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The Role of Energy in a Real-business-cycle Model with an Endogenous Capital Utilization Rate and a Government Sector: Lessons from Bulgaria (1999-2016)

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The role of energy in a real-business-cycle model with an endogenous capital utilization rate and a government sector: lessons from Bulgaria (1999-2016)

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Abstract

We introduce a pro-cyclical endogenous utilization rate of physical capita1 stock into a real-business-cycle model augmented with a detailed government sector. We calibrate the model to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2016). We investigate the quantitative importance of the endogenous depreciation rate, and the capital utilization mechanism working through the use of energy for cyclical fluctuations in Bulgaria. In particular, a positive shock to energy prices in the model works like a negative technological shock. Allowing for variations in factor utilization and the presence of energy as a factor of production improves the model performance against data, and in addition this extended setup dominates the standard RBC model framework with constant depreciation and a fixed utilization rate of physical capital, *e.g.*, Vasilev (2009).

Keywords: Business fluctuations, capital utilization rate, endogenous depreciation rate, energy use, energy prices, Bulgaria

JEL Classification Codes: E32, E22, E37

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1 Introduction and Motivation

The average labor productivity in Bulgaria in the period following the introduction of currency board (1997) is highly pro-cyclical.[1] The classical explanation for this stylized fact, which is observed in many developed economies, as presented in Jorgenson and Griliches (1967), is that the major inputs of production, labor and capital, are used more intensively during periods of expansions as compared to periods of recessions. In order to quantitatively rationalize this phenomenon, and gain a deeper understanding of the transmission mechanism responsible for economic fluctuations, we introduce an endogenous utilization rate of physical capital stock into a relatively standard real business-cycle model with a detailed government sector. We examine the quantitative importance of the variability in capital utilization and its relevance to generate plausible cyclical movements in aggregate variables. More specifically, we investigate whether allowing for cyclical capital utilization helps our augmented real-business-cycle model match the empirical business cycles in Bulgaria in the period after the introduction of the currency board arrangement. The period of our investigation was chosen due to the fact that the introduction of the hard exchange-rate peg achieved macroeconomic stability in Bulgaria, and thus the time series have good statistical properties.

The other novelty in this paper is the particular way capital utilization enters the model. We follow Finn (2000) by adopting the empirical regularity that capital utilization requires energy, and argue for the importance of the energy in the transmission of technological shocks.[2] In turn, there are two costs to the capital utilization decision that are at play in the current model: a cost in terms of higher energy use, and a cost in terms of a higher depreciation rate of physical capital stock. The first is a direct effect working through the production function, and following from fact that energy becomes a *de facto* factor of production through the link with utilization rate and capital stock. The second is an indirect channel, which

[1] This stylized fact is similar to the finding documented in Bils and Cho (1994) for the US as well.

[2] Earlier studies, *e.g.* Kydland and Prescott (1988), Greenwood, Hercowitz, and Huffman (1988), and Finn (1995) have incorporated varying capital utilization in real-business-cycle models but without featuring the energy use channel. Still, they make use of the Keynes (1936) notion of "user cost of capital" but in a neoclassical framework, where the changes in capital utilization affect the marginal efficiency of capital. We make use of that notion in the current paper as well.

is one of the novelties in this paper. This effect occurs due the presence of a depreciation cost of utilization and the linkage between it and the use of energy, which works indirectly through the accumulated stock of physical capital. We then use this artificial economy with endogenous capital utilization through energy use as a laboratory in order to study the importance of energy price shocks on the main aggregate variables. In order to be able to draw plausible quantitative predictions, we calibrate the theoretical economy to approximate Bulgaria in the period 1999-2016. We find that a positive shock to energy prices is akin to a negative technological shock, and propose an explanation for a technological shock.[3]

It comes as no surprise that unexpected changes in world energy prices are very important for an energy-intensive production in Bulgaria, a small open economy. Energy price hikes or drops can have important real effects on the economy the fact that Bulgaria, imports most of its energy inputs (oil and natural gas in particular) from the Russian federation. Next, from the perspective of the Bulgarian economy, the price of the aggregate energy input is taken as given. This, the industry structure of the energy production is not of central importance and will be ignored in this paper. More specifically, we abstract away from the issue, as it is of limited relevance for the international transmission of how changes in the price of imported energy inputs effect Bulgarian economy.[4] Instead, what takes a central stage in this paper is the fact that energy prices directly affect the productivity and the profitability of all sectors in the economy, and thus aggregate output. Overall, the model with endogenous utilization rate through the use of energy performs better than earlier real-business-cycle models vis-a vis data for Bulgaria. In particular, consistent with observed cyclical fluctuations in Bulgaria, total hours follow output movement. Nevertheless, as with the standard RBC model, the model with endogenous utilization rate of capital falls far short of generating wage variability as in data, and the wage rate in the model is very strongly

[3] The novelty, however, is that the transmission mechanism of energy price shocks stems from a relatively little explored relationship between energy usage and services provided by physical capital, and described in this paper. Put differently, since energy enters the production function only because it is essential to the utilization of capital, the endogenous variations in utilization and energy use would be inter-related.

