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Abstract

Suppose an identical regulatory reform is adopted simultaneously across a number of countries. We argue that the reformers will grow differently after the reform. To understand the reasons behind the eventual outcome divergence, we set up a tractable general equilibrium (GE) model to study how firms of different size grow after a regulatory reform. The reform reduces the costs to start, operate and close a business. The regulatory cost is modeled as lost labor hours. The model predicts that larger firms will grow faster than smaller firms after the reform. We then take the model predictions to the largest global publicly available firm-level data set, the Enterprise Surveys (ES), which encompasses 121,991 firm-level observations from 2006 to 2015 in 136 countries and territories and 211 country-years. We merge the ES data with the Doing Business indicators. We then test the theoretical predictions, which are broadly confirmed in the data. Thus, based on the notable differences of firm-size distributions across countries which are fairly stable over time, identical reforms may produce a variety of growth outcomes across countries.

Keywords: reform-growth puzzle, deregulation, firm size

JEL Codes: D21, D22, D58, E02, L11, L25

Highlights:

- Identical reforms influence firms of different size differently
- Firm size distributions (FSDs) across the globe are different
- Identical reforms produce a variety of reforms outcomes based on the FDS differences
- We build a tractable general equilibrium model to study the above effects
- We merge the Doing Business and the Enterprise Surveys data to test the predictions

1 Introduction

Suppose an identical regulatory reform is adopted simultaneously across a number of countries. Will the reformers be affected identically? This paper argues they will not, and looks for the reasons behind an eventual outcome divergence.

The explanation offered here is that regulatory reforms – i.e., the reforms aimed at reducing the costs to start, operate and close a business – affect firms of different size differently. Then, if two countries go through identical reforms but their firm size distributions are *ex-ante* different, the two economies will grow differently after the reform. Naturally, the argument extends to more than two economies and to more than one regulatory reform. It also produces a variety of reform outcomes across countries and possibly over time. Therefore, studying the reform outcomes across firms of different size has notable policy implications which motivates us to look at the reform effects from this angle.

Our approach is to set up a tractable general equilibrium (GE) model and study how output of firms of different size grows after regulatory costs are reduced. The theoretical results suggest that larger firms would grow faster than smaller firms after the reform. This turns into our main hypothesis. We test it on the largest global publicly available firm-level data set, and broadly

confirm its validity.

Our theory is in the spirit of Luttmer (2007) and Boedo and Mukoyama (2012) who also use GE models to numerically study the effects of entry and labor regulations on productivity and employment but is set apart in two important ways. First, our paper still entails a micro-founded GE approach but is computationally far less intensive and allows for a tractable analytical solution without losing explanatory virtue. Second, none of the theoretical works considers the different effects of regulations across firm size to explain the variety of reform outcomes across countries.

The remarkable variation of reform outcomes has been studied in the literature before, most notably for transition economies. While some papers place the measurement of reforms at the center of explaining it (Campos and Horváth, 2012), other use a meta-analytical approach (Babecký and Campos, 2011; Babecký and Havránek, 2014). Our approach contributes to the empirical literature as well, exactly because it advances the firm size distribution (FSD) argument which seems to have evaded attention so far.

The next section illustrates the differences observed in the firm-size distributions across the globe. It also argues why those differences matter for producing a variety of reform outcomes across countries.

2 Firm-Size Distributions Across Countries

Providing credible evidence of a variety of reform outcomes across countries hinges on several important questions. First, are there significant differences in the firm-size distributions (FSDs) across countries? If the FSDs are the same, then the reform outcomes across countries would hardly be significantly different, even if small and large firms are found to grow differently after the reform. Second, do reforms influence those distributions? If FSDs are influenced by the reforms over short periods of time, then the FSDs themselves would be endogenous to the reforms. Therefore, it is important to know whether one can take the FSDs as exogenous at least in a cross-sectional setting. Third, are the cross-country growth differences affected by the differences in the FSDs? If they are, then a reform could not only have a different effect on firms of different size but it could also bring aggregate reform implications across countries. This part of the paper addresses each of these questions.

Over recent decades there have been substantial efforts to explain the statistical regularities behind FSDs both within and across countries, and over time. Gabaix (2011) reviews the evidence that FSDs in developed countries

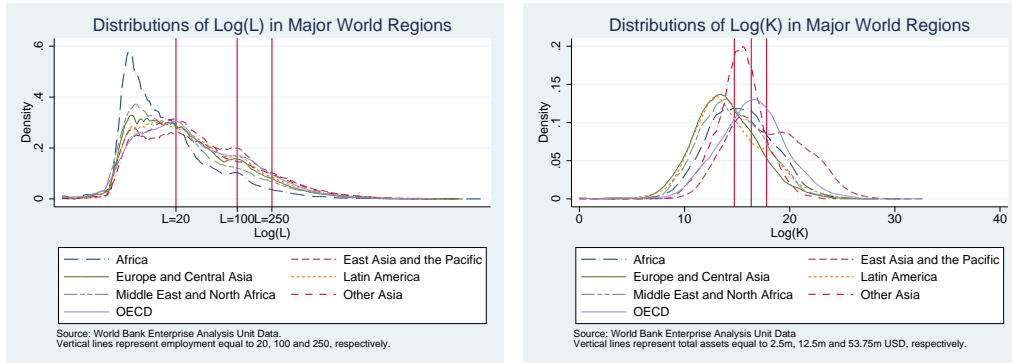
are found to have a Zipf distribution, at least in their upper tails.¹ However, among most developing and some developed countries this regularity in FSDs is harder to observe (Alfaro et al., 2008; Kaizoji et al., 2006).

In addition, looking at the figures below, which are based on the Enterprise Surveys data, it is obvious that there are marked differences in FSDs across major regions of the world, especially in the small-firm segments of the distributions. Those differences are also observed within each of those regions. Admittedly, the figures are a sample of the universe of firms across countries and global regions. However, the Enterprise Surveys sampling methodology has been designed to be representative of the underlying FSDs.²

The FSD differences across countries may be explained by two arguments. First, many young firms operate in the small-firm segment. The growth of those firms is more volatile (Alexander, 1949; Samuels and Smyth, 1968). Second, they grow faster as well but are also more likely to fail (Mansfield, 1962; Jovanovic, 1982; Dunne et al., 1989; Mata, 1994). The snapshots of FSDs in Figure 1(a) and Figure 1(b) capture marked differences in the FSDs

¹Following Gabaix (2011), the Zipf distribution in firm size essentially means that the probability of a firm size S being greater than x is inversely proportional to x .

²See p.2 in the Sampling Methodology notes on the ES website: <http://www.enterprisesurveys.org/methodology>



(a) Log(L)

(b) Log(K)

Figure 1: Firm-Size Distributions of Employment and Assets

across major world regions exactly in the small firms segment (below 20 employees in Figure 1(a) and below USD 2.5m in assets in Figure 1(b)). Then, we can note sizeable differences in FSDs across countries.

