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Abstract

We introduce an environmental dimension into a real-business-cycle model augmented with a detailed government sector. We calibrate the model to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2016). We investigate the quantitative importance of utility-enhancing environmental quality, and the mechanics of environmental ("carbon") tax on polluting production, as well as the effect of government spending on pollution abatement over the cycle. In particular, a positive shock to pollution emission in the model works like a positive technological shock, but its effect is quantitatively very small. Allowing for pollution as a by-product of production improves the model performance against data, and in addition this extended setup dominates the standard RBC model framework, *e.g.*, Vasilev (2009).

JEL classification: E32, C68, Q2, Q4, Q54, Q58

Keywords: Business cycles, pollution, environmental quality, environmental tax, abatement spending

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1 Introduction and Motivation

Despite being viewed as a microeconomic field, there has been a recent interest into matters connected to the environment among macroeconomists as well, e.g. Fischer and Heutel (2013). Since pollution levels follow a certain dynamic, modern quantitative economics can utilize the tools of dynamic optimization to analyze the importance of environment quality for aggregate economic activity. There are relatively few many papers in this new "environmental-Real-Business-Cycles" (RBC) literature: Fischer and Heutel (2013) provide an excellent survey of macroeconomic analysis and propose several modeling approaches of environmental issues. Their work is an important contribution, as it brings together two strands of literatures - the macroeconomic RBC literature, and the environmental economics one.

A suitable case study for the aggregate effects of environmental policies is Bulgaria, a former communist country, and a current EU member state. We will focus on the period after the introduction of the currency board arrangement (1999-2016), which is a period of macroeconomic stability. One aspect of the communist heritage was the over-reliance on heavy manufacturing, and the disregard of environmental norms. In particular, the energy-intensive industry was a major polluter of the environment. This had to change as Bulgaria joined the EU in 2007, and Bulgaria had to start abiding by different environmental standards. One such effect was the shrinking of industrial production, as installing green technologies turned out to be quite costly. Additionally, closing coal mines and power plants running on coal, which provided substantial employment for the population turned out to be politically costly as well, and governments were avoiding the hot potato as they were afraid of social unrest.

The analysis of environmental policies in a macroeconomic context requires a dynamic stochastic general equilibrium framework (DSGE), in order to capture the endogenously-produced interaction between real activity and pollution emission, and to be consistent with the fact that both production and pollution processes feature a certain degree of uncertainty. More specifically, in this paper, we augment an otherwise standard RBC model with utility-enhancing environmental quality, and a detailed government sector. In other words, the population will value ”clean air” as well. However, the quality of environment will be eroding over time due to emitted pollution, but ”environmental cleanliness” can be maintained through the use of government spending on abatement.[1] Often pollution is a negative externality, as producers often do not take it under consideration when choosing their production levels. Such external effects then necessitate government action to improve allocative efficiency through taxes and spending. Therefore, there will be a government in the model, which, in addition to the other taxes levied, will impose an environmental tax on dirty production, and the revenue collected will be spend on environmental abatement (cleaning).

Our study extends Angelopoulos *et al.* (2013) by allowing for an endogenous labor supply, and augment it with a more detailed government sector. We believe this to be important, as this choice variable is going to generate additional interaction among the model variables. Furthermore, labor income constitutes two-thirds of total income in the economy, so it is interesting in its own merit to study how the presence of an environmental dimension, with polluting industry, and an environmental taxation, and spending on environmental cleaning can affect the cyclical fluctuations in the labor market. To keep things as simple as possible, we will stay within the closed-economy setup, retain the representative agent assumption, and the stand-in firm simplification, and choose to ignore the global implications that pollution produced in Bulgaria may have internationally. In this sense, we will focus on the local effect of pollution on the territory of Bulgaria.

The study would also differ non-trivially from Heutel's (2012) treatment and modelling of pollution in a RBC framework, and will abstract away from the optimal climate policy issue. In addition, we will not focus on pollution permits, caps and emission targets, as they are likely not to be relevant for business cycle fluctuations.[2] Lastly, in contrast to Xepapadeas (2004), who focuses on qualitative growth effects in continuous-time framework, here we

[1] In this paper we will refer to ”pollution” as an abstract category, and will not distinguish between carbon monoxide, carbon dioxide, nitrogen dioxide, or sulfur dioxide. For practical purposes, we will focus on emission of carbon dioxide. As pointed in Heutel (2013), CO_2 emissions are more problematic, as SO_2 emissions have a much shorter half-life.