[4] In another line of research, Rotemberg and Woodford (1996a) present a theory based on imperfect competition in the oil market to explain business cycle fluctuations in the US economy. Hamilton (1983, 1985, 1996) studies the effect of oil price on real output in the US.

pro-cyclical, while wages are acyclical in data.

The rest of the paper is organized as follows: Section 2 describes the model framework and describes the decentralized competitive equilibrium system, Section 3 discusses the calibration procedure, and Section 4 presents the steady-state model solution. Sections 5 proceeds with the out-of-steady-state dynamics of model variables, and compared the simulated second moments of theoretical variables against their empirical counterparts. Section 6 concludes the paper.

2 Model Description

There is a representative households which derives utility out of consumption and leisure. The time available to households can be spent in productive use or as leisure. In addition, the household chooses optimally the rate at which capital stock is being utilized. The government taxes consumption spending and levies a common tax on all income, in order to finance wasteful purchases of government consumption goods, and government transfers. On the production side, there is a representative firm, which hires labor and utilized capital to produce a homogenous final good, which could be used for consumption, investment, government purchases, or energy consumption. Depreciation rate is endogenous, and is a function of the endogenous capital utilization rate, and depends on the energy use.

2.1 Household's problem

There is a representative household, which maximizes its expected utility function, as in Finn (2000):

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{[c_t^{\psi}(1-h_t)^{1-\psi}]^{1-\sigma}}{1-\sigma} \right\}, \tag{2.1}$$

where E_0 denotes household's expectations as of period 0, c_t denotes household's private (non-energy) consumption in period t, h_t are hours worked in period t, $0 < \beta < 1$ is the discount factor, $0 < \psi < 1$ is the relative weight that the household attaches to consumption, and $\sigma > 0$ is the curvature of the utility function.[5]

The household starts with an initial stock of physical capital $k_0 > 0$, and has to decide how much to add to it in the form of new investment, as well as the rate at which the stock of physical capital is being utilized. As a result, every period physical capital depreciates at an endogenous rate, which depends on the level of utilitization rate u_t chosen by the household, so $0 < \delta(u_t) < 1$. Following Taubman and Wilkinson (1970), Greenwood, Hercowitz, and Huffman (1988), and Finn (1995, 2000), the functional form for the endogenous depreciation rate is as follows:

$$\delta(u_t) = \omega_0 \frac{u_t^{\omega_1}}{\omega_1} \in (0,1), \tag{2.2}$$

where $\omega_0 > 0$, $\omega_1 > 1$. This depreciation function is consistent with Keynes's (1936) notion of the "user cost of capital," which argues that higher utilization causes faster depreciation, at an increasing rate, because of faster "wear and tear" on the aggregate physical capital stock. In addition, as in Finn (2000), we assume that capital utilization requires energy, e_t.[6] More specifically, it will be postulated that energy spending complements the service flow from physical capital as follows:

$$\frac{e_t}{k_t} = a(u_t) = \nu_0 \frac{u_t^{\nu_1}}{\nu_1}, \tag{2.3}$$

where $\nu_0 > 0$, $\nu_1 > 1$. The technical relationship function, $a(.)$, the same as those developed by Finn (1995), postulates that energy is essential to the utilization of capital, with increases in utilization requiring more energy usage per unit of capital, at an increasing rate.[7] The

[5] This utility function is equivalent to a specification with a separable term containing government consumption, e.g. Baxter and King (1993). Since in this paper we focus on the exogenous (observed) policies, and the household takes government spending as given, the presence of such a term is irrelevant. For the sake of brevity, we skip this term in the utility representation above.

[6] This channel is missing from earlier studies, such as Taubman and Wilkinson (1970), Greenwood, Hercowitz, and Huffman (1988), and is one of the novelties of this paper.

[7] This modelling choice could be traced back to Jorgen and Grilliches (1967), who find that capital and electricity are complements in production. In addition, after some algebra, one can show that

$$u_t = \left(\frac{e_t}{k_t}\right)^{\frac{1}{\nu_1}} \left(\frac{\nu_1}{\nu_0}\right)^{\frac{1}{\nu_1}} \tag{2.4}$$

law of motion for physical capital is then

$$k_{t+1} = i_t + (1 - \delta(u_t))k_t, \tag{2.5}$$

and the real interest rate is r_t, hence the before-tax effective (utilized) physical capital income of the household in period t equals $r_t u_t k_t$. In addition to capital income, the household can generate labor income. Hours supplied to the representative firm are rewarded at the hourly wage rate of w_t, so pre-tax labor income equals $w_t h_t$. Lastly, the household owns the firm in the economy and has a legal claim on all the firm's profit, π_t.