Despite the notable cross-industry differences in FSDs (Lotti and Santarelli, 2004; Rossi-Hansberg and Wright, 2007), and despite the documented underlying evolutionary process towards an equilibrium FSD *within an industry* (Hashemi, 2000), the within-country distributions are relatively stable, as found by Cabral and Mata (2003) and Henly and Sánchez (2009). Cabral and Mata (2003) also note that the FSD of a given cohort of firms changes slowly over time, while Henly and Sánchez (2009) add that the within-industry FSD changes over long periods of time and the within-country FSD stays unchanged. Doi and Cowling (1998) assert that in some countries (e.g., Japan) the share of output and employment across size classes is relatively constant

over long periods of time, while in others (e.g., the UK) they change only slowly in favor of smaller firms. Axtell (2001) also concludes that FSDs are stable over time, at the same time being robust to the definition of firm size. Then, it can be assumed that cross-country FSD differences are notable and stable over relatively long periods of time, and are not affected by reforms in the short-run.

The above assumption does not mean the within-country and within-industry FSDs do not evolve. For example, Sutton (1997, 2007) present an extensive discussions on the FSD evolution. However, it is more likely that the differences in FSDs across countries come from an underlying difference in some fundamental factor rather than a given reform. Lucas (1978) argues that FSD is underlined by a distribution of managerial talent. Thus, different countries end up having different FSDs depending on the international allocation of talent. At the same time, countries with lower quality of institutions and enforcement of property rights have a different allocation of talent into productive and rent-seeking occupations (Murphy et al., 1991). Thus, the observed cross-country differences in FSDs have more to do with the longer-term differences in the underlying institutions and property rights systems which are far more stable over time than with the regulatory reforms. As a result, we can safely assume that the FSD are exogenous to regulatory reforms, at least over a short period of time.

Finally, the FSD literature concludes that FSDs are correlated with cross-country income differences (Alfaro et al., 2008; Gabaix, 2011). This evidence contributes to the understanding that FSDs are an important determinant of cross-country differences in the growth effects of reforms.

In a nutshell, both the firm-level data used in this work and the size distribution literature point to significant differences in FSDs across countries. In addition, policy reforms seem to do little to affect the evolution of FSDs over short periods of time within a country. Rather, FSDs are more likely to be driven by fundamentals that affect industry specialization than with policies. Therefore, it is legitimate to assume both the FSD within a country and the cross-country differences in FSDs as given, at least in a short panel, and especially in a cross-sectional data setting. However, the variation in the FSDs also affects the cross-country income differences. Thus, it is intuitive to advance the argument that an identical reform would produce a variety of reform outcomes across countries based on the underlying differences in the FSDs. This argument seems to have evaded both the theoretical and the empirical literature so far, and stands at the center of our approach.

The next section demonstrates the basic elements of the GE model which is set up to study how an identical regulatory reform produces different effects across firms of different size. The section also presents the empirical framework.

3 Methodology

3.1 Overview of the Model

There is a representative household deriving utility from consumption and leisure, where aggregate consumption is a bundle of all varieties available in the economy. There is a unit mass of monopolistically-competitive variety producers facing regulatory costs of production. More specifically, we extend the framework by Dixit and Stiglitz (1977) by introducing a regulatory cost as in Luttmer (2007).

The regulatory cost, \bar{h} , is measured in terms of management time spent dealing with regulations. Those may be start-up procedures or procedures to register property, obtain electricity, comply with contracts or close a business. Thus, \bar{h} has a predictive power over the entire life cycle of a firm. In the firm's problem we demonstrate that \bar{h} is identical to a combination of both fixed and variable labor cost. As regulations induce both fixed and variable costs, the model is able to explain the behavior of firms of different size after any regulatory reform.

To further extend the literature, a government regulatory agency ("regulator") is added. The regulator chooses the level of regulations, and enforces them through the use of employed bureaucrats, whose wages are paid out of the raised tax revenue. The model concludes with empirical predictions

about how regulation affects firms of different size which are then taken to the data.

3.2 Household's Problem

In this model, all varieties $\{c_i\}_{i=1}^N$ are equally weighted as seen from the household's utility function, which is of the form:

$$\max_{c_i, h_i} \ln\left(\left[\int_0^N c_i^\rho di\right]^{1/\rho}\right) + \ln\left(1 - \int_0^N h_i di\right), \quad (1)$$

where $\rho \in (0, 1)$. Note that total time is normalized to unity. In order to generate income to finance consumption of varieties, the household can supply hours $\{h_i\}_{i=1}^N$ from its time endowment to the firms. The market hourly wage rate is w , and labor services are assumed to be homogenous across firms (hence the single wage rate prevailing in the economy). Therefore, total labor income is

$$\int_0^N w_i h_i di = w \int_0^N h_i di = wH, \text{ where } H = \int_0^N h_i di. \quad (2)$$

In addition, the agent will have a claim on all firms' profits ($\Pi = \int_0^N \pi_i di$), where Π and $\pi(i)$ denote aggregate and individual profits, respectively. Note that with free entry, there will be only one firm producing a particular variety and equilibrium profits will be zero. The household's budget constraint then

becomes

$$\int_0^N p_i c_i di = wH + \Pi - \tau, \quad (3)$$

where p_i denotes the price of variety i , and τ is the amount of lump-sum taxes owed. Taking $\{p_i\}_{i=1}^N$ and w as given, the household then chooses $\{c_i, h_i\}_{i=1}^N$ to maximize Eq.(1) s.t. Eqs. (2)-(3). The resulting FOCs are as follows:

$$c_i : \frac{c_i^{\rho-1}}{\int_0^N c_i^\rho di} = \lambda p_i, \quad (4)$$

$$c_j : \frac{c_j^{\rho-1}}{\int_0^N c_j^\rho dj} = \lambda p_j, \quad (5)$$

$$h_i : \frac{1}{1 - \int_0^N h_i di} = \lambda w, \quad (6)$$

$$h_j : \frac{1}{1 - \int_0^N h_j di} = \lambda w. \quad (7)$$

It follows that $h_i = h_j = h$ as both satisfy the same FOC, and thus $H = Nh$. Intuitively, if the hourly wage rate is the same across firms, and the labor supplied is homogenous, then in equilibrium the household will work the same number of hours in each firm.