[2] For those interested, Fischer and Springborn (2011) provide a valuable survey on the topic.

focus instead on the quantitative properties of business cycle fluctuations in discrete time, cross-correlations-, and auto-correlation comparison, and in particular, we study the relative volatility and cyclicality of pollution with respect to output in Bulgaria.

The rest of the paper is organized as follows: Section 2 describes the model setup, Section 3 describes the model calibration, Section 4 characterizes the steady-state, Section 5 describes the model dynamics out-of-the steady state, and compares the relative volatilities, the cross-correlation functions (CCFs) and auto-correlation functions (ACFs) obtained from simulated data against the empirical counterparts. Section 6 concludes.

2 Model Description

There is a representative household in the model economy, which derives utility out of consumption, leisure and environmental quality. On the production side, there is a stand-in firm, which produces a homogeneous final good, and pollution as a by-product, which in turn lowers the level of environmental quality. The government imposes a carbon tax on output, and in addition can spend on pollution abatement activities. The government also has access to consumption and income taxation, and returns the surplus revenue back to the household in a lump-sum fashion. The final good which could be used for consumption, investment, or government pollution abatement spending.

2.1 Household

The representative one-member household values consumption, leisure, and environmental quality:

$$E_t \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \theta \ln(1-h_t) + \gamma \ln q_t \right\}, \tag{2.1}$$

where E_0 is the expectations operator as of period 0, $0 < \beta < 1$ is the discount factor, c_t denotes household's consumption in period t, h_t denote hours worked, and q_t is preference for clean environment ("environment quality"). Parameter $\theta > 0$ reflects the relative (to consumption) weight attached to leisure, while $\gamma > 0$ denotes the relative weight that the household attaches to environment quality. As in in Angelopoulos *et al.* (2013), we define the last term as a "good" (or absence of pollution, hence "more is better"), and not as a "bad" (stock of pollution). This is done to preserve the positive monotonicity in household's preferences. In addition, environmental quality will possess all the features of a public good.

The household starts with an initial stock of physical capital $k_0 > 0$, and has to decide how much to add to it in the form of new investment. Every period physical capital depreciates at a rate δ^k, where $0 < \delta^k < 1$. The law of motion for physical capital is then

$$k_{t+1} = i_t + (1 - \delta^k)k_t, \tag{2.2}$$

and the real interest rate is r_t, hence the before-tax capital income of household i in period t equals $r_t k_t$. The household also owns the firm in the economy, and has a legal claim on the firm's profit, π_t. In addition to capital income, each household can generate labor income by working in the representative firm. The hourly wage rate is w_t, so before-tax labor income equals $w_t h_t$. The household's budget constraint is as follows:

$$(1 + \tau_t^c)c_t + k_{t+1} - (1 - \delta^k)k_t = (1 - \tau_t^y)[w_t h_t + r_t k_t + \pi_t] + g_t^t, \tag{2.3}$$

where τ_t^c is the consumption tax rate, τ_t^y is the common (labor and capital) income tax rate, and g_t^t denotes government lump-sum transfers.

The household takes initial capital stock k_0, environmental quality $\{q_t\}_{t=0}^{\infty}$, prices $\{w_t, r_t\}_{t=0}^{\infty}$, profits $\{\pi_t\}_{t=0}^{\infty}$, and policy variables $\{\tau_t^c, \tau_t^y, g_t^t\}_{t=0}^{\infty}$ as given, and chooses $\{c_t, h_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize Eq. (2.1) s.t (2.2)-(2.3). The first-order optimality conditions (FOCs), and the boundary (transversality) condition for physical capital, are as follows:

$$c_t \quad : \quad \frac{1}{c_t} = \lambda_t, \tag{2.4}$$

$$h_t \quad : \quad \frac{\theta}{1 - h_t} = \lambda_t(1 - \tau_t^y)w_t \tag{2.5}$$

$$k_{t+1} \quad : \quad \lambda_t = \beta E_t \lambda_{t+1}[1 + (1 - \tau_{t+1}^y)r_{t+1} - \delta^k] \tag{2.6}$$

$$TVC \quad : \quad \lim_{t \to \infty} \beta^t \lambda_t k_{t+1} = 0. \tag{2.7}$$

The interpretation of the conditions above is standard; The first FOC equates the marginal benefit from an additional unit of consumption and the shadow price of wealth. The second equation balances the disutility of labor and the benefit in terms of after-tax wage, and weighted by the price in terms of consumption. The third one is a dynamic optimality condition, which states how capital should be allocated in any two congruent periods. The last one is a boundary condition, imposed to rule out explosive solution paths.