Next, the household's problem can be now simplified to

$$\max_{\{c_t,u_t,h_t,k_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{[c_t^{\psi}(1-h_t)^{1-\psi}]^{1-\sigma}}{1-\sigma} \right\}, \tag{2.6}$$

s.t.

$$(1+\tau^c)c_t + k_{t+1} - (1-\delta(u_t))k_t + p_t e_t = (1-\tau^y)[w_t h_t + r_t u_t k_t] + g_t^t + \pi_t, \tag{2.7}$$

where τ^c is the tax on consumption, τ^y is the proportional income tax rate $(0 < \tau^c, \tau^y < 1)$, levied on both labor and capital income, p_t is the relative (to the aggregate consumption price index) energy price, e_t denotes energy use in period t, and g_t^t denotes government transfers. The household takes the two tax rates $\{\tau^c, \tau^y\}$, government spending categories, $\{g_t^c, g_t^t\}_{t=0}^{\infty}$, profit $\{\pi_t\}_{t=0}^{\infty}$, the realized technology process $\{A_t\}_{t=0}^{\infty}$, prices $\{p_t, w_t, r_t\}_{t=0}^{\infty}$, and chooses $\{c_t, h_t, u_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize its utility subject to the budget constraint.[8] The constraint optimization problem generates the following optimality conditions:

$$c_t \; : \; [c_t^{\psi}(1-h_t)^{1-\psi}]^{-\sigma}\psi c_t^{\psi-1} = \lambda_t(1+\tau^c) \tag{2.8}$$

$$h_t \; : \; [c_t^{\psi}(1-h_t)^{1-\psi}]^{-\sigma}(1-\psi)(1-h_t)^{-\psi} = \lambda_t(1-\tau^y)w_t \tag{2.9}$$

$$u_t \; : \; \delta'(u_t) + p_t a'(u_t) = \omega_0 u_t^{\omega_1 - 1} + p_t \nu_0 u_t^{\nu_1 - 1} = (1-\tau^y)r_t \tag{2.10}$$

$$k_{t+1} \; : \; \lambda_t = \beta E_t \lambda_{t+1}[1 + (1-\tau^y)r_{t+1}u_{t+1} - \delta(u_{t+1}) - p_{t+1}a(u_{t+1})] \tag{2.11}$$

$$TVC \; : \; \lim_{t\to\infty} \beta^t \lambda_t k_{t+1} = 0, \tag{2.12}$$

[8]Note that by choosing k_{t+1} the household is implicitly setting investment i_t optimally. Similarly, by choosing the utilization rate, the household is determining the time-varying depreciation rate. Lastly, by choosing the level of physical capital and the rate of capital utilization, that determines optimally the level of energy use.

where λ_t is the Lagrangean multiplier attached to household's budget constraint in period t.

The interpretation of the first-order conditions above is as follows: the first one states that for each household, the marginal utility of consumption equals the marginal utility of wealth, corrected for the consumption tax rate. The second equation states that when choosing labor supply optimally, at the margin, each hour spent by the household working for the firm should balance the benefit from doing so in terms of additional income generates, and the cost measured in terms of lower utility of leisure. The third equation describes the optimal utilization rate, which requires that the change in the depreciation rate, or the marginal cost in terms of an increased depreciation rate resulting from utilizing capital at a higher rate, together with the marginal cost in terms of additional energy used in the capital utilization, equal the after tax return on utilized capital. In other words, the marginal benefit resulting from physical capital services should balance with the user cost of capital at the margin. The fourth equation is the so-called "Euler condition," which describes how the household chooses to allocate physical capital over time. The last condition is called the "transversality condition" (TVC): it states that at the end of the horizon, the value of physical capital should be zero.

2.2 Firm problem

There is a representative firm in the economy, which produces a homogeneous product. The price of output is normalized to unity. The production technology is Cobb-Douglas and uses both utilized (effective) physical capital, $u_t k_t$, and labor hours, h_t, to maximize static profit

$$\Pi_t = A_t (u_t k_t)^{\alpha} h_t^{1-\alpha} - r_t u_t k_t - w_t h_t, \tag{2.13}$$

where A_t denotes the level of technology in period t. Since the firm rents the capital from households, the problem of the firm is a sequence of static profit maximizing problems. In equilibrium, there are no profits, and each input is priced according to its marginal product, *i.e.*:

$$u_t k_t : \alpha \frac{y_t}{u_t k_t} = r_t, \tag{2.14}$$

$$h_t : (1-\alpha) \frac{y_t}{h_t} = w_t. \tag{2.15}$$

In addition, using the link between energy, capital, and utilization we can express output as follows:

$$y_t = A_t h_t^{1-\alpha}\left[k_t^{(1-\frac{1}{\nu_1})} e_t^{\frac{1}{\nu_1}} (\frac{\nu_1}{\nu_0})^{\frac{1}{\nu_1}}\right]^{\alpha} \tag{2.16}$$

The equation specifies output as a function of labor, capital, and energy, showing the direct effect of energy on output.[9]

2.3 Government

In the model setup, the government is levying taxes on labor and capital income, as well as consumption, in order to finance spending on wasteful government purchases, and government transfers. The government budget constraint is as follows:

$$g_t^c + g_t^t = \tau^c c_t + \tau^y [w_t h_t + r_t u_t k_t] \tag{2.17}$$

Tax rates and government consumption-to-output ratio would be chosen to match the average share in data, and government transfers would be determined residually in each period so that the government budget is always balanced.

