Center for
Economic Theories and Policies
Sofia University St. Kliment Ohridski
Faculty of Economics and Business Administration

ISSN: 2367-7082

Bulgarian Economic Papers

Search and Matching Frictions and Business Cycle Fluctuations in Bulgaria

Aleksandar Vasilev

BEP 03-2016
Publication: February 2016

Search and matching frictions and business cycle fluctuations in Bulgaria

Aleksandar Vasilev*

February 14, 2016

Abstract

In this paper we investigate the quantitative importance of search and matching frictions in Bulgarian labor markets. This is done by augmenting an otherwise standard real business cycle model a la Long and Plosser (1983) with both a two-sided costly search and fiscal policy. This introduces a strong propagation mechanism that allows the model to capture the business cycles in Bulgaria better than earlier models. The model performs well vis-a-vis data, especially along the labor market dimension, and in addition dominates the market-clearing labor market framework featured in the standard RBC model, e.g Vasilev (2009), as well as the indivisible labor extension used in Hansen (1985).

JEL classification: D51, E24, E32, J40

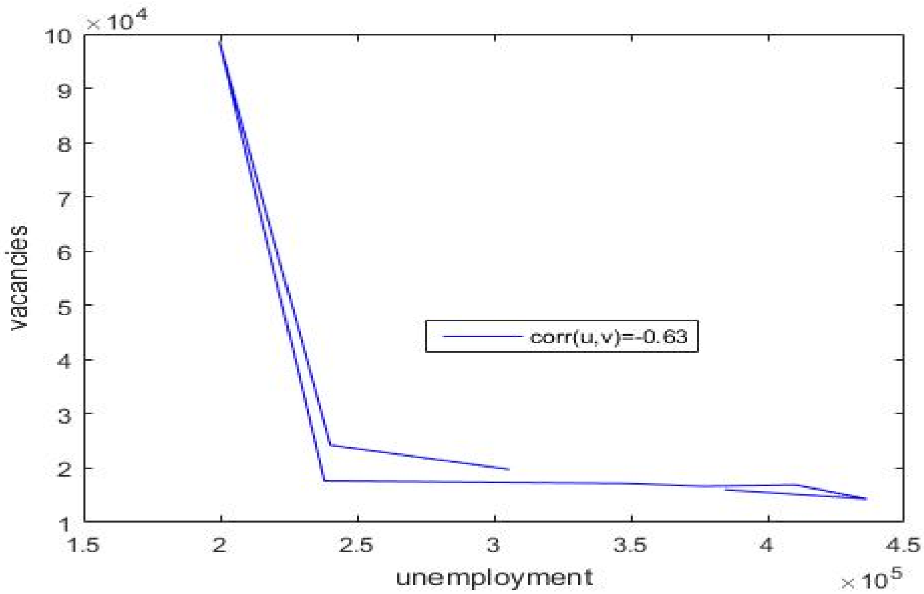
Keywords: general equilibrium, unemployment and wages, business cycles, fiscal policy

*Asst. Professor and CERGE-EI Affiliate Fellow, Department of Economics, American University in Bulgaria, 1 Georgi Izmirliev Sq., Blagoevgrad 2700, Bulgaria. Tel: 00 359 73 888 482. E-mail: avasilev@aubg.bg.

1 Introduction

The standard real business cycle model was shown to be unable to capture the dynamics in the labor markets in Bulgaria, e.g. Vasilev (2009). One dimension that the setup with a perfectly competitive market for the labor input is not able to explain the presence of involuntary behavior. Another aspect of data that such a framework cannot capture is the stylized fact documented in many developed economies, the so-called "Beveridge curve," the strong negative relationship between open positions (vacancies) and unemployment.[1] A strong negative correlation between vacancies and unemployment has also been observed in Bulgaria, a former transition country and as of 2007, a member state of the European Union. Figure 1 below plots the behavior of the two labor variables since the EU accession.

Figure 1: Beveridge curve in Bulgaria 2006-14



Several researchers have proposes that the failure of standard RBC models (*e.g.* Kydland and Prescott (1982) and Long and Plosser (1983) for the US, and Vasilev (2009) for Bulgaria) to capture adequately labor market dynamics might necessitates abandoning of the Walrasian frictionless market-clearing paradigm, at least when it comes to the way labor markets work

[1] The curve is named in Blanchard and Diamond (1989), in recognition of W.H. Beveridge's (1944) original contribution.

in reality, and introduce some impediments when it comes to finding a job, or informational frictions that preclude both individuals and firms from being able to perfectly observe all prospects and unoccupied positions available at any moment in time. Lucas (1987) took the task seriously and models the transition out of unemployment to produce a neoclassical general equilibrium model that is able to generate involuntary unemployment in equilibrium.[2] Diamond (1982) and Pissarides (1985) show the relevance of a search-and matching model in macroeconomic context, when the separation rate is taken to be exogenous.