Next, the FOCs for c_i and c_j are divided side by side to obtain

$$c_i = \left(\frac{p_i}{p_j}\right)^{\frac{1}{\rho-1}} c_j. \quad (8)$$

This expression is then plugged into the household's budget constraint to yield

$$\int_0^N p_i \left(\frac{p_i}{p_j}\right)^{\frac{1}{\rho-1}} c_j di = w \int_0^N h_i di = wH + \Pi - \tau. \quad (9)$$

Let $wH + \Pi - \tau = Y$, i.e., total income equals total real output. Rearranging (9) produces

$$c_j p_j^{\frac{1}{1-\rho}} \int_0^N p_i^{\frac{\rho}{\rho-1}} di = Y. \quad (10)$$

Denote aggregate price index as $P \equiv \int_0^N p_i^{\frac{\rho}{\rho-1}} di$, then divide output by the price index P , and call the ratio B to obtain $B = Y/P = c_j p_j^{\frac{1}{1-\rho}}$, or

$$c_j = B p_j^{\frac{1}{\rho-1}} \quad (11)$$

that is, demand for each variety is isoelastic. Without loss of generality, (11) can be written out with the index i , which will help later when defining equilibrium.

3.3 Government Regulator's Problem

There is a government regulator, whose objective function is positively monotone in the amount of regulation passed, \bar{h} , and negatively-related to the amount of lump-sum taxes (τ) that needs to be raised to pay bureaucrats' wages in return to producing and enforcing the regulations. Taxes are set so that the budget constraint is balanced every period. The regulator's problem is then to

$$\max_{\bar{h}, \tau} \bar{h}^\theta - \tau^2, \text{ s.t. } w\bar{h} = \tau. \quad (12)$$

The utility derived from regulation is assumed to feature positive, but decreasing marginal benefit of regulation, hence $0 < \theta < 1$. At the same time, as in Barro (1979), the regulator wants to minimize the distortionary effect of changing taxes, hence the tax term will be described via a convex cost function.

We normalize the wage rate to unity and substitute τ from the budget constraint to obtain the optimal level of regulation: $\bar{h} = (2/\theta)^{2-\theta}$. In the data \bar{h} is pinned down by the time senior management spends dealing with regulations.

3.4 Variety-Producing Firms

Each of the N firms in the economy would produce a single variety, which will be differentiated from the other $N - 1$ goods. The production function of each variety will be Cobb-Douglas in labor and capital, and the total factor productivity (TFP) will enter multiplicatively, *i.e.*,

$$c_i = A_i K_i^\alpha (h_i - \bar{h})^{(1-\alpha)}, \quad (13)$$

where A_i is the productivity shift parameter, K_i denotes the capital input used by firm i , and $\alpha, 1 - \alpha$ denote the capital and labor shares, respectively. Physical capital is assumed to be pre-installed in this set-up, and thus will be treated as a fixed input. In the presence of regulatory costs, positive output

will be produced only in case hours worked (h_i) exceed the hours spent dealing with regulations (\bar{h}). This production function features increasing returns to scale: with fixed costs, and capital being a pre-installed input, which does not change in the short-run, charging a price equal to the marginal cost leads to profits, and thus a perfectly competitive equilibrium cannot exist.

Further, all labor is paid w , thus total labor cost is the sum of unproductive and productive labor, i.e., $wh_i = w\bar{h} + w(h_i - \bar{h})$. Alternatively, the first expression can be regarded as a fixed entry/exit cost, while the second could be viewed as a variable cost dependent on doing business regulations.

3.5 Symmetric Monopolistically Competitive Equilibrium

For tractability purposes, the model will focus on a symmetric case: it is assumed that all firms use the same technology and capital. Thus, TFP can be normalized to unity, $A_i = 1$, and $K_i = K$. In other words, all firms will be identical in productivity, capital, and employment. The crucial advantage of symmetry is that it produces an easily tractable model at the expense of somewhat empirically naïve first-order conditions. To remedy this issue, we extend the model in the Technical Appendix with the asymmetric case. In it, we show that the comparative statics will depend explicitly on the firm

size.

As a perfectly competitive equilibrium cannot exist in this framework, the equilibrium concept is relaxed to allow for monopolistically competitive producers and free entry. In that case, the firm's objective is to maximize profit by taking the demand for its product as given, and choosing the price of its variety, $\{p_i\}$. By optimally setting the price, the firm will optimally set its output as well:

$$\max_{p_i} p_i B p_i^{\frac{1}{\rho-1}} - w h_i. \quad (14)$$

Given that

$$c_i = K^\alpha (h_i - \bar{h})^{(1-\alpha)}, \quad (15)$$

it follows that

$$h_i = c_i^{\frac{1}{1-\alpha}} K^{-\frac{\alpha}{1-\alpha}} + \bar{h} = \left[B p_i^{\frac{1}{\rho-1}} \right]^{\frac{1}{1-\alpha}} K^{-\frac{\alpha}{1-\alpha}} + \bar{h} \quad (16)$$

Plug in the expression for h_i in firm's optimization problem to obtain

$$\begin{aligned} \max_{p_i} p_i \left[B p_i^{\frac{1}{\rho-1}} \right]^{\frac{1}{1-\alpha}} K^{-\frac{\alpha}{1-\alpha}} - w \left[\left[B p_i^{\frac{1}{\rho-1}} \right]^{\frac{1}{1-\alpha}} K^{-\frac{\alpha}{1-\alpha}} + \bar{h} \right] = \\ \max_{p_i} \left[B p_i^{\frac{1}{\rho-1}} \right]^{\frac{1}{1-\alpha}} K^{-\frac{\alpha}{1-\alpha}} [p_i - w] - w \bar{h} \end{aligned} \quad (17)$$

From the FOC it follows that

$$p_i = w/\rho > w, \quad (18)$$

or that each price is a fixed mark-up over the wage, hence

$$p = p_i = p_j \quad (19)$$

and also

$$c = c_i = c_j. \quad (20)$$

Impose the symmetry into the FOCs for consumption and hours, and divide them side by side to obtain

$$\frac{\frac{c^{\rho-1}}{Nc\rho}}{\frac{1}{1-H}} = \frac{1}{\rho}, \quad (21)$$

or

$$\frac{1-H}{Nc} = \frac{1}{\rho}. \quad (22)$$

Note that aggregate consumption equals

$$Nc = NK^\alpha(h_i - \bar{h})^{(1-\alpha)} = (NK)^\alpha(Nh_i - N\bar{h})^{(1-\alpha)} = (NK)^\alpha(H - N\bar{h})^{(1-\alpha)}. \quad (23)$$

Plug this expression in (22) and simplify to obtain

$$\rho(1-H) = (NK)^\alpha(H - N\bar{h})^{(1-\alpha)}. \quad (24)$$

We can solve the non-linear equation above for H as a function of parameters \bar{h}, ρ, K, α , then obtain hours worked per firm $h = H/N$, consumption of each variety, and using free entry condition, determine the number of the

firms. Unfortunately, in the general case, there is no closed-form solution, and numerical exercises for different parameters have to be executed to study comparative statics. Only for the special case when $\alpha = 0$, i.e., shutting down the physical capital and collapsing the production function to becoming linear in labor, the model can be solved analytically. We will present that particular solution, in which the comparative statics are transparent.