2.4 Dynamic Competitive Equilibrium (DCE)

For a given process followed by technology $\{A\}_{t=0}^{\infty}$ average tax rates $\{\tau^c, \tau^y\}$, initial capital stock k_0, the decentralized dynamic competitive equilibrium is a list of sequences $\{c_t, i_t, k_t, u_t, e_t, h_t\}_{t=0}^{\infty}$ for the household, a sequence of government purchases and transfers $\{g_t^c, g_t^t\}_{t=0}^{\infty}$, and input prices $\{p_t, w_t, r_t\}_{t=0}^{\infty}$ such that (i) the household maximizes its utility function subject to its budget constraint; (ii) the representative firm maximizes profit; (iii) government budget is balanced in each period; (iv) all markets clear.

[9] Note that if the depreciation rate is held constant, then the transmission of energy price shocks is restrained only to the effect of energy input on output through the production function channel. However, when depreciation rate is endogenous and depends on the utilization of physical capital, and then in turn through it on the use of energy, then energy has an additional indirect effect on output, which operates through the capital stock. As we show later in the paper, the combination of those direct and indirect effects produces important difference in the dynamics of model variables.

3 Data and Model Calibration

To characterize business cycle fluctuations with an endogenous depreciation rate in Bulgaria, we will focus on the period following the introduction of the currency board (1999-2016). Quarterly data on output, consumption and investment was collected from National Statistical Institute (2017), while the real interest rate is taken from Bulgarian National Bank Statistical Database (2017). The calibration strategy described in this section follows a long-established tradition in modern macroeconomics: first, as in Vasilev (2016), the discount factor, $\beta = 0.982$, is set to match the steady-state capital-to-output ratio in Bulgaria, $k/y = 13.964$, in the steady-state Euler equation. The labor share parameter, $1-\alpha = 0.571$, is obtained as in Vasilev (2017d), and equals the average value of labor income in aggregate output over the period 1999-2016. This value is slightly higher as compared to other studies on developed economies, due to the overaccumulation of physical capital, which was part of the ideology of the totalitarian regime, which was in place until 1989. Next, the average income tax rate was set to $\tau^y = 0.1$. This is the average effective tax rate on income between 1999-2007, when Bulgaria used progressive income taxation, and equal to the proportional income tax rate introduced as of 2008. Similarly, the tax rate on consumption is set to its value over the period, $\tau^c = 0.2$.

In terms of parameters characterizing the household's preferences, the curvature of the utility function is set to $\sigma = 2$, as in Hansen and Singleton (1983). Note that this parameter does not enter steady-state computation, and only affects cyclical fluctuations. Next, the relative weight attached to the utility out of leisure in the household's utility function, ψ, is calibrated to match that in steady-state consumers would supply one-third of their time endowment to working. This is in line with the estimates for Bulgaria (Vasilev 2017a) as well over the period studied. Net, the steady-state depreciation rate of physical capital in Bulgaria, $\delta = 0.013$, was taken from Vasilev (2016). It was estimated as the average quarterly depreciation rate over the period 1999-2014. In addition, the steady-state capital utilization rate is normalized to unity, thus $\omega_0 = 0.013$. The curvature papameter, ω_1, does not enter the steady state, and only matters for cyclical fluctuations. As in Finn (2000), we set $\omega_1 = 1.25$. Next, the scale parameter ν_0 set to average value of energy-to-physical capital ratio, e/k. Again, the curvature papameter of the energy-utilization function, ν_1, does not enter the steady state, and only matters for cyclical fluctuations. As in Finn (2000), we set $\nu_1 = 1.61$.

Finally, the processes followed by TFP processes and energy prices, are estimated from the detrended series by running an AR(1) regression and saving the residuals. Table 1 below summarizes the values of all model parameters used in the paper.