This paper continues the tradition and aims to model the labor market in Bulgaria after the introduction of the currency board (1997) in an equilibrium real business cycle model with fiscal policy.[3] For that purpose, the setup in Merz (1995) is augmented with fiscal policy a la Cristiano and Eichenbaum (1992).[4] The two-sided costly labor search and matching frictions introduced in the model setup creates an inefficient outcome in the labor market due to the search and congestion externalities. This rigidity could be driven by heterogeneity of workers' skill level, or the time cost involved due to the imperfect information possessed by either side of the market.[5] After all, it takes time and effort to find the best match, and involuntary unemployment is exactly the case when an individual is not able to find work at a prevailing wage.[6] Therefore, as Shimer (2010) argues, the search and matching mechanism could be regarded as the process captured by the ad hoc convex labor adjustment costs inroduced in some macroeconomic models, e.g Hansen and Sargent (1988), which helped the framework amplify employment fluctuations.

In the model, search and recruiting activities are viewed as costly investment activities that help eventually augment the number of jobs created ("matches"), which in turn increase total

[2]Other approaches include Hansen(1985) and Rogerson (1988) indivisible labor idea, where workers participate in lotteries and can buy insurance against becoming unemployed.

[3]Earlier periods are excluded due to the low-quality of data from the early 1990s and the volatile time period of the financial crisis from 1996-97.

[4]Wright (1986) and Howitt (1988) show how search problems may at least partially resolve the labor market puzzles that RBC models with perfectly-competitive labor markets exhibit.

[5]Kennan (2006) emphasizes the importance of private information to explain unemployment over the business cycle.

[6]For an excellent treatment on how earlier search literature merged with the RBC literature, the interested reader is referred to Ramey (2011).

employment. The vacancies that are posted by employer could be viewed as an as asset that generates a value when the position is filled with a suitable candidate. The market tightnes, defined as the ratio of vacancies-to-unemployment, causes the search and congestion externalities. With trade frictions in the market for labor, the search effort is suboptimal, thus the labor input is rationed. Since this rationing is stochastic (due to the limited information about candidates on the market and available positions), the price, *i.e.*, the wage rate, is not the only allocative mechanism. Therefore, the inefficiency cannot be eliminated by wage adjustments alone.

On the worker side, working is generally more valuable than being unemployment. However, under certain conditions, unemployment may be an optimal outcome, if it is not to the worker's or the employer's advantage to continue the employment contract. Thus the model is able to produce involuntary employment in equilibrium. More specifically, in each period matches are destroyed with some exogenous probability, and any employed person faces a risk of being laid off.[7,8] The search and matching approach in the real business cycle (RBC) literature has potential to replace earlier paradigm, as it can quantitatively explain several dimensions of labor markets that the standard RBC model has troubles fitting.[9] The process of trading the labor input in an environment featuring imperfect information, or equivalently, the search and matching frictions present in the labor markets provide a tractable mechanism which is both realistic and plausible. Still, the output of the search and matching process, the number of matches (described by a matching function), will be taken as a stylized empirical construct, and thus as an ad hoc modelling approach.

The paper proceeds to evaluate the quantitative importance of search and matching fric-

[7]The assumption of an exogenous job destruction rate is mainly for analytical convenience and model tractability. Endogeneizing the separation rate, as pointed out in Pissarides (2000), does not alter fundamentally the job creation and job search processes. Den Haan *et al.* (2000) show that this feature adds more persistence to the model variables and helps the setup capture better the volatility in job creation and job destruction.

[8]Adding on-the-job search also does not change qualitatively the results, as it does not affect the main theory of the search and matching frictions.