2.2 Stand-in firm

There is a representative firm in the economy, which produces a homogeneous product. Total production value is taxed at a rate τ_t^E. The price of output is normalized to unity. The production technology is Cobb-Douglas and uses both physical capital and labor hours to maximize static profit

$$\pi_t = (1 - \tau_t^E) A_t k_t^{\alpha} h_t^{1-\alpha} - r_t k_t - w_t h_t, \tag{2.8}$$

where A_t denotes the level of technology in period t, and τ^E denotes the time-varying proportional environmental tax on revenue. In equilibrium, profit is zero ($\pi_t = 0$), and each input is priced according to its tax-adjusted marginal product, *i.e.*:

$$k_t \ : \ r_t = (1 - \tau_t^E)\alpha \frac{y_t}{k_t}, \tag{2.9}$$

$$h_t \ : \ w_t = (1 - \tau_t^E)(1 - \alpha) \frac{y_t}{h_t}. \tag{2.10}$$

The carbon/energy tax acts like a tax on inputs, and in many aspects similar to an income tax, but born by producer (like a payroll tax).

2.3 Pollution and environmental quality

In this paper, the stock of environmental quality is equivalent to "absence of pollution." As in Angelopoulos *et al.* (2013), and Economides and Phillipopulos (2007), environmental quality evolves according to the following law of motion:

$$q_{t+1} = (1 - \delta^q)\bar{q} + \delta^q q_t - p_t + \nu g_t^E \tag{2.11}$$

where $\bar{q} > 0$ denotes the steady-state stock of environmental quality, $0 < \delta^q < 1$ is the persistence parameter of environment quality. p_t denotes the level of emitted pollution in period t, which decreases environmental quality. To offset the effect of pollution, government can spend resources on pollution abatement (clean-up policy), and the efficiency of that technology is captured by parameter $\nu > 0$. lastly, in the model, pollution p_t is generated as a by-product of production, or, in other words:

$$p_t = \phi_t y_t = \phi_t A_t k_t^{\alpha} h_t^{1-\alpha}, \tag{2.12}$$

where $0 < \phi_t < 1$ is the time-varying measure of the pollution technology that maps (say CO_2) emissions as a function of aggregate output.[3] Note that when we solve for the decentralized competitive equilibrium, the firm will maximize profit independently of the level of pollution emitted, and would produce a level of output that is larger than the socially optimal amount. In that sense, there will be a negative externality effect in the competitive equilibrium in the model, and the allocations will be inefficient.

2.4 Government

In the model setup, the government is levying taxes on labor and capital income, taxes production, as well consumption in order to its finance spending on transfers and pollution-decreasing (abatement) activities. The government budget constraint is as follows:

$$g_t^t + g_t^E = \tau_t^c c + \tau_t^E y + \tau_t^y [w_t h_t + r_t k_t] \tag{2.13}$$

For simplicity, taes will be set to their average effective rates in data. Government spending on abatement-to-output ratio would be chosen to match the average share in data, and government transfers would be determined residually in each period so that the government budget is always balanced.[4]

2.5 Stochastic processes

Total factor productivity, A_t, is assumed to follow an AR(1) process in logs, in particular

$$\ln A_{t+1} = (1 - \rho_a) \ln A + \rho_a \ln A_t + \epsilon_{t+1}^a,$$

where $A > 0$ is steady-state level of the total factor productivity process, $0 < \rho_a < 1$ is the first-order autoregressive persistence parameter and $\epsilon_t^a \sim iidN(0, \sigma_a^2)$ are random shocks

[3]This way of modelling is very close in spirit to Heutel (2013), who works with output net of pollution, or $(1 - \phi_t)y_t$.

[4]From the government constraint it is clear that carbon taxes are an additional burden on labor and capital income.

to the total factor productivity progress. Hence, the innovations ϵ^a_t represent unexpected changes in the total factor productivity process.