In addition to simplifying the algebra, the analytical solution allows for tractable comparative statics. By using the fact that $\rho + N\bar{h} = (1 + \rho)H$, the equilibrium outcomes from the simplified model then become:

$$H = \frac{\rho + N\bar{h}}{1 + \rho} \quad (25)$$

$$h = \frac{\rho + N\bar{h}}{N(1 + \rho)} \quad (26)$$

$$c = h - \bar{h} = \frac{\rho + N\bar{h}}{N(1 + \rho)} - \bar{h} = \frac{\rho(1 - N\bar{h})}{N(1 + \rho)} \quad (27)$$

From the free entry condition ($\pi = pc - wh = 0$), and after some algebra, it follows that

$$N = \frac{1 - \rho}{2\bar{h}}. \quad (28)$$

That is, the total number of varieties will be endogenously determined, and it will depend on the model parameters, including regulation. More specifically, reducing the regulatory costs \bar{h} increases the number of varieties N :

$$\frac{dN}{d\bar{h}} = -\frac{1-\rho}{2\bar{h}^2} < 0. \quad (29)$$

Next, total labor supply can be obtained as:

$$H = \frac{\rho + \frac{1-\rho}{2}}{1+\rho} = \frac{2\rho + 1 - \rho}{2(1+\rho)} = \frac{1+\rho}{2(1+\rho)} = \frac{1}{2}, \quad (30)$$

where the peculiar result is due to the log-log specification for the utility function. Individual hours are then

$$h = \frac{H}{N} = \frac{1}{2N} = \frac{\bar{h}}{1-\rho}, \quad (31)$$

hence positively related to \bar{h} . In other words, the higher the regulatory cost, the higher the hours supplied to each firm (in order to produce a positive quantity of a variety). Finally, output for each variety is

$$c_i = c = \frac{\rho(1 - \frac{1-\rho}{2})}{(1+\rho)^{\frac{(1-\rho)}{2h}}} = \frac{\rho\bar{h}}{1-\rho}. \quad (32)$$

The last equation, together with (29), produces the following empirical implications. Deregulation (reducing \bar{h}) will make each existing firm produce less, and at the same time will create more varieties to choose from. We can further differentiate (32) with respect to \bar{h} , and then again to ρ , to obtain the differences in the deregulation effects across firms of different size:

$$\frac{dc}{d\bar{h}} = \frac{\rho}{1-\rho} > 0, \quad (33)$$

$$\frac{dc^2}{d\bar{h}d\rho} = \frac{1}{(1-\rho)^2} > 0. \quad (34)$$

That is, deregulation reduces output and its effect will depend on ρ . Further, the effect on output of existing firms with higher ρ is stronger than the effect on firms with smaller ρ .

As smaller firms typically have more substitutes, hence higher ρ , our **empirical implication** from the model is that smaller firms will reduce output more than bigger firms after a regulatory reform aimed at lowering the costs and time to do business. In other words, bigger firms will benefit more from deregulation.

Our proxy for firm size in the symmetric set-up is admittedly crude. To bring more realism to the model, in the Technical Appendix we present the asymmetric equilibrium where firms differ in several standard characteristics: TFP, capital stock, employment and sales. The algebra to derive the deregulation effects across firms of different size then becomes tedious. This extension is one of our contributions, as it generates comparative statics that explicitly depend on firm size (TFP, capital, employment, sales) of the particular firm. Still, the qualitative results are identical to the symmetric case. The only difference is that the effect of \bar{h} then depends on the size of A_i and K_i . For sufficiently large A_i and/or K_i , the effect is positive. Firms with high productivity and/or firms with a lot of assets are usually larger firms (that have achieved economies of scale).

3.6 Data

There are two main sources of data to test the model predictions: the Enterprise Surveys data and the World Bank Doing Business data. The Enterprise Surveys data set is produced by the Enterprise Analysis Unit (EAU) at the World Bank. As of June 06, 2016, it encompasses firm-level data from 2006 to 2015 in 136 countries and territories and 211 country-years. The data set has 121,991 firm-level observations from 15 industries in each of those country-years. The firm-level frequency in each of those industries is presented in Table 1.

The Enterprise Surveys data set is probably the largest publicly available firm-level data set which is suitable for policy analysis. To reduce the number of empty industry-country cells, we drop any industry with less than 1000 observations, and any country with less than 100 observations. We also refrain from using the built-in *subjective* self-reported evaluations of regulations in the EAU data, as it would be challenging to extract the exogenous variation in the regulatory measures from the survey data.

The second data source – The World Bank Doing Business data base – contains numerous *objective* measures of regulations across most countries and territories in the world since 2005. We choose 5 measures of regulatory policies which arguably capture the regulatory burden over the entire life

cycle of a firm. Those are: costs to start up; costs to register property; costs to get electricity; cost to enforce a contract; recovery rate after insolvency.

We take those variables not only because they match the life cycle of a typical firm but also because these measures vary notably across countries *and* over time which could bring out policy implications from reforms. Summary statistics for the above reforms are presented in Table 2, and the definitions of variables are presented in Table 3. The details of the empirical model follow.

3.7 Empirical Model

The theoretical model has demonstrated that firms of different size grow differently after identical regulatory reforms. In order to test this prediction, we estimate the following model:

$$\begin{aligned} \Delta \log Y_{ikt} = & \alpha_1 + \alpha_2 \Delta \log L_{ikt} + \alpha_3 \Delta SU_{kt} S_{ikt} + \alpha_4 \Delta P_{kt} S_{ikt} + \alpha_5 \Delta E_{kt} S_{ikt} + \\ & \alpha_6 \Delta C_{kt} S_{ikt} + \alpha_7 \Delta I_{kt} S_{ikt} + \mathbf{Z}'_{ikt} \alpha + f_s + f_k + f_{st} + f_{kt} + \Delta \epsilon_{ikt} \end{aligned} \quad (35)$$

where $\log Y_{ikt}$ stands for either sales, $\log SAL_{ikt}$, or sales per worker, $\log SPW_{ikt}$, of firm i in country k in period t . In addition, $\log L_{ikt}$ is the number of employees,³ respectively, to estimate the impact of the main factors of production; SU_{kt} , P_{kt} , E_{kt} , C_{kt} and I_{kt} are the Doing Business measures of costs

³Labor costs is another option for L_{ikt} . However, the data contains about 20,000 fewer observations on labor costs than number of employees. This is an obvious reason to prefer

to start up, costs to register property, costs to get electricity, cost to enforce a contract, and the recovery rate after insolvency, respectively, that firm i has to deal with in country k in period t . In the data, the time difference between t and $t - 1$ is 3 years.

Note that all Doing Business measures vary on country level only. This brings two implications: first, including them in the model *separately* in either levels or differences does not make sense, as their effects are captured by either the country fixed effects or the country-year effects which are included in the model; second, the firm-level variation in the regulatory measures is brought by their interaction with the size of the firm.