[9]As pointed out in Andolfatto (1996), the search and matching paradigm is an isomorphic representation to the labor hoarding mechanism, as described in Burnside and Eichenbaum (1993).

tions in the case of the Bulgarian business cycle after the introduction of the currency board arrangement, which, when complemented with other reforms, brought aggregate stability to the economy. Those real rigidities in the labor markets introduce history dependence in the employment status, which makes employment, unemployment, and output more persisten. Such real rigidities in the labor market could be thus regarded as a qualitatively important propagation mechanism that can replicate data behavior, especially along the labor market dimension.[10] Overall, the search and matching model, and the trade frictions in particular, generates persistence in output and both employment and unemployment, and is able to respond to the criticism in Nelson and Plosser (1992), Cogley and Nason (1995) and Rotemberg and Woodford (1996), who argue that the RBC model does not have a strong internal propagation mechanism besides the strong persistence in the TFP process. Incorporating search leads to labor productivity in the model leading employment over the business cycle, which is what we observe in data as well. The standard market clearing (Walrasian) setting cannot account for this fact, as in the standard RBC model a technology shock leads to an increase in both productivity (wage) and hours contemporaneously. In contrast, the search model breaks the one-to-one relationship between productivity and employment, Productivity leads employment, as matching takes time. The very low dynamic correlation between wages and employment in Bulgaria is well-approximated in the model, mostly due to the fact that the wage rate is determined through Nash-bargaining procedure. Finally, the dynamic correlation between vacancies and unemployment in Bulgaria is also well-captured by the model: That is also a dimentison that the standard RBC model (Vasilev 2009) cannot capture, since vacancies are not featured there.

The rest of the paper is organized as follows. Section 2 describes the model setup. Setup 3 outlines the model parameterization and the calibration strategy employed. Section 4 presents the steady state results. Section 5 discusses the impulse responses, compares simulated to empirical moments and evaluates the model's overall goodness-of-fit. Section 6 concludes.

[10] Therefore, such rigidities in a model provide a useful "laboratory"' to study labor market fluctuations in Bulgaria.

2 Model Setup

The structure of the model economy is as follows: There is a unit mass of households, as well as a representative firm. The households own the physical capital and labor, who are supplied to the firm. Employment depends on both the probability of matching, and the search effort of households. The firm produces output using labor and capital. It posts a vacancy to advertise an available position. Thus, the labor market is characterized by a costly two-sided search. The wage rate is decided via a Nash bargaining procedure. The government uses tax revenues from labor and capital income to finance the wasteful government consumption and the lump-sum government transfers.

2.1 Households

There is a unit mass of homogeneous one-member households, who derive utility out of consumption and leisure

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \ln C_t + \phi \ln(1 - N_t) \Bigg\}, \tag{2.1}$$

where E_0 denotes the expectations operator as of time 0, C_t, N_t denote consumption and hours (employment)[11] in period t, $0 < \beta < 1$ is the discount factor. As in Merz (1995) households will be pooling together all resources and in this way achieve full insurance against the contingency of unemployment. As a result, consumption will be identical across households regardless of the employment status.

Households own all the capital in the economy. Aggregate physical capital evolves according to the following law of motion:

$$K_{t+1} = I_t + (1 - \delta)K_t, \tag{2.2}$$

where $0 < \delta < 1$ is the depreciation rate. Households will rent the capital to the firm at the rate r_t, generating $r_t K_t$ in before-tax capital income.

[11] This equivalence follows from the normalization of the mass of households, as well as setting total time endowment equal to unity.

Another source of income for the households is the labor income. Aggregate employment evolves according to[12]

$$N_{t+1} = (1-\psi)N_t + p_t S_t (1 - N_t), \tag{2.3}$$

where $0 \leq \psi \leq 1$ denotes the transition rate from employment to unemployment, and $p_t \geq 0$ denotes the probability of a match in period t, which depends on the tightness of the labor market.[13,14] Households take the probability p_t at which the aggregate search effort produces a match as given. Aggregate before-tax labor income is then $w_t N_t$, where w_t is the hourly wage rate in the economy.

Households can decide to use time and effort to improve their chances of forming a match. As in Merz (1995), the search cost function is assumed to be monotone in the search intensity and of the form

$$b_0 S_t^{\eta}(1 - N_t), \tag{2.4}$$

where $b_0 > 0$, $\eta > 1$, and $S_t > 0$.[15] That is, the cost of searching for a job is $b_0 S_t^{\eta}$ per household, and the mass of unemployed households is $1 - N_t$. Since search cost produces a waste of resources in the economy, total search cost will be accounted for as an output cost.[16] Households own the firm in the economy and claim all the profit. Households' budget constraint is then

$$C_t + K_{t+1} - (1-\delta)K_t + b_0 S_t^{\eta}(1 - N_t) \leq (1 - \tau^k) r_t K_t + (1 - \tau^l) w_t N_t + \Pi_t + G_t^{tr}, \tag{2.5}$$

where $\{\tau^k, \tau^l\}$ are the effective tax rates on capital and labor income, Π_t denote firm's profits, and G_t^{tr} are government transfers.