Pollution technology rate, ϕ_t, is also assumed to follow an AR(1) process in logs:

$$\ln \phi_{t+1} = (1-\rho_\phi)\ln\phi + \rho_\phi \ln\phi_t + \epsilon^\phi_{t+1},$$

where $\phi > 0$ is steady-state rate of pollution technology parameter, $0 < \rho_\phi < 1$ is the first-order autoregressive persistence parameter and $\epsilon^\phi_t \sim iidN(0, \sigma^2_\phi)$ are random shocks to the pollution technology. Hence, the innovations ϵ^ϕ_t represent unexpected changes in the pollution technology.

2.6 Dynamic Competitive Equilibrium (DCE)

For a given process followed by technology $\{A_t, \phi_t\}^\infty_{t=0}$, average tax rates $\{\tau^c, \tau^y, \tau^E\}$, initial capital stock k_0, initial environmental quality $\{q_0\}$, the decentralized dynamic competitive equilibrium is a list of sequences $\{c_t, i_t, k_t, p_t, q_t, h_t\}^\infty_{t=0}$ for the household, a sequence of government purchases and transfers $\{g^t_t, g^E_t\}^\infty_{t=0}$, and input prices $\{w_t, r_t\}^\infty_{t=0}$ such that (i) the household maximizes its utility function subject to its budget constraint; (ii) the representative firm maximizes profit; (iii) government budget is balanced in each period; (iv) pollution and environmental quality follow their laws of motion; (v) all markets clear.

3 Data and Model Calibration

To characterize business cycle fluctuations with pollution and environmental taxation in Bulgaria, we will focus on the period following the introduction of the currency board (1999-2016). Quarterly data on output, consumption and investment was collected from National Statistical Institute (2018), while the real interest rate is taken from Bulgarian National Bank Statistical Database (2018). The calibration strategy described in this section follows a long-established tradition in modern macroeconomics: first, as in Vasilev (2016), the discount factor, $\beta = 0.982$, is set to match the steady-state capital-to-output ratio in Bulgaria, $k/y = 13.964$, in the steady-state Euler equation. The labor share parameter, $1-\alpha = 0.571$, is obtained as in Vasilev (2017b), and equals the average value of labor income in aggregate output over the period 1999-2016. This value is slightly higher as compared to other studies on developed economies, due to the overaccumulation of physical capital, which was part of the ideology of the totalitarian regime, which was in place until 1989. Next, the average income tax rate was set to $\tau^y = 0.1$. This is the average effective tax rate on income between 1999-2007, when Bulgaria used progressive income taxation, and equal to the proportional income tax rate introduced as of 2008. Similarly, the tax rate on consumption is set to its value over the period, $\tau^c = 0.2$. Carbon tax rate was set to its average effective rate $\tau^E = 0.024$, measured as tax payment relative to output value, and spending on abatement is on average $g^E = 0.01$, or one percent of aggregate output.

Next, the relative weight attached to the utility out of leisure in the household's utility function, $\theta = 1.243$, is calibrated to match that in steady-state consumers would supply one-third of their time endowment to working. This is in line with the estimates for Bulgaria (Vasilev 2017a) as well over the period studied. The relative weight attached to environmental quality, $\gamma = 0.25$, which is in line with the weight attached to public goods in Bulgaria (Vasilev 2018). Next, the depreciation rate of physical capital in Bulgaria, $\delta^k = 0.013$, was taken from Vasilev (2016). It was estimated as the average quarterly depreciation rate over the period 1999-2014.

The steady-state level of environmental quality, $\bar{q}$ is normalized to unity, as in Angelopoulos *et al.* (2013). The degree of persistence of environmental quality is also set to a high value, $\delta^q = 0.9$, as environmental quality is not just something that pertains to Bulgarian territory. Next, since we do not have any data on the efficiency of abatement technology, we normalize $\nu = 1$ as in Economides and Phillipopoulos (2008); In other words the cleaning technology is identical to the government spending on abatement, which is not a very strong assumption. Next, for pollution technology, $\phi = 0.067$ was set as the average ratio of carbon dioxide emissions to output. Finally, the processes followed by TFP process is estimated from the detrended series by running an AR(1) regression and saving the residuals. Due to the lack of data, the moments of the pollution technology will be set identical to that of TFP. Given the multiplicative way pollution technology interacts with the production function, that makes perfect sense.Table 1 on the next page summarizes the values of all model parameters used in the paper.