The size of the firm S_{ikt} is measured by the log-number of employees; \mathbf{Z}'_{ikt} is a vector of other firm observables, including whether the firm has obtained an ISO certification, to capture some differences in the performance of firms with different levels of technology and more sophisticated management procedures, legal structure, age of the firm and top manager experience. For parsimony, we report the estimates of age and top manager experience only.

Further, f_s , f_k , f_{st} and f_{kt} are the sector- and country fixed and time-varying effects. Including those is motivated by the firm-level evidence by Commander and Svejnar (2011) on the reform outcomes in Central and Eastern Europe. In their paper, the country fixed effects turn out to be more

the latter, especially given the correlation between the two is 0.60 and highly significant.

important than reforms in determining firm performance, at least in firms from Central and Eastern Europe and the former Soviet Union.

Finally, $\Delta \log K_{ikt}$ is conspicuously missing from the growth equation. This is because of data limitations. As there is no measure of the lagged level of capital in the Enterprise Surveys data (apart from its panel component which is very small), we need to assume the growth of sales and sales per worker depends on the growth of labor rather than both the growth of labor and capital. To address this data limitation, we can run the model in levels instead as data on K_{ikt} does exist. However, the results would not have a clear reform interpretation so we favor running the model in differences.

Naturally, excluding the change in capital introduces an omitted variable bias (OVB) in all estimates. However, if such a bias indeed exists, it would bias all the estimates in the same way, as capital is missing from all the regression equations. Since we are more interested in the sign of the parameters rather than their magnitudes, we believe the OVB is not crucial in interpreting the results.

The lack of K_{ikt-1} data presents another important limitation of the model. Potentially, the results depend on how the size of the firm has been defined. If there was data on K_{ikt-1} , we could substitute capital for labor in the interaction terms to check if the results crucially depended on the definition of firm size. At the moment, unfortunately that is not possible.

Apparently, even the richest publicly available firm-level data set to date has important limitations. Despite those, the results broadly confirm our hypothesis. Their presentation follows.

4 Results

The results are presented in two tables corresponding to the two firm-level performance indicators: sales and sales per worker. The former is a measure which has implications about the growth of firms, while the latter is a better gauge of labor productivity. As both have growth and development implications, we run separate estimations for each of the two performance indicators.

Tables 4 and 5 present the results. They demonstrate a somewhat nuanced effect of regulatory reforms on firms of different size. While the effect of start-up and exit regulations is independent of firm size, reforming other doing business regulations leaves a significantly different mark across firm size.

Specifically, both tables demonstrate that all significant parameter estimates on output growth and labor productivity growth are negative. This means reducing the regulatory burden on registering property, obtaining electricity and enforcing contracts benefits larger firms more than smaller firms.

This difference is particularly notable for enforcing contracts where the effect is highly significant. It is noteworthy that a highly significant multicollinearity is found between the interaction terms. That is why the estimates in the last column of both Table 4 and Table 5 are insignificant.

We further study the significant estimates to **derive policy implications**. First, making property registration cheaper by 5.8 percentage points of the property value (i.e., moving from the 75-th percentile to the 25-th percentile of registering property distribution) would bring about half a percentage point margin in sales and labor productivity growth for each unit increase in firm size. As the measurement unit of firm size is $\text{Ln}(\text{No.employees})$, the above result would trigger non-linear effects across size classes. However, those can be calculated, and are presented in Table6 for each of the reforms and size classes.⁴

Table 6 implies that moving from the 75-th to the 25-th percentile of the property registration distribution would make a firm with 20 employees grow about 17.4 percentage points faster than a firm with only 1 employee, and a firm with 250 employees grow about 5.3 percentage points faster than a firm

⁴For example, the precise estimate of $\alpha_5 = -0.00000451$. Then, the $\text{IQR} = 1571.65$. Therefore, moving from the 75th to the 25th percentile induces an effect of about $+0.007$ for each unit increase in the size of the firm. The increase in firm size for the different firms is also given in Table 6.

with a 100 employees. Next, moving from the 75-th to the 25-th percentile of the getting electricity distribution would make a firm with 20 employees grow about 2.1 percentage points faster than a firm with only 1 employee, and a firm with 250 employees grow about 0.6 percentage points faster than a firm with a 100 employees. Finally, moving from the 75-th to the 25-th percentile of the enforcing contracts distribution would make a firm with 20 employees grow about 10.0 percentage points faster than a firm with only 1 employee, and a firm with 250 employees grow about 3.0 percentage points faster than a firm with a 100 employees.

In any case, those are very large reforms. Typically, in any given year a country does a tiny fraction of the reforms needed to trigger the above effects, if it does any reform at all. Potentially, however, those effects exist and our estimates show they do favor larger firms more.

These results are in line with our theoretical predictions, which suggested larger firms will grow faster after a regulatory reform. They can be explained by the intuition by Aghion et al. (2007). They claim that deregulation would make the incumbent firms innovate more to prevent further entry, especially in technologically advanced industries. Incumbent firms in those industries would therefore grow faster.

As advanced countries also have more technologically advanced firms which are typically larger, then countries with a high share of large firms

will develop faster than countries with a high share of small firms, especially after one of the above reforms. The results also demonstrate that firm size is one of the factors behind the variety of regulatory reform outcomes across countries.

5 Conclusion

The abundance of evidence on the impact of regulatory reforms motivates us to build a micro-founded GE model which explains the variety of reform outcomes across countries. Our approach is based on the notable yet stable differences in the firm size distributions (FSDs) across countries and over time. The argument we advance is that if two countries go through identical reforms but their FSDs are *ex-ante* different, then those two economies will grow differently after the reform.

Our model has two versions: a symmetric and an asymmetric one. The former is presented for simplicity, while the latter, which is left for the Technical Appendix, is more realistic. Both versions predict larger firms to grow faster after a regulatory reform aimed at reducing the hours spent dealing with regulations. The model predictions are then tested on the Enterprise Surveys data merged with the Doing Business data, both produced by the World Bank.

The empirical results conform well with the theory predictions when it comes to the impact of some regulatory reforms. Specifically, both sales and labor productivity of larger firms grow faster than those of smaller firms after reducing the costs to register property, obtain electricity and enforce contracts. However, our estimates show that firms of different size do not grow differently after a reform of entry or exit regulations.

The paper extends the recent literature in several ways. First, our micro-founded GE model is tractable. Previous work relies either on numerical solutions or on extending existing empirical evidence. Second, the majority of the literature misses the importance of looking at the divergent effects of reforms across firms of different size. Those micro-level differences in the reform impact may also produce a variety of reform outcomes across countries, and this avenue for research has been largely underestimated so far.

The diverging growth and labor productivity outcomes stem from the notable differences in the size distribution of firms across countries. Our FSD argument then holds rich policy implications for size-contingent regulatory reforms in the developing and emerging economies striving to climb back on the convergence path. In addition, it offers a novel explanation for the remarkable variation of reform outcomes across countries.