Table 1: Model Parameters

Parameter	Value	Description	Method
β	0.982	Discount factor	Calibrated
α	0.429	Capital Share	Data average
$1-\alpha$	0.571	Labor Share	Calibrated
ψ	0.873	Relative weight attached to consumption	Calibrated
σ	2.000	Curvature parameter, utility function	Set
δ	0.013	Depreciation rate on physical capital	Data average
ω_0	0.013	Scale parameter, depreciation function	Calibrated
ω_1	1.250	Curvature parameter, depreciation function	Set
ν_0	0.0143	Scale parameter, energy utilization function	Data average
ν_1	1.610	Curvature parameter, energy utilization function	Set
τ^y	0.100	Average tax rate on income	Data average
τ^c	0.200	VAT/consumption tax rate	Data average
A	0.604	Steady-state value of TFP process	Calibarated
p	1.000	Steady-state energy price level	Calibrated
ρ_a	0.701	AR(1) persistence coefficient, TFP process	Estimated
ρ_p	0.980	AR(1) persistence coefficient, energy price process	Estimated
σ_a	0.044	st. error, TFP process	Estimated
σ_p	0.013	st. error, energy process	Estimated

4 Steady-State

Once the values of model parameters were obtained, the steady-state equilibrium system solved, the "big ratios" can be compared to their averages in Bulgarian data. The results are reported in Table 2 below. The steady-state level of output was normalized to unity (hence the level of technology A differs from one, which is usually the normalization done in other studies), which greatly simplified the computations. Next, the model matches consumption-to-output and government purchases ratios by construction; The investment ratios are also closely approximated, despite the closed-economy assumption and the absence of foreign trade sector. The shares of income are also identical to those in data, which is an artifact of the assumptions imposed on functional form of the aggregate production function. The after-tax return, where $\bar{r} = (1-\tau^y)r - \delta$ is also relatively well-captured by the model. Lastly, given the absence of debt, and the fact that transfers were chosen residually to balance the government budget constraint, the result along this dimension is understandably not so close to the average ratio in data.

Table 2: Data Averages and Long-run Solution

Variable	Description	Data	Model
y	Steady-state output	N/A	1.000
c/y	(non-energy) Consumption-to-output ratio	0.624	0.624
i/y	Investment-to-output ratio	0.201	0.175
pe/y	Energy consumption-to-output ratio	0.151	0.151
g^t/y	Government transfers-to-output ratio	0.220	0.149
wh/y	Labor income-to-output ratio	0.571	0.571
ruk/y	Capital income-to-output ratio	0.429	0.429
h	Share of time spent working	0.333	0.333
$\bar{r}$	After-tax net return on capital	0.014	0.016

5 Out of steady-state model dynamics

Since the model does not have an analytical solution for the equilibrium behavior of variables outside their steady-state values, we need to solve the model numerically. This is done by log-linearizing the original equilibrium (non-linear) system of equations around the steady-state. This transformation produces a first-order system of stochastic difference equations. First, we study the dynamic behavior of model variables to an isolated shock to the total factor productivity process, and then we fully simulate the model to compare how the second moments of the model perform when compared against their empirical counterparts.

5.1 Impulse Response Analysis

This subsection documents the impulse responses of model variables to a 1% surprise innovation to technology, as well as an unexpected one-percent change in energy prices.[10] The impulse response functions (IRFs) are presented in Fig. 1 and Fig.2 on the next page.

5.1.1 Impulse Responses to Technology Shocks

As a result of the one-time unexpected positive shock to total factor productivity, output increases upon impact. This expands the availability of resources in the economy, so used of output - consumption, investment, energy use and government consumption also increase contemporaneously. At the same time, the increase in productivity increases the after-tax return on the two factors of production, labor and capital. The representative households then respond to the incentives contained in prices and start accumulating capital, and supplies more hours worked. In turn, the increase in capital input feeds back in output through the production function and that further adds to the positive effect of the technology shock. Lastly, the utilization rate increases as well, following the increase in the return on capital, but this also increases the endogenous depreciation rate. In the labor market, the wage rate increases, and the household increases its hours worked. In turn, the increase in total hours further increases output, again indirectly.

[10]This price is to be interpreted as an aggregate category, comprising electricity, coal, natural gas, and petroleum.

Over time, as capital is being accumulated, its after-tax marginal product starts to decrease, which lowers the households' incentives to save. As a result, physical capital stock eventually returns to its steady-state, and exhibits a hump-shaped dynamics over its transition path. Final private consumption also exhibits the same shape in its dynamic pattern. The rest of the model variables return to their old steady-states in a monotone fashion as the effect of the one-time surprise innovation in technology dies out.

