The Road to Socioeconomic Fractality

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Abstract: Modelling socioeconomic phenomena is a challenge because of the difficulty to relate abstract conceptual structures with complex empirical data. The standard econometric approach takes whatever insight there exists, and simplifies it to fit into regression equations. However, developing economic ideas and empirical models separately may foster a tendency for science to diverge from reality, especially when those ideas originate in another discipline. This paper suggests a stochastic-optimization-based mapping of concepts from any domain on concepts from economics and management science. Such an approach could potentially alleviate the divergence problem by outsourcing part of the researcher’s task to the computational intelligence. By way of example I discuss the opportunities to use the field of mathematical neuroscience as a source of knowledge to be transferred to socioeconomic research.

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1. Introduction

It is fashionable to lament about the discrepancy between economic theory and how the economy and business actually function. The tone is set, as usual, by high profile figures, including Nobel laureates. Amartya Sen proclaimed long ago that the axioms used in economic and political theory need revision (Sen, 1997). The late Ronald Coase, a centenarian, concluded that, “Economics as currently presented in textbooks and taught in the classroom does not have much to do with business management, and still less with entrepreneurship. The degree to which economics is isolated from the ordinary business of life is extraordinary and unfortunate.” (Coase, 2012). This list of authorities and truisms could be made very long, but two examples suffice to see where this chain of thought is pointing. However, there seems to be less agreement on what should be done to get out of these dire straits.

Here I suggest – with no intention to criticize – that part of the problem may lie with the adopted modelling practice in the economic science. Because it is immensely difficult to grasp quantitatively the complexity of socioeconomic life, researchers often resort to specific combinations of mathematical and computational methods that serve as “crutches”. The result may look like a piece of theoretical advancement, yet the price paid would most likely be a thinned bond with reality. I discuss briefly the problem in Section 2, and then propose in Section 3 a new idea about how better to conduct modelling research, aided by modern computational techniques. Equipped with innovative armory, one could venture into the uncharted territory of socioeconomic fractality. There, a new kind of plough meets a virgin soil to produce a new kind of fruit. This is the topic of the concluding Section 4.

2. On the nature of quantitative modelling in economics

It is standard practice in the natural sciences to seek concepts, definitions, and measurements that, when linked by laws, would explain phenomena in the observable world directly and with a tolerable amount of error. In a sequence of refinements of both theory and method, virtually everything amenable to discovery within the dominant paradigm is eventually discovered. Then comes a crisis, a scientific revolution resolves it and the story continues with a new set of concepts and methods.

However, social sciences are so complicated that they sometimes travel a more elliptic road to discovering truths. Because direct link between theory and empirical observation is often
unfeasible, the tools of econometrics lend a helping hand. Take this example from economics. In 1962, Jan Tinbergen introduced the gravity model of trade, positing that trade between two countries is more intense when they are geographically closer and have bigger economies – just like the attraction between two celestial bodies. Compare, however, Newton’s formula for gravitation with the equation used in economics. The former is:

\[ F = g \frac{m_1 m_2}{r^2}. \]  

In Eq. (1), \( F \) is the force of attraction between two bodies with masses \( m_1 \) and \( m_2 \), \( g \) is the gravitational constant, and \( r \) is the distance between the centres of the masses. What could qualify as the equivalent of physical mass in economics: perhaps GDP? It is trickier to define “distance”. It could be geographical distance, but the two trading countries may have common border. They may have well developed railways and highways and so transport between them would be easy. They may even speak the same language. Their political systems may be similar, their legal systems may be of high quality and able to efficiently resolve contract disputes, and all of these would contribute to diminishing the “distance”. On the other hand, there might exist tariffs, quotas, and institutional barriers. It is hard to imagine putting them all in a single parameter, but here comes econometrics and constructs an entire regression equation around the idea. This is one example (Mengova, 2012):

\[
\ln T_{ijc} = \ln \alpha_c + \beta_{1c} \ln (GDP_i) + \beta_{2c} \ln (GDP_j) + \gamma_{1c} \ln (GDP_i/Population_i) \\
+ \gamma_{2c} \ln (GDP_j/Population_j) + \delta_{1c} \ln (Dist_{ij}) + \delta_{2c} \ln (Area_i \times Area_j) \\
+ \theta_{1c} \ln (I_i) + \theta_{2c} \ln (I_j) + \varepsilon_c (Adj_{ij}) + \zeta_c (Language_{ij}) \\
+ \eta_c (Conv) + \lambda Year + u_{ijc}, \text{ where } c = 1, 2.
\]  