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7 Tables

Table 1: Number of Firm-level Observations by Industry

Industry	No. obs.
Textiles	5,879
Leather	1,107
Garments	8,133
Food	13,042
Metals and machinery	10,374
Electronics	1,953
Chemicals and pharmaceuticals	5,438
Wood and furniture	2,876
Non-metallic and plastic materials	7,659
Auto and auto components	1,053
Other manufacturing	9,429
Retail and wholesale trade	28,109
Hotels and restaurants	6,572
Other services	12,629
Other: Construction, Transportation, etc	7,214
Total	121,467

Notes: The table presents the number of firm-level observations within each of the industries featured in the World Bank Enterprise Analysis Unit data, as of June 6, 2016. We cleaned the data from 443 observations which did not have a definite industry affiliation, and from further 81 observations which had missing data on their industry affiliation.

Table 2: Summary Statistics for Doing Business Policies

Policy	Mean	Median	St. Dev.	Variance	IQR
Start-Up Costs	47.57	16.00	95.87	9190.54	42.00
Costs to Register Property	6.16	5.00	4.93	24.29	5.80
Costs to Obtain Electricity	1874.61	513.55	4014.66	1.61e+07	1571.65
Costs to Enforce a Contract	34.18	27.50	24.15	582.99	16.70
Recovery Rate	33.81	30.50	24.32	591.69	26.60

Notes: The table presents summary statistics for each of the explanatory variables included in the empirical model and taken from the World Bank Doing Business data. IQR (inter-quartile range) is the change of the respective variable from the 25th to the 75th quartile. The data has been downloaded on June 24, 2016 from <http://databank.worldbank.org>, and its last update has been on 23 Nov. 2015.

Table 3: Definitions of Variables

Variable	Definition	Source
Sales	Total annual sales in last fiscal year for each firm	EAU
Sales per worker	Sales divided by the total no. of permanent, full-time employees in the last fiscal year	EAU
Log(L)	The natural logarithm of total number of permanent, full-time employees in the last fiscal year	EAU
Start-Up Costs	Costs to start up a business, expressed as a share of annual income per capita	DB
Costs to Register Property	Costs to register property, expressed as a share of property value	DB
Costs to Obtain Electricity	Costs to get electricity, expressed as a share of annual income per capita	DB
Costs to Enforce a Contract	Cost to enforce a contract, expressed as a share of the claim	DB
Recovery Rate	Recovery rate after insolvency, expressed as cents on the dollar	DB
Mgr. Exp.	Years of experience which the top manager of this company has	EAU
Firm Age	The year of data collection minus the year in which the firm was set-up.	EAU
Quality cert.	A dummy variable indicating whether the firm has an internationally recognized quality certificate (=1) or not (=0).	EAU
Legal	The firm's current legal status: publicly listed company; privately held, limited liability company; sole proprietorship; partnership; limited partnership; other	EAU

Notes: The table presents the definitions of the variables used in the empirical model, as well as their source. DB stands for Doing Business, EAU stands for Enterprise Analysis Unit.

Table 4: Regulatory Reforms and $\Delta\text{Log}(\text{SAL})$ across Firms of Different Size

	Dependent variable: $\Delta\text{Log}(\text{SAL})$					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \text{Log}(\text{L})$.413*** (.025)	.463*** (.033)	.439*** (.038)	.476*** (.030)	.365*** (.038)	.434*** (.059)
$\Delta \text{Start*Size}$.000 (.000)					-.000 (.000)
$\Delta \text{Prop*Size}$		-.010* (.006)				-.005 (.005)
$\Delta \text{Electr*Size}$			-.000** (.000)			-.000 (.000)
$\Delta \text{Contract*Size}$				-.002*** (.001)		.000 (.001)
$\Delta \text{Insolv*Size}$.002 (.001)	.001 (.001)
Mgr. Exp.	-.001** (.001)	-.001** (.001)	-.001 (.001)	-.001** (.001)	-.001** (.001)	-.001 (.001)
Firm Age	-.001*** (.000)	-.001*** (.000)	-.002*** (.000)	-.001*** (.000)	-.001*** (.000)	-.003*** (.000)
Const.	.319*** (.089)	.058 (.070)	.178*** (.049)	.250** (.105)	.324*** (.087)	.124 (.089)
Observations	41095	41028	18742	41095	41095	18742
Adjusted R^2	.163	.164	.108	.164	.164	.108

Notes: The table presents results from OLS estimations of the difference in $\text{Log}(\text{Sales})$ on the difference in $\text{Log}(\text{No. of employees})$, on other observables from the World Bank Enterprise Analysis Unit firm-level data, and on objective reform data, measured by The Doing Business Database interacted with the firm size measured by the $\text{Log}(\text{No. of employees})$. All estimations include the age of the firm, its legal status, an indicator of a quality certificate, industry-, industry-year, country- and country-year effects. Standard errors are clustered on country-year and are given in parentheses. Symbols: * $p < .10$, ** $p < .05$, *** $p < .01$

Table 5: Regulatory Reforms and $\Delta\text{Log}(\text{SPW})$ across Firms of Different Size

	Dependent variable: $\Delta\text{Log}(\text{SPW})$					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \text{Log}(L)$	-.587*** (.025)	-.537*** (.033)	-.561*** (.038)	-.524*** (.030)	-.635*** (.038)	-.566*** (.059)
$\Delta \text{Start*Size}$.000 (.000)					-.000 (.000)
$\Delta \text{Prop*Size}$		-.010* (.006)				-.005 (.005)
$\Delta \text{Electr*Size}$			-.000** (.000)			-.000 (.000)
$\Delta \text{Contract*Size}$				-.002*** (.001)		.000 (.001)
$\Delta \text{Insolv*Size}$.002 (.001)	.001 (.001)
Mgr. Exp.	-.001** (.001)	-.001** (.001)	-.001 (.001)	-.001** (.001)	-.001** (.001)	-.001 (.001)
Firm Age	-.001*** (.000)	-.001*** (.000)	-.002*** (.000)	-.001*** (.000)	-.001*** (.000)	-.003*** (.000)
Const.	.319*** (.089)	.058 (.070)	.178*** (.049)	.250** (.105)	.324*** (.087)	.124 (.089)
Observations	41095	41028	18742	41095	41095	18742
Adjusted R^2	.170	.171	.112	.171	.170	.112

Notes: The table presents results from OLS estimations of the difference in $\text{Log}(\text{Sales per worker})$ on the difference in $\text{Log}(\text{No. of employees})$, on other observables from the World Bank Enterprise Analysis Unit firm-level data, and on objective reform data, measured by The Doing Business Database interacted with the firm size measured by the $\text{Log}(\text{No. of employees})$. All estimations include the age of the firm, its legal status, an indicator of a quality certificate, industry-, industry-year, country- and country-year effects. Standard errors are clustered on country-year and are given in parentheses. Symbols: * $p < .10$, ** $p < .05$, *** $p < .01$