Table 1: Model Parameters

Parameter	Value	Description	Method
β	0.982	Discount factor	Calibrated
α	0.429	Capital Share	Data average
$1-\alpha$	0.571	Labor Share	Calibrated
θ	1.243	Relative weight attached to leisure	Calibrated
γ	0.250	Relative weight attached to env. quality	Set
δ^k	0.013	Depreciation rate on physical capital	Data average
δ^q	0.900	Persistence, environmental quality	Set
τ^y	0.100	Average tax rate on income	Data average
τ^E	0.024	Average tax rate on production	Data average
τ^c	0.200	VAT/consumption tax rate	Data average
A	0.604	Steady-state value of TFP process	Calibrated
q	1.000	Steady-state value of env.quality	Set
ν	1.000	Efficiency, abatement spending	Set
ϕ	0.067	Steady-state pollution technology	Data Average
ρ_a	0.701	AR(1) persistence coefficient, TFP process	Estimated
ρ_ϕ	0.701	AR(1) persistence coefficient, pollution process	Set
σ_a	0.044	st. error, TFP process	Estimated
σ_ϕ	0.044	st. error, pollution process	Set

4 Steady-State

Once the values of model parameters were obtained, the steady-state equilibrium system solved, the "big ratios" can be compared to their averages in Bulgarian data. The results are reported in Table 2 on the next page. The steady-state level of output was normalized to unity (hence the level of technology A differs from one, which is usually the normalization done in other studies), which greatly simplified the computations. Next, the model overestimates consumption-to-output, as there is no government consumption in the model. The investment ratio is also closely approximated, despite the closed-economy assumption. The shares of income are also identical to those in data, which is an artifact of the assumptions imposed on functional form of the aggregate production function. The after-tax return, where $\bar{r} = (1 - \tau^y)r - \delta$ is also relatively well-captured by the model.

Table 2: Data Averages and Long-run Solution

Variable	Description	Data	Model
y	Steady-state output	N/A	1.000
c/y	Consumption-to-output ratio	0.624	0.815
i/y	Investment-to-output ratio	0.201	0.175
g^E/y	Public spending on abatement-to-output ratio	0.010	0.010
wh/y	Labor income-to-output ratio	0.571	0.571
rk/y	Capital income-to-output ratio	0.429	0.429
h	Share of time spent working	0.333	0.333
$\bar{r}$	After-tax net return on capital	0.014	0.016

5 Out of steady-state model dynamics

Since the model does not have an analytical solution for the equilibrium behavior of variables outside their steady-state values, we need to solve the model numerically. This is done by log-linearizing the original equilibrium (non-linear) system of equations around the steady-state. This transformation produces a first-order system of stochastic difference equations. First, we study the dynamic behavior of model variables to an isolated shock to the total factor productivity process, then an isolated shock to the pollution technology process, and then we fully simulate the model to compare how the second moments of the model perform when compared against their empirical counterparts.

5.1 Impulse Response Analysis

This subsection documents the impulse responses of model variables to a 1% surprise innovation to technology, as well as an unexpected one-percent change in the pollution technology process. The impulse response functions (IRFs) are presented in Fig. 1 and Fig.2 on the next page.

5.1.1 Impulse Responses to Technology Shocks

As a result of the one-time unexpected positive shock to total factor productivity, output increases. This expands the availability of resources in the economy, so consumption, investment, energy use and government consumption also increase upon impact. At the same time, the increase in productivity increases the after-tax return on the two factors of production, labor and capital. All households respond to the incentives contained in prices and start accumulating capital, and supplying more hours worked. In turn, the increase in capital input feeds back in output through the production function and further adds to the positive effect of the technology shock. In the labor market, wages increase, and the household increases hours worked. In turn, the increase in hours further increases output. In the environmental dimention, pollution and abatement dynamics follows that of output. As a result, initially environmental quality falls, but then as the technology shock decreases, output falls, pollution falls, and environmental quality returns to its steady-state within a decade following the shock.

Over time, as capital is being accumulated, its marginal product starts to decrease, which lowers the households' incentives to save. As a result, capital eventually returns to its steady-state, and exhibits a hump-shaped dynamics over the transition path. Consumption also exhibits the same shape in its dynamic pattern. The rest of the variables return to their old steady-states in a monotone fashion as the effect of the one-time surprise innovation in technology dies out.