Here, \( T_{ijc} \) is the volume of (nominal) trade flows between country \( i \) and country \( j \), i.e., \( T_{ij} = X_{ij} + M_{ij} \), or exports plus imports between \( i \) and \( j \). The indicator \( c = 1 \) stands for homogeneous products, and \( c = 2 \) for differentiated (or complex) products. \( Dist_{ij} \) is the geographical distance between country \( i \) and country \( j \). Further, \( Area_i \times Area_j \) measures the common area of the two countries, \( Adj_{ij} \) accounts for common land border between country \( i \) and country \( j \) and takes a value of 1 if they share one, and a value of 0 if they do not. They account for two different measures of trade costs – the first one measuring transportation costs, the second the ease of
trade exchange. \( \text{Language}_{ij} \) takes a value of 1 if country \( i \) and country \( j \) share common language and is 0 otherwise. Variable \( u_{ijc} \) is error term associated with the dependent variable \( T_{ijc} \). Variables \( \text{Conv}, l_i \), and \( l_j \) account for the quality of the legal systems of the two countries and the degree of convergence between those systems (Mengova, 2012). All Greek letters are regression coefficients.

Apparently the gravity model idea is captured, yet Eq. (2) is no longer Newton’s formula. All meaningful conclusions are drawn on the basis of the regression equation. Today’s economic literature is full of similar examples where a theoretical insight leads to certain mathematical model, which is in the end abandoned for a conventional regression model. This is sad because the insight had often been economic rather than coming from another science, and its mathematical derivation might have been quite sophisticated. Therefore, one of the claims here is that developing two separate streaks – one of theory, and another of regressions – may help, if not create, a tendency for the notorious divergence between economic science and reality. Of course regression keeps analysis to the ground and related to reality. But its limits, mostly in imagination, inevitably hamper the advancement of science. On the other hand, ‘high theory’ is free to roam, unrestricted by the need to say something precise about the empirical world. Ultimately, we have nominal theoretical achievements, but not much of a progress in understanding the economy. However, today’s computational methods may offer an unexpected help in improving the usefulness of high theory.

3. A computer-aided search for knowledge
Computational intelligence gained respect in a number of applied sciences and its use in economics is hardly surprising. Here I seek to present a novel application that might – at least in some cases – ‘squeeze the lemon’ of theory much better than previously thought. Consider Figure 1. Following Torgerson (1958), it gives an abstract overview of the relation between scientific models and the observable world. Circles \( C_1 \) – \( C_5 \) represent the theoretical constructs in each discipline. For example, in various branches of physics that can be length, force, electric current etc. In every science, it is necessary that at least some theoretical constructs be defined so as to have a clear connection with phenomena from the real world. That is how a science gets linked to empirical observations. In Figure 1 these ‘hooks’ are the constructs \( C'_1 \), \( C'_2 \), and \( C'_3 \). Their definitions, shown with double lines, contain a prescription of actions that always yield the same result when conducted by an expert. The sequence of
particular operations and procedures makes up the operational definition, and shows how exactly a scientific theory connects with the observable world.

![Diagram](image)

**Figure 1.** Computer-aided search for knowledge. Blue panel: a theoretical model with scientific concepts $C_1, C_2, \ldots$ and relations among them (the black lines). Circles $C'_1, C'_2,$ and $C'_3$ represent scientific concepts for which empirical measurement is possible. The blue bunch of arrows designates the opportunity to find relations among theoretical and operational concepts. See text for more detail.

Other constructs are defined only theoretically and together with the connections among them form theoretical models. When a branch of science contains theoretical models of connected constructs, and at the same time for some of the latter exist operational definitions, there is a
theory that can be empirically verified. Eventually, such theories are refined and become useful in explaining and predicting phenomena from the surrounding world.