Table 6: Growth Margins Across Size Classes after Reforms

L	Log(L)	$\Delta \text{Log}(L)$	Prop. $\Rightarrow \Delta Y(\%)$	Electr. $\Rightarrow \Delta Y(\%)$	Contr. $\Rightarrow \Delta Y(\%)$
$L = 1$	0.00	-	-	-	-
$L = 20$	3.00	3.00	17.4	2.1	10.0
$L = 50$	3.91	0.91	5.3	0.6	3.0
$L = 100$	4.61	0.70	4.1	0.5	2.3
$L = 250$	5.52	0.91	5.3	0.6	3.0
$L = 1000$	6.91	1.39	8.1	1.0	4.6
$L = 10000$	9.21	2.30	13.3	1.6	7.7

Notes: The table presents estimates of the marginal effects of the Doing Business reforms on sales and labor productivity growth ($\Delta Y(\%)$) across size classes. The estimates are based on the main parameter estimates given in Table 4 and Table 5, and on increasing L from 1 to 20, from 20 to 50, from 50 to 100, etc. The magnitude of the reform used to produce the effects on $\Delta Y(\%)$ above is to move from the 75-th to the 25-th percentile in the respective reform distribution.

Technical Appendix: Asymmetric equilibrium

In this extended setup, the firms are differentiated by size. First, we will allow the TFP level to differ in order to distinguish between firms of different efficiency: A_i will be different, so for the same input of labor, output will be different:

$$c_i = A_i K_i^\alpha (h_i - \bar{h})^{(1-\alpha)}. \quad (\text{A.1})$$

However, as seen from the equation above, differences in size might not be triggered by difference in TFP alone. Differences in capital input and employment levels can also account for that. Therefore, in the analysis to follow, all those three – TFP and capital and labor inputs – will be used as proxies for firm size.

In addition, the consumer side will be slightly amended as well to accommodate the asymmetric solution. In the absence of symmetry, the representative agent should be allowed to supply different number of hours to different firms. In order for such a choice to be optimal, the setup must allow for different wage rates across firms. One simple way to model this consistently is to allow for labor to be heterogeneous. Total labor supply will be then a weighed average of individual hours, rather than just the sum of

those, namely

$$H = \int_0^N a_i h_i di, \quad (\text{A.2})$$

where a_i will be the weight attached to the hours supplied to firm i , with $a_i > 0$, $\int_0^N a_i di = 1$. That is, working hours are not valued identically by the consumer because different jobs require different effort. The weights $0 < a_i < 1$ given by consumers to hours worked in different firms will be taken as given. The implication of this modelling choice is to allow for different wages at different firms. In addition, the specification of the utility of leisure may be rationalized by the fact that certain labor tasks may require different skill level, are performed in hazardous environment to one's health, or lead to excessive amount of stress, and thus decrease the consumer's utility of leisure much faster than other types of labor.

When solving the model extension described above, the different wage rates w_i , and the utility weights attached to hours, a_i , will show up in the equilibrium result in a non-linear way, which complicates the comparative statics. Therefore, in order to isolate the size from the productivity effect, after taking the FOCs all utility weights will be set equal (thus wages and hours across firms becoming equal), allowing only individual firms' TFP to differ. This is a valid approach, as such collapsing of the model is done after every unit in the model has optimized. The comparative static expressions

will then simplify greatly, and the sign being contingent on the particular firm's TFP level.

Consumer's problem: As in the symmetric case, the representative consumer problem is to maximize utility subject to the budget constraint, or

$$\max_{c_i, h_i} \ln\left(\int_0^N c_i^\rho di\right)^{1/\rho} + \ln\left(1 - \int_0^N a_i h_i di\right) \quad (\text{A.3})$$

s.t.

$$\int_0^N p_i c_i di = \int_0^N w_i h_i di + \Pi - \tau. \quad (\text{A.4})$$

FOCs:

$$c_i : \frac{c_i^{\rho-1}}{\int_0^N c_i^\rho di} = \lambda p_i, \quad (\text{A.5})$$

$$c_j : \frac{c_j^{\rho-1}}{\int_0^N c_j^\rho dj} = \lambda p_j, \quad (\text{A.6})$$

$$h_i : \frac{1}{1 - \int_0^N a_i h_i di} = \lambda w_i, \quad (\text{A.7})$$

$$h_j : \frac{1}{1 - \int_0^N a_i h_i di} = \lambda w_j. \quad (\text{A.8})$$

Divide the FOCs for h_i and h_j to obtain

$$\frac{w_i}{w_j} = \frac{a_i}{a_j}. \quad (\text{A.9})$$

Wages are proportional to the corresponding utility weights attached to hours. Next, divide the FOCs for c_i and c_j to obtain

$$c_i = \left(\frac{p_i}{p_j}\right)^{\frac{1}{\rho-1}} c_j. \quad (\text{A.10})$$

Now plug this expression into the budget constraint to obtain:

$$\int_0^N p_i \left(\frac{p_i}{p_j}\right)^{\frac{1}{\rho-1}} c_j = \int_0^N w_i h_i di + \Pi - \tau. \quad (\text{A.11})$$

Let $\int_0^N w_i h_i di + \Pi - \tau = Y$ (again, with free entry profit income is zero).

Then

$$c_j = p_j^{\frac{1}{\rho-1}} \frac{Y}{\int_0^N p_i^{\frac{\rho}{\rho-1}} di}. \quad (\text{A.12})$$

Analogously to the symmetric case, define

$$P \equiv \int_0^N p_i^{\frac{\rho}{\rho-1}} di. \quad (\text{A.13})$$

to be the aggregate price index. Also, let $B = Y/P$ and derive individual demand for variety j as:

$$c_j = B p_j^{\frac{1}{\rho-1}}. \quad (\text{A.14})$$

Again, the demand for each variety is isoelastic.