[12] In this model we do not distinguish between unemployed and out of the labor force.

[13] As argued in Pissarides (2000), endogeneizing the job destruction rate does not change much the qualitative properties of the model.

[14] Note also that the equation above produces a steady state employment rate of $N^* = M^*/\psi$, where M^* are steady-state jobs created ("matches"), and thus there is a natural rate of unemployment $U^* = 1 - N^* = 1 - M^*/\psi$.

[15] Similarly, Seater (1979) also argues that the search cost function should be increasing at the margin.

[16] Other authors that take this approach, are Phelps *et al.* (1970), Pissarides (1988), and Pissarides (1990).

Taking the tax rates $\{\tau^k, \tau^n\}$, prices $\{w_t, r_t\}_{t=0}^{\infty}$, profit $\{\Pi_t\}_{t=0}^{\infty}$, government transfers $\{G_t^{tr}\}_{t=0}^{\infty}$, the process followed by total factor productivity $\{A_t\}_{t=0}^{\infty}$ and initial conditions for capital K_0 and employment N_0 as given, households choose $\{C_t, N_{t+1},$
$S_t, K_{t+1}\}_{t=0}^{\infty}$ to maximize (2.1) s.t. (2.2)-(2.5). The resulting first-order optimality conditions (FOCs), and the transversality condition (TVC) are as follows:

$$C_t \quad : \quad \frac{1}{C_t} = \lambda_t \tag{2.6}$$

$$K_{t+1} \quad : \quad \lambda_t = \beta E_t \lambda_{t+1}[(1-\tau^k)\alpha \frac{Y_{t+1}}{K_{t+1}} + (1-\delta)] \tag{2.7}$$

$$S_t \quad : \quad \lambda_t b_0 \eta S_t^{\eta-1} = \mu_t p_t \tag{2.8}$$

$$N_{t+1} \quad : \quad \frac{b_0 \eta S_t^{\eta-1}}{C_t} = p_t \beta E_t \Bigg\{ \frac{1}{C_{t+1}} \Bigg[(1-\tau^l) w_{t+1} + b_0 S_{t+1}^{\eta} \Bigg] + \frac{\phi}{1-N_{t+1}} + \frac{b_0 \eta S_{t+1}^{\eta-1}}{C_{t+1} p_{t+1}} [(1-\psi) - p_{t+1} S_{t+1}] \Bigg\} \tag{2.9}$$

$$(TVC) \quad : \quad \lim_{t\to\infty} \frac{1}{C_t} K_{t+1} = 0 \tag{2.10}$$

where λ_t, μ_t are the Lagrangean multipliers of the budget constraint, and employment dynamics, respectively.

The first-order optimality conditions obtained above have standard interpretations. The first is the optimality condition for consumption, which requires that the marginal utility from consumption equals the marginal utility of wealth. The second is the so-called Euler condition, which describes how households would choose capital in two congruent periods in order to smooth consumption. The static optimality condition for the search effort balances the costs and benefits from searching for a job. A similar logic applies to employment. We can think of it as determining the labor supply. However, in this case choosing employment is a dynamic problem, as the value of a match extends to more than one periods. Each unemployed household chooses the level of search effort in order to balance the costs and benefits at the margin. The benefit is the discounted payoff from the labor income and the foregone search cost minus the disutility from working. As in Merz (1995), this benefit is conditioned on "any additional search effort leading to a job match with probability p_t." The TVC is a boundary condition on capital, which guarantees that explosive solutions are ruled out.

2.2 Firm

There is a representative firm in the setup using a Cobb-Douglas production function,[17] which uses both capital and labor

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{2.11}$$

where $0 < \alpha < 1$ measures capital share. With search externalities, $1-\alpha$ is no longer the labor share. Still, the production function features constant returns to scale.

The firm chooses how much capital to rent, how many to employ, and how many vacancies to advertise. Firm's problem now becomes dynamic due to the value of the match, and the fact that if a vacancy is filled, then the firm can economize on advertising the position.