The economic variable *inflation* is a good example to illustrate the difference between theoretical and operational definition. The former comes from Fisher’s monetary equation, dealing with variables like price level, total stock of money, velocity of money circulation etc. On the other hand, an operational definition of inflation is the consumer price index (CPI). Apparently, the latter definition changes over time, unlike its theoretical counterpart. The consumer basket evolves due to social progress and technological innovation, bringing out new products to the market. One can view circles $C_1$ and $C'_1$ in Figure 1 as an illustration of the two definitions, with the dotted line indicating the problematic relationship between them.

How about a theoretical model that comes from a field unrelated to economics – just like the gravity model – and is considered useful for it. Which concepts from either science ought to get paired, is the cardinal question. Now even the dotted lines do not exist. And that is where computational methods become relevant. Today’s technology is advanced enough to suggest methods like stochastic optimization, which can find the best fit of variables from another field of science onto economic concepts.

Exactly that automated process is depicted with the bright blue bunch of arrows in Figure 1. Its purpose is to suggest that the search for variable match may be somewhat arbitrary. Many traditionally minded economists may be puzzled by this seemingly erratic mapping between sets of concepts, but they need not be. Figure 2 and Figure 3 show that a process of stochastic optimization – in this case simulated annealing – can do the matchmaker’s job automatically. Two points are relevant here. First, a theorem guarantees that if the stochastic optimization process is slow enough, the system of equations will find its global minimum of error with respect to the empirical data at hand. That is, if the data are adequate and fully describing the process, the algorithm will match in an optimal way the elements from empirical economics onto theoretical concepts from the other field.

In particular, Figure 2 shows an example of an objective/error function being minimized along the way. A complementary look is provided by Figure 3, which shows how a particular parameter, one among many, finds its optimal value after several thousand rounds of
computation. So the first point is that efficient algorithms can fit pieces of the two domains as smoothly as possible.

Figure 2. An excerpt from a stochastic optimization process. See text for more detail.

Figure 3. Narrowing the interval of a parameter subjected to stochastic optimization.
The second point balances the first one. While an algorithm may do an extremely good job, it can never compensate for a conceptual deficiency. If the model is not suitable to match, only a modest result would be achieved. It is the role of the researcher to come up with an insightful idea why such a mapping makes sense in the first place.

Of course, there is no limit to human imagination with regard to the set of possible mappings. For example, one might try Newton’s formula and use stochastic optimization to map it directly onto available data about international trade. Another example for creative mapping, developed in the age before computational stochastics, was the Navier–Stokes equations for heat and mass transfer that were adapted to describe stock options in the early 1970s. Physics has been perhaps the most frequent source of inspiration for economic modelling. Other sciences have also had a share. An example for potential future crossbreeding is given in the next Section.

4. Socioeconomic fractality

A transfer of theoretical ideas among distant branches of science has for long been widespread. The point has always been to take knowledge, really fitting well in the target domain and thus invigorating it by producing some impressive small result and offering immense opportunities for further research. Hardly any limitation other than the conceptual ones could hinder the success of such research (but conceptual limitations are perhaps the toughest).

In a few recent publications I have advocated the use of mathematical and computational neuroscience in economics and management science (Mengov, 2013; 2014; 2015). This suggestion is far less exotic than it might appear at first sight. Economics has routinely drawn inspiration from biology-related disciplines such as evolutionary biology, ethology, and of course psychology. With regard to the last one, the sway of behavioral economics shows no signs of fading for decades already. It might even be argued that its presence, most visible in behavioral finance, in our time is in fact gaining momentum (Sedlarski & Dimitrova, 2014; Gerunov, 2013; 2014). Neuroscience is a kin discipline which also has some innovative insights to offer.
To begin with, it is immediately obvious that, to quote Bruno Apolloni, “The social network is a fractal extension of our brain networks” (Apolloni, 2013). Due to Facebook, LinkedIn, and the likes, we are connected with one another better than ever and this has a profound effect on the way we conduct our business. Neuroscience, with its century-long tradition, could be a natural source of knowledge for the economic behaviour of the modern humans.