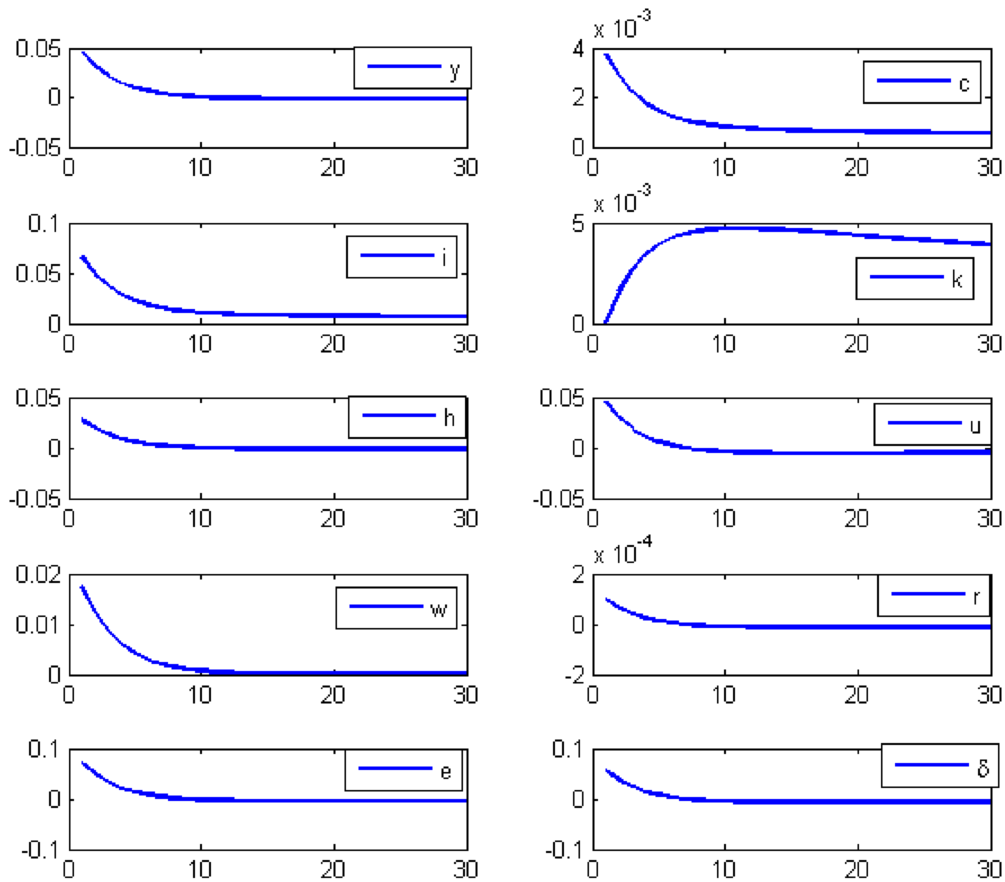

Figure 1: Impulse Responses to a 1% surprise innovation in technology

5.1.2 Impulse Responses to Unanticipated Energy Prices

As a result of an unexpected one-time increase in the price of the aggregate energy input, illustrated in Fig. 2 on the next page, the consumption of energy decreases, while its substitute, the non-energy private consumption, increases. Due to the relative scarcity of energy, illustrated in the increased valuation of energy, capital utilization rate increases. In turn, the time-varying endogenous depreciation rate increases as well, which in turn decreases capital accumulation, and investment. As a result of the lower capital availability, real interest rate goes up.

