in Nelson and Plosser (1992), Cogley and Nason (1995) and Rotemberg and Woodford (1996), who argue that this class of models do not have a strong internal propagation mechanism besides the strong persistence in the TFP process. Furthermore, the Calvo nominal wage mechanism dominates other non-Walrasian models such as Vasilev (2016, 2017b,d).

Next, as seen from Table 5 on the next page, over the business cycle, in data labor productivity leads employment. The model nominal wage persistence, however, cannot account for this fact. In this model, as well as in the standard RBC model a technology shock can

Table 4: Autocorrelations for Bulgarian data and the model economy

		k			
Method	Statistic	0	1	2	3
Data	$corr(u_t, u_{t-k})$	1.000	0.765	0.552	0.553
Model	$corr(u_t, u_{t-k})$	1.000	0.818	0.629	0.442
	(s.e.)	(0.000)	(0.035)	(0.063)	(0.084)
Data	$corr(h_t, h_{t-k})$	1.000	0.484	0.009	0.352
Model	$corr(h_t, h_{t-k})$	1.000	0.818	0.629	0.442
	(s.e.)	(0.000)	(0.035)	(0.063)	(0.084)
Data	$corr(y_t, y_{t-k})$	1.000	0.810	0.663	0.479
Model	$corr(y_t, y_{t-k})$	1.000	0.815	0.625	0.438
	(s.e.)	(0.000)	(0.037)	(0.067)	(0.091)
Data	$corr(a_t, a_{t-k})$	1.000	0.702	0.449	0.277
Model	$corr(a_t, a_{t-k})$	1.000	0.814	0.624	0.437
	(s.e.)	(0.000)	(0.038)	(0.070)	(0.096)
Data	$corr(c_t, c_{t-k})$	1.000	0.971	0.952	0.913
Model	$corr(c_t, c_{t-k})$	1.000	0.815	0.625	0.438
	(s.e.)	(0.000)	(0.037)	(0.067)	(0.091)
Data	$corr(i_t, i_{t-k})$	1.000	0.810	0.722	0.594
Model	$corr(i_t, i_{t-k})$	1.000	0.816	0.624	0.434
	(s.e.)	(0.000)	(0.038)	(0.069)	(0.095)
Data	$corr(w_t, w_{t-k})$	1.000	0.760	0.783	0.554
Model	$corr(w_t, w_{t-k})$	1.000	0.816	0.627	0.441
	(s.e.)	(0.000)	(0.036)	(0.065)	(0.087)

be regarded as a factor shifting the labor demand curve, while holding the labor supply curve constant. Therefore, the effect between employment and labor productivity is only a contemporaneous one. Still, the model with nominal wage persistence a la Calvo (1983) is a clear improvement over the perfectly-competitive labor market paradigm used in Vasilev (2009).

Table 5: Dynamic correlations for Bulgarian data and the model economy

		k						
	Statistic	-3	-2	-1	0	1	2	3
Data	$corr(h_t, (y/h)_{t-k})$	-0.342	-0.363	-0.187	-0.144	0.475	0.470	0.346
Model	$corr(h_t, (y/h)_{t-k})$	0.034	0.033	0.029	0.983	0.030	0.034	0.035
	(s.e.)	(0.732)	(0.643)	(0.531)	(0.098)	(0.534)	(0.645)	(0.734)
Data	$corr(h_t, w_{t-k})$	0.355	0.452	0.447	0.328	-0.040	-0.390	-0.57
Model	$corr(h_t, w_{t-k})$	0.034	0.033	0.029	0.983	0.030	0.034	0.035
	(s.e.)	(0.732)	(0.643)	(0.531)	(0.098)	(0.534)	(0.645)	(0.734)

### 6 Conclusions

We augment an otherwise standard DSGE model with a richer government sector, and add monopolistic competition in the product market, and rigid prices, as well as rigid wages a la Calvo (1983) in the labor market. This specification with the nominal wage rigidity, when calibrated to Bulgarian data after the introduction of the currency board (1999-2016), allows the framework to reproduce better observed variability and correlations among model variables, and those characterizing the labor market in particular. These results suggest that technology shocks seem to be the dominant source of economic fluctuations, but nominal wage rigidities a la Calvo (1983), as well as the monopolistic competition in the product market, might be important factors of relevance to the labor market dynamics in Bulgaria, and such imperfections should be incorporated in any model that studies cyclical movements in employment and wages.

Still, the model suffers from some of the usual shortcomings inherent in this class of DSGE models. As a suggestion for future research, the model might be extended to accommodate other important (and real) frictions in the labor market, possibly along the lines of Vasilev (2016, 2017b, 2017d).

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