Firm's problem: As in the symmetric case, taking the demand for its variety as given, the firm producing each variety will set its price optimally to maximize profit, or

$$\max_{p_i} K_i^{-\frac{\alpha}{1-\alpha}} [B p_i^{\frac{\rho}{\rho-1}}]^{\frac{1}{1-\alpha}} [p_i - w_i] - w_i \bar{h}. \quad (\text{A.15})$$

FOC:

$$p_i = w_i / \rho > w_i. \quad (\text{A.16})$$

Prices are again a fixed mark-up over the wage. However, in the asymmetric case the price of variety i is a mark-up over the wage rate paid for labor services supplied to firm i . Using the proportionality between prices and wages, it follows that

$$\frac{p_i}{p_j} = \frac{w_i}{w_j} = \frac{a_i}{a_j}, \quad (\text{A.17})$$

$$p_i = \frac{a_i}{a_j} p_j. \quad (\text{A.18})$$

Similarly, consumption is also proportional to the utility weights ratio:

$$c_i = \left(\frac{a_i}{a_j}\right)^{\frac{1}{\rho-1}} c_j. \quad (\text{A.19})$$

Next, construct the marginal rate of substitution between consumption and hours:

$$\frac{c_j^{\rho-2} (1 - \int_0^N a_i h_i di) \left(\frac{a_i}{a_j}\right)}{\int_0^N \left(\frac{a_i}{a_j}\right)^{\frac{1}{\rho-1}} di} = \frac{\lambda p_i}{\lambda w_i}. \quad (\text{A.20})$$

Simplify to obtain

$$\frac{[A_j K_j^\alpha (h_j - \bar{h})^{(1-\alpha)}] \rho - 2 (1 - \int_0^N a_i h_i di) \left(\frac{a_i}{a_j}\right)}{\int_0^N \left(\frac{a_i}{a_j}\right)^{\frac{1}{\rho-1}} di} = \frac{1}{\rho}. \quad (\text{A.21})$$

The equation above can be solved implicitly for h_j as a function of model parameters $(A_j, K_j, a_j, \bar{h}, N, \rho)$. In particular, using the Implicit Function Theorem (IFT) we can compute comparative statics.

To simplify the derivations further, we will normalize all utility weights to unity, $a_j = a_i = 1$. That is not a crucial assumption, as the normalization

is done after FOCs are derived. With the normalization in place, it follows that $h_i = h_j = h$. More specifically, the labor input effect $h_i - \bar{h}$ in the production function is now isolated, and the factors driving difference in firm size will be the level of TFP and physical capital. The equation of interest then is recast in the following form:

$$F \equiv \rho A_j^{\rho-2} K_j^{\alpha(\rho-2)} (h_j - \bar{h})^{(1-\alpha)(\rho-2)} (1 - Nh_j) - N = 0. \quad (\text{A.22})$$

To obtain the effect of the setup cost (regulation) on labor supplied to individual firm, apply IFT to obtain:

$$\frac{dh_j}{d\bar{h}} = -\frac{F_{\bar{h}}}{F_{h_j}} > 0, \quad (\text{A.23})$$

since

$$\begin{aligned} F_{h_j} &= \rho A_j^{\rho-2} K_j^{\alpha(\rho-2)} (1 - \alpha)(\rho - 2)(h_j - \bar{h})^{(1-\alpha)(\rho-2)-1} (1 - Nh_j) \\ &\quad + \rho A_j^{\rho-2} K_j^{\alpha(\rho-2)} (h_j - \bar{h})^{(1-\alpha)(\rho-2)} (-N) < 0 \end{aligned} \quad (\text{A.24})$$

$$F_{\bar{h}} = \rho A_j^{\rho-2} K_j^{\alpha(\rho-2)} (1 - \alpha)(\rho - 2)(-1)(h_j - \bar{h})^{(1-\alpha)(\rho-2)-1} (1 - Nh_j) > 0. \quad (\text{A.25})$$

In other words, the higher the setup cost, the higher the labor supply to an individual firm. The size of the firm does not matter in this case, as the $A_j^{\rho-2} K_j^{\alpha(\rho-2)}$ term will cancel out.

In turn, given that consumption is monotone in h , it is easy to show that the higher the setup cost, the higher the output, or:

$$\frac{dc_j}{d\bar{h}} = A_j \left(\frac{dh_j}{d\bar{h}} - 1 \right) > 0, \quad (\text{A.26})$$

as we have shown above that

$$\frac{dh_j}{d\bar{h}} > 1. \quad (\text{A.27})$$

Note that the size of the effect will be proportional to the size of the firm (as represented by A_j).

The next comparative static to be explored is the dependence between the labor supplied to firm i and total number of varieties/firms N (or entry):

$$\frac{dh_j}{dN} = -\frac{F_N}{F_{h_j}} < 0 \quad (\text{A.28})$$

since

$$\begin{aligned} F_{h_j} &= \rho A_j^{\rho-2} K_j^{\rho-2} (1-\alpha)(\rho-2)(h_j - \bar{h})^{(1-\alpha)(\rho-2)-1} (1 - Nh_j) \\ &\quad + \rho A_j^{\rho-2} K_j^{\rho-2} (h_j - \bar{h})^{(1-\alpha)(\rho-2)} (-N) < 0, \end{aligned} \quad (\text{A.29})$$

$$F_N = \rho A_j^{\rho-2} K_j^{\rho-2} (h_j - \bar{h})^{(1-\alpha)(\rho-2)} (-h_j) - 1 < 0. \quad (\text{A.30})$$

In addition,

$$\frac{dc_j}{dN} = A_j \frac{dh_j}{dN} < 0. \quad (\text{A.31})$$

The higher the entry, or the larger the number of varieties, the lower the output of each variety. Again, the size of the effect will be proportional to the size of the firm.

Next, the effect of the degree of substitutability on labor supplied to individual firm is as follows:

$$\frac{dh_j}{d\rho} = -\frac{F_\rho}{F_{h_j}}, \quad (\text{A.32})$$

where

$$F_{h_j} = \rho A_j^{\rho-2} K_j^{\rho-2} (1-\alpha)(\rho-2)(h_j - \bar{h})^{(1-\alpha)(\rho-2)-1} (1 - Nh_j) \\ + \rho A_j^{\rho-2} K_j^{\rho-2} (h_j - \bar{h})^{(1-\alpha)(\rho-2)} (-N) < 0, \quad (\text{A.33})$$

$$F_\rho = A_j^{\rho-2} K_j^{\rho-2} (h_j - \bar{h})^{(1-\alpha)(\rho-2)} (1 - Nh_j) \\ + \rho(1 - Nh_j) A_j^{\rho-2} K_j^{\rho-2} (h_j - \bar{h})^{(1-\alpha)(\rho-2)} \ln[A_j K_j^\alpha (h_j - \bar{h})^{(1-\alpha)}]. \quad (\text{A.34})$$

The first term of F_ρ is positive, the second is ambiguous. More specifically, the second term can be split into two parts - the first is positive, but the log part is unclear. Ultimately, it all depends on the size of $A_j K_j^\alpha$: if A_j , or/and K_j is/are large enough, the term will positive.⁵ Hence the effect of ρ on h_j is ambiguous, and so is the effect of ρ on c_j . For a large enough firm (sufficiently high enough A_i , and/or K_j), the effects are positive:

$$\frac{dh_j}{d\rho} = -\frac{F_\rho}{F_{h_j}} > 0. \quad (\text{A.35})$$

Otherwise, both effects are negative. Therefore, the degree of substitutability will produce a result which is conditional on the firm size, as proxied by TFP and capital stock.

To sum up the results from the model, both the symmetric and the asymmetric cases suggest that identical reforms across countries would make larger firms grow faster than smaller firms.

⁵Note that $h_j - \bar{h} > 0$ but small, as total labor supply is much less than 1.