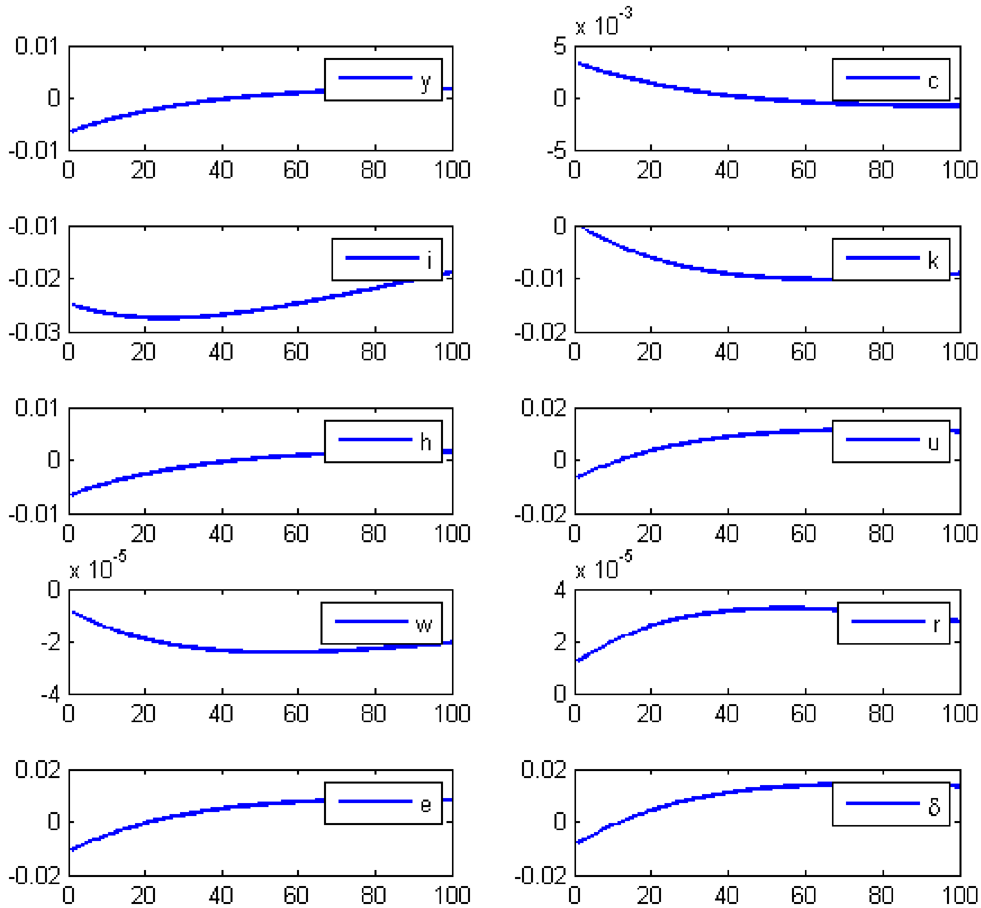

Figure 2: Impulse Responses to a 1% surprise innovation in energy price

Next, from the complementarity of capital and labor in the Cobb-Douglas production function, hours fall as well, and the wage rate in the economy increases. Interestingly, aggregate output falls as well upon impact of the energy price shock, so an increase in energy prices is akin to a negative productivity shock, as energy could be expressed as a direct input in the production function. Next, as government consumption and transfers follow private output,both fall as well. Over time, all variables return to their steady-state, but the negative effects from one-time unexpected increases in energy prices has a long-term negative effect on the economy.

5.2 Simulation and moment-matching

As in Vasilev (2017b), we will now simulate the model 10,000 times for the length of the data horizon. Both empirical and model simulated data is detrended using the Hodrick-Prescott (1980) filter. Table 3 on the next page summarizes the second moments of data (relative volatilities to output, and contemporaneous correlations with output) versus the same moments computed from the model-simulated data at quarterly frequency.[11] To minimize the sample error, the simulated moments are averaged out over the computer-generated draws. As in Vasilev (2016, 2017b, 2017c), the model matches quite well the absolute volatility of output and investment. By construction, government consumption in the model varies as much as output. However, the model in this paper underestimates the variability in consumption, due to the presence of energy consumption, which acts as a substitute for non-energy consumption. Still, the model is qualitatively consistent with the stylized fact that consumption generally varies less than output, while investment is more volatile than output.

With respect to the labor market variables, the variability of employment predicted by the model is almost identical to that in data, but the variability of wages in the model is much lower than that in data. This is yet another confirmation that the perfectly-competitive assumption, e.g. Vasilev (2009), does not describe very well the dynamics of labor market variables. In addition, as in Vasilev (2017b, 2017c),the model fails in matching unemployment volatility, which in this model varies as much as the employment rate.[12] Next, in terms of contemporaneous correlations, the model systematically over-predicts the pro-cyclicality

[11] The model-predicted 95 % confidence intervals are available upon request.

[12] The reason behind this mismatch could be driven by several possible explanatory factors: the fact that the model misses the ”out-of the-labor-force” segment, as well as the significant emigration to the older EU member states.

Table 3: Business Cycle Moments

	Data	Model
σ_y	0.05	0.05
σ_c/σ_y	0.55	0.14
σ_i/σ_y	1.77	1.97
σ_g/σ_y	1.21	1.00
σ_h/σ_y	0.63	0.63
σ_w/σ_y	0.83	0.39
$\sigma_{y/h}/\sigma_y$	0.86	0.39
σ_u/σ_y	3.22	0.63
σ_w/σ_h	1.32	1.61
$corr(c,y)$	0.85	0.47
$corr(i,y)$	0.61	0.75
$corr(g,y)$	0.31	1.00
$corr(h,y)$	0.49	0.96
$corr(w,y)$	-0.01	0.97
$corr(u,y)$	-0.47	-0.96
$corr(h,y/h)$	-0.14	0.92

of the main aggregate variables - consumption, investment, and government consumption. This, however, is a common limitation of this class of models. However, along the labor market dimension, the contemporaneous correlation of employment with output, and unemployment with output, is relatively well-matched. With respect to wages, the model predicts strong cyclicality, while wages in data are acyclical. This shortcoming is well-known in the literature and an artifact of the wage being equal to the labor productivity in the model.

In the next subsection, as in Vasilev (2016), we investigate the dynamic correlation between labor market variables at different leads and lags, thus evaluating how well the model matches the phase dynamics among variables. In addition, the autocorrelation functions (ACFs) of empirical data, obtained from an unrestricted VAR(1) are put under scrutiny and compared and contrasted to the simulated counterparts generated from the model.

5.3 Auto- and cross-correlation

This subsection discusses the auto-(ACFs) and cross-correlation functions (CCFs) of the major model variables. The coefficients empirical ACFs and CCFs at different leads and lags are presented in Table 4 below against the averaged simulated AFCs and CCFs. Following Canova (2007), this is used as a goodness-of-fit measure.