5.1.2 Impulse Responses to Unanticipated Pollution Technology Process

As a result of an unexpected one-time increase in pollution technology process, illustrated in Fig.2 on the next page, pollution increases, and the level of environmental quality decreases. Since output does not change, spending on abatement does not change, so the only the dynamics of environmental quality is affected. Over time, as the pollution technology effect

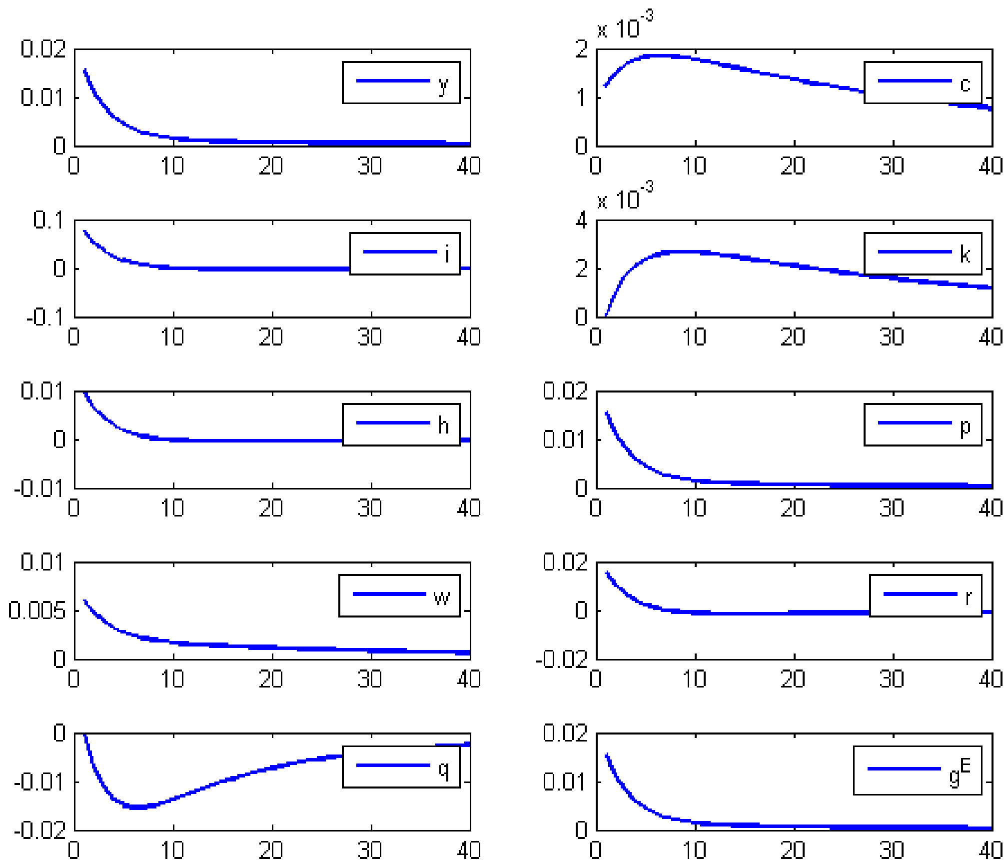

Figure 1: Impulse Responses to a 1% surprise innovation in total factor productivity

dies, the effect on the quality of the environment dies as well. The other variables in the model are not affected, so shocks to the pollution technology are unlikely to be the leading factor for business cycle fluctuations.

5.2 Simulation and moment-matching

We will now simulate the model 10,000 times for the length of the data horizon. Both empirical and model simulated data is detrended using the Hodrick-Prescott (1980) filter. Table 3 on the next page summarizes the second moments of data (relative volatilities to output, and contemporaneous correlations with output) versus the same moments computed from