One particular kind of neural system seems especially good candidate to model social interactions. It is called Adaptive resonance theory (ART) neural network and was introduced by Grossberg (1976a, 1976b). It was further refined and extended in hundreds of publications over the ensuing decades, with a recent overview published in Grossberg (2013).

In brief, adaptive resonance theory describes how the human mind learns new knowledge without forgetting the existing one. According to this theory, all knowledge is stored in connections among neurons in the brain whereby, to a first approximation, three neural layers,

![Diagram](image.png)

Figure 4. Adaptive resonance theory (ART) neural network.
F₁, F₂, and F₃ are instrumental. They exchange signals in two directions: a “bottom-up” stream comes in from the senses and provokes a “top-down” response of associations, based on previous knowledge. Both streams are compared and matched to produce “impressions”, which, if found adequate in a certain mathematical sense, are eventually memorized. These interactions are shown in Figure 4. The theory posits that the brain is “plastic” as it is able to accommodate change, and at the same time “stable” as it retains what has been learned already. This is the solution to the famous “stability-plasticity dilemma”. The term “adaptive resonance” denotes information processing analogous to physical resonance in mechanical and electrical systems. It is information that “resonates” as multidimensional signals are exchanged between layers F₁ and F₂. Responding to an incoming image, the neural network instantly scans its memories in F₂ to find a sufficiently close match. If one exists, all related neurons are activated to exchange signals with the impressions layer F₁. This process is called adaptive resonance. Until it lasts, knowledge update takes place. The interaction is local as it affects a limited number of synaptic connections. If no good match is found, then the incoming pattern is deemed to be entirely new and is learned in a new set of connections among neurons in F₁ and F₂. Again, a resonance takes place.

It is intriguing that the operations in the ART neural network have been identified as analogous to some processes in a broad class of hierarchical social systems, existing in many strata of human society (Mengov, 2013; 2015). The main idea is summarized in Figure 5, showing how typical parliamentary procedures in a democratic establishment resemble the ART functionality. There are many similar details between the two sequences of events. Precisely that is what I call socioeconomic fractality (Mengov, 2015) – the projection of events from the millisecond-to-second time span in a neural circuit, over a social system operating on a months-to-years scale.

Let us look at the left column in Figure 5. It describes procedures in a parliamentary democracy, but with some adaptation it would just as easily fit totalitarian states. Similar in principle are the ways in which corporations and all kinds of institutions, large and small, replace their chief executive officers, presidents, commanders-in-chief, etc. Hardly different, though simpler, is the power handover in the animal world, where each species has developed its own rituals, often brutal, to elect the next leader of the group.
Figure 5. A fractal-type analogy between the operation of an ART neural network and the functioning of a parliamentary democracy. (Adapted from Mengov (2013; 2015)).

The right column in Figure 5 describes what is happening in the ART neural network. There are many details in the similarity and none is accidental. Its origin and rationale are discussed extensively in Mengov (2013; 2015). The ART system looks like a strong candidate for
computational mapping between socioeconomic theory and neuroscience as described earlier. Moreover, as this effort one day produces fruit, it would become relevant to seek bridges to more traditional economic fields such as new institutional economics, and especially its modern treatments, accommodating contributions from behavioral economics (cf. Sedlarski, 2013).

The fractality concept is in no way limited to the ART example. That was first, but other analogies across scale are immediately obvious. Many communities, markets, and other collectives of people – physical or virtual, are good candidates to be modelled by the Gated Dipole neural network (Grossberg, 1972), which is a mathematical model of the brain circuit, responsible for emotion generation. Further, when a social or socioeconomic system of human beings evolves and develops a kind of emotional memory – known perhaps by other names in different scientific disciplines, then an augmented version of the dipole, i.e., the Recurrent Associative gated Dipole (READ) would be a better model. In the more distant future one could foresee going not only in the direction of more simplicity, but also in the direction of more complexity – once the simpler models have explained many phenomena, more complicated ones would become relevant.

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