Table 4: Autocorrelations for Bulgarian data and the model economy

		k			
Method	Statistic	0	1	2	3
Data	$corr(u_t, u_{t-k})$	1.000	0.765	0.552	0.553
Model	$corr(u_t, u_{t-k})$	1.000	0.955	0.901	0.837
	(s.e.)	(0.000)	(0.027)	(0.051)	(0.073)
Data	$corr(n_t, n_{t-k})$	1.000	0.484	0.009	0.352
Model	$corr(n_t, n_{t-k})$	1.000	0.955	0.901	0.837
	(s.e.)	(0.000)	(0.027)	(0.051)	(0.074)
Data	$corr(y_t, y_{t-k})$	1.000	0.810	0.663	0.479
Model	$corr(y_t, y_{t-k})$	1.000	0.955	0.901	0.836
	(s.e.)	(0.000)	(0.027)	(0.050)	(0.073)
Data	$corr(a_t, a_{t-k})$	1.000	0.702	0.449	0.277
Model	$corr(a_t, a_{t-k})$	1.000	0.955	0.900	0.836
	(s.e.)	(0.000)	(0.026)	(0.050)	(0.072)
Data	$corr(c_t, c_{t-k})$	1.000	0.971	0.952	0.913
Model	$corr(c_t, c_{t-k})$	1.000	0.955	0.903	0.845
	(s.e.)	(0.000)	(0.026)	(0.050)	(0.073)
Data	$corr(i_t, i_{t-k})$	1.000	0.810	0.722	0.594
Model	$corr(i_t, i_{t-k})$	1.000	0.954	0.901	0.841
	(s.e.)	(0.000)	(0.026)	(0.050)	(0.073)
Data	$corr(w_t, w_{t-k})$	1.000	0.760	0.783	0.554
Model	$corr(w_t, w_{t-k})$	1.000	0.920	0.900	0.836
	(s.e.)	(0.000)	(0.026)	(0.050)	(0.073)

As seen from Table 4 on the previous page, the model compares relatively well vis-a-vis data. Empirical ACFs for output and investment are slightly outside the confidence band predicted by the model, while the ACFs for total factor productivity and household consumption are well-approximated by the model. The persistence of labor market variables are also relatively well-described by the model dynamics. Overall, the model with energy-utilization channel generates too much persistence in output and both employment and unemployment, and is subject to the criticism in Nelson and Plosser (1992), Cogley and Nason (1995) and Rotemberg and Woodford (1996b), who argue that the RBC class of models do not have a strong internal propagation mechanism besides the strong persistence in the TFP process. In those models, e.g. Vasilev (2009), and in the current one, labor market is modelled in the Walrasian market-clearing spirit, and output and unemployment persistence is low.

Next, as seen from Table 5 below, over the business cycle, in data labor productivity leads employment. The model, however, cannot account for this fact.[13]

Table 5: Dynamic correlations for Bulgarian data and the model economy

		k						
Method	Statistic	-3	-2	-1	0	1	2	3
Data	$corr(n_t, (y/n)_{t-k})$	-0.342	-0.363	-0.187	-0.144	0.475	0.470	0.346
Model	$corr(n_t, (y/n)_{t-k})$	0.123	0.195	0.292	0.918	0.288	0.221	0.171
	(s.e.)	(0.320)	(0.286)	(0.250)	(0.146)	(0.243)	(0.281)	(0.317)
Data	$corr(n_t, w_{t-k})$	0.355	0.452	0.447	0.328	-0.040	-0.390	-0.57
Model	$corr(n_t, w_{t-k})$	0.123	0.195	0.292	0.918	0.288	0.221	0.171
	(s.e.)	(0.320)	(0.286)	(0.250)	(0.146)	(0.243)	(0.281)	(0.317)

[13] As in the standard RBC model a technology shock can be regarded as a factor shifting the labor demand curve, while holding the labor supply curve constant. Therefore, the effect between employment and labor productivity is only a contemporaneous one.

6 Conclusions

We introduce a pro-cyclical endogenous utilization rate of physical capita1 stock into a real-business-cycle model augmented with a detailed government sector. We calibrate the model to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2016). We investigate the quantitative importance of the endogenous depreciation rate, and the capital utilization mechanism working through the use of energy for cyclical fluctuations in Bulgaria. In particular, a positive shock to energy prices in the model works like a negative technological shock. Allowing for variations in factor utilization and the presence of energy as a factor of production improves the model performance against data, and in addition this extended setup dominates the standard RBC model framework with constant depreciation and a fixed utilization rate of physical capital, *e.g.*, Vasilev (2009).

Still, the failure of the model along the labor market dimension - the high pro-cyclicality of wages, and the low variability of the price of labor relative to that observed in data both suggest that the setup should depart from the perfectly-competitive paradigm. As a suggestion for future research, the model should focus on the important frictions in the labor market, which forms almost two-thirds of total income (and much quantitatively much more important than the share of capital and energy), and extend the model along the lines of Vasilev (2016, 2017b, 2017c).

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