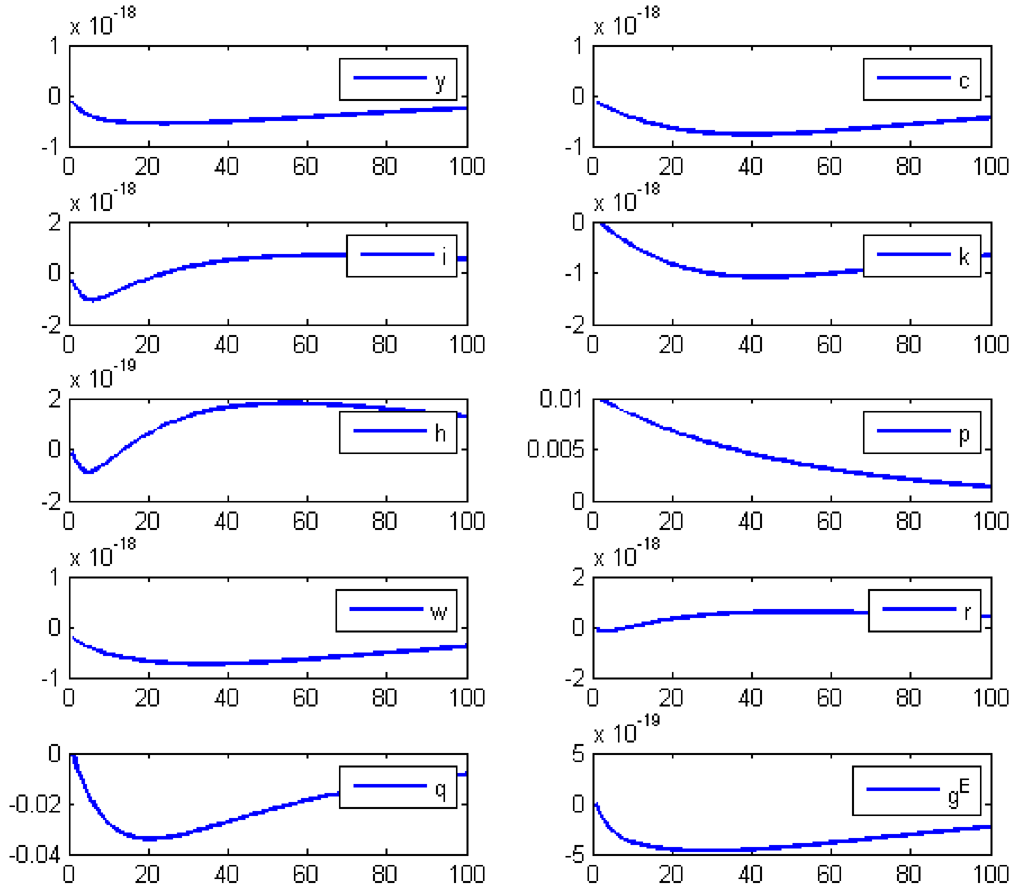

Figure 2: Impulse Responses to a 1% surprise innovation in pollution technology

the model-simulated data at quarterly frequency.[5] To minimize the sample error, the simulated moments are averaged out over the computer-generated draws. The model matches quite well the absolute volatility of output but overestimates the variability of investment. In addition, the model slightly underestimates the variability in consumption; Still, the model is qualitatively consistent with the stylized fact that consumption generally varies less than output, while investment is more volatile than output.

With respect to the labor market variables, the variability of employment predicted by the model is a bit lower than that in data, but the variability of wages in the model is much

[5]The model-predicted 95 % confidence intervals are available upon request.

Table 3: Business Cycle Moments

	Data	Model
σ_y	0.05	0.05
σ_c/σ_y	0.55	0.47
σ_i/σ_y	1.77	4.30
σ_h/σ_y	0.63	0.53
σ_p/σ_y	0.26	1.23
σ_w/σ_y	0.83	0.58
σ_w/σ_h	1.32	1.09
$corr(c, y)$	0.85	0.56
$corr(i, y)$	0.61	0.92
$corr(h, y)$	0.49	0.87
$corr(w, y)$	-0.01	0.89
$corr(p, y)$	-0.31	0.78
$corr(q, y)$	0.31	-0.48
$corr(h, y/h)$	-0.14	0.58

lower than that in data. This is yet another confirmation that the perfectly-competitive assumption does not describe very well the dynamics of labor market variables. Next, in terms of contemporaneous correlations, the model systematically over-predicts the pro-cyclicality of investment, but under-predicts consumption pro-cyclicality. This, however, is a common limitation of this class of models. However, along the labor market dimension, the contemporaneous correlation of employment with output, is relatively well-matched. With wages, the model predicts strong cyclicality, while wages in data are acyclical. This shortcoming is well-known in the literature and an artifact of the wage being equal to the labor productivity in the model. Along the environmental dimension, the match is poor, as Bulgaria has been implementing structural changes to modernize its polluting industry. That is why as output was growing, the share of manufacturing is going down, and there is an entry of "green" firms, while the model predicts a strong procyclicality of pollution. In turn, the model predicts a degradation of environmental quality during expansions, while Bulgaria has been following the EU regulations for emission levels.

5.3 Auto- and cross-correlation

This subsection discusses the auto-(ACFs) and cross-correlation functions (CCFs) of the major model variables. The coefficients of the empirical ACFs and CCFs at different leads and lags are presented in Table 4 below against the averaged simulated AFCs and CCFs. Following Canova (2007), this is used as a goodness-of-fit measure. As seen from Table 4 on next page, the model compares relatively well vis-a-vis data. Empirical ACFs for output and investment are slightly outside the confidence band predicted by the model, while the ACFs for total factor productivity and household consumption are well-approximated by the model. The persistence of labor market variables are also relatively well-described by the model dynamics. Next, as seen from Table 5 on the next page, over the business cycle, in data labor productivity leads employment. The model, however, cannot account for this fact. Being a version of the standard RBC model, where a technology shock is a factor shifting the labor demand curve only, the effect between hours and labor productivity (wage rate) is only a contemporaneous one.

6 Conclusions

We introduce an environmental dimension into a real-business-cycle model augmented with a detailed government sector. We calibrate the model to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2016). We investigate the quantitative importance of utility-enhancing environmental quality, and the mechanics of environmental (”carbon”) tax on polluting production, as well as the effect of government spending on pollution abatement over the cycle. In particular, a positive shock to pollution emission in the model works like a positive technological shock, but its effect is quantitatively minute on the model variables. Allowing for pollution as a by-product of production improves the model performance against data, and in addition this extended setup dominates the standard RBC model framework, *e.g.*, Vasilev (2009).

Still, the failure of the model along the environmental dimension, and the cyclicality of pollution and environmental quality is somethind that requires additional research. One possible extension is to model the structural transformation of the economy where the share

Table 4: Autocorrelations for Bulgarian data and the model economy

		k			
Method	Statistic	0	1	2	3
Data	$corr(h_t, h_{t-k})$	1.000	0.484	0.009	0.352
Model	$corr(h_t, h_{t-k})$	1.000	0.955	0.900	0.836
	(s.e.)	(0.000)	(0.027)	(0.053)	(0.077)
Data	$corr(y_t, y_{t-k})$	1.000	0.810	0.663	0.479
Model	$corr(y_t, y_{t-k})$	1.000	0.955	0.902	0.841
	(s.e.)	(0.000)	(0.026)	(0.050)	(0.072)
Data	$corr(a_t, a_{t-k})$	1.000	0.702	0.449	0.277
Model	$corr(a_t, a_{t-k})$	1.000	0.955	0.901	0.838
	(s.e.)	(0.000)	(0.027)	(0.052)	(0.072)
Data	$corr(c_t, c_{t-k})$	1.000	0.971	0.952	0.913
Model	$corr(c_t, c_{t-k})$	1.000	0.958	0.911	0.859
	(s.e.)	(0.000)	(0.023)	(0.044)	(0.063)
Data	$corr(i_t, i_{t-k})$	1.000	0.810	0.722	0.594
Model	$corr(i_t, i_{t-k})$	1.000	0.955	0.900	0.836
	(s.e.)	(0.000)	(0.027)	(0.052)	(0.076)
Data	$corr(w_t, w_{t-k})$	1.000	0.760	0.783	0.554
Model	$corr(w_t, w_{t-k})$	1.000	0.957	0.907	0.851
	(s.e.)	(0.000)	(0.024)	(0.046)	(0.067)

of services increases at the expense of the diminishing manufacturing sector. Another direction would be to compare the environmental fiscal policy with that from the optimal (Ramsey) case. We leave those for future work.

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Table 5: Dynamic correlations for Bulgarian data and the model economy

		k						
Method	Statistic	-3	-2	-1	0	1	2	3
Data	$corr(h_t, (y/h)_{t-k})$	-0.342	-0.363	-0.187	-0.144	0.475	0.470	0.346
Model	$corr(h_t, (y/h)_{t-k})$	0.066	0.072	0.080	0.574	0.074	0.022	-0.014
	(s.e.)	(0.352)	(0.307)	(0.253)	(0.028)	(0.220)	(0.271)	(0.306)
Data	$corr(n_t, w_{t-k})$	0.355	0.452	0.447	0.328	-0.040	-0.390	-0.57
Model	$corr(n_t, w_{t-k})$	0.066	0.072	0.080	0.574	0.074	0.022	-0.014
	(s.e.)	(0.352)	(0.307)	(0.253)	(0.028)	(0.220)	(0.271)	(0.306)

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