The advertising cost incurred equals aV_t, $a > 0$. Those are considered as part of production costs, and thus will be deducted from the firm's profit. The firm takes the aggregate law of employment as a constraint when maximizing its discounted profit:

$$N_{t+1} = (1-\psi)N_t + q_t V_t \tag{2.12}$$

The firm takes the endogenous probability that a vacancy is filled, $\{q_t\}$, as given.

FOCs:

$$K_t \;:\; \alpha \frac{Y_t}{K_t} = r_t \tag{2.13}$$

$$N_t \;:\; \beta_t \left[(1-\alpha) \frac{Y_{t+1}}{N_{t+1}} - w_{t+1} + \frac{a(1-\psi)}{q_{t+1}} \right] = \frac{a}{q_t} \tag{2.14}$$

The first one is the usual optimality condition for capital, saying that the input is rewarded its marginal product. The optimality condition for labor is different from the one in standard RBC models. In the literature, the second optimality condition is also referred to as the job creation condition (JCC).[18] On the right-hand side is the effective cost of a vacancy, which is the product of the advertising cost per opened vacancy, a, and the expected time on average

[17] The assumption of constant returns to scale is a useful one, as it allows to think of the stand-in form as an aggregation of single-vacancy outlets.

[18] Vacancies are already chosen optimally, and the optimality condition was plugged in the JCC equation.

that this vacancy stays unfilled, $1/q_t$. The expression on the left-hand side is the expected discounted benefit from a vacancy: when filled, the return to the firm is the difference between the marginal product of labor less the wage, plus the saved cost on not advertizing a vacancy, weighted by the probability of the match not being destroyed.

2.3 Matching technology

Aggregate job matches are assumed to be generated by the following production function:

$$M_t = V_t^{1-\gamma}[S_t(1-N_t)]^\gamma, \tag{2.15}$$

where $0 \leq \gamma \leq 1$ measures the elasticity of job matches with respect to search effort, and V_t is the number of vacancies available in period t. This type of modelling is based on the empirical findings of Blanchard and Diamond (1989) and Pissarides (1986). Mortensen (1982) and Hosios (1990) also argue that search effort should be also included as in input in the aggregate matching function, hence the specification used above.

In addition, this type of modelling matches as described above implies endogenous probabilities for the transition from unemployment to employment, defined as

$$p_t = \frac{M_t}{S_t(1-N_t)} = \left(\frac{\theta_t}{S_t}\right)^{1-\gamma}, \tag{2.16}$$

where

$$\theta_t = \frac{V_t}{1-N_t} = \frac{V_t}{U_t} \tag{2.17}$$

represents the tightness of the labor market. More specifically, when the market is tight, the probability of finding a job (and filling a position) will be low. Thus, the job-finding rate can be expressed as a function of θ, or

$$p(\theta_t) = \frac{M_t}{1-N_t} = \left(\frac{\theta_t}{S_t}\right)^{1-\gamma}. \tag{2.18}$$

That is, the probability of making a transition from being unemployed to becoming employed decreases with the congestion caused by either increase in unemployment or the search effort. Lastly,

$$q_t = \frac{M_t}{V_t} = \left(\frac{S_t}{\theta_t}\right)^\gamma \tag{2.19}$$

is the transition probability from an unfilled vacancy to a filled one. It is increasing in the search effort, the amount of vacancies and unemployment, and decreasing in market tightness, since

$$q(\theta_t) = \left(\frac{S_t}{\theta_t}\right)^{\gamma}. \tag{2.20}$$

Alternatively, the inverse of the transition probability from unemployment to employment,

$$\frac{1}{q_t} = \frac{1}{q(\theta_t)}, \tag{2.21}$$

can be interpreted as the expected duration of a vacancy.

2.4 Wage determination

The wage rate will be determined as an outcome from a Nash bargaining protocol, where the worker and the firm will negotiate over the distribution of the rents arising from the value of the match.[19] In technical terms,

$$w_t = \arg\max_{w} [W_t - U_t]^{\lambda} [J_t - Q_t]^{1-\lambda}, \tag{2.22}$$

where the surplus to the household is the difference between W_t, the value to the household from being employed, and U_t, the value when unemployed. From the employer perspective, the surplus from the match is the difference between the value J_t from filling a vacancy and Q_t is the value from an unfilled vacancy.[20]

It is a standard result (Shimer 2010) that the wage rate obtained is[21]

$$w_t = \lambda\left[(1-\alpha)\frac{Y_t}{N_t} + a\frac{V_t}{1-N_t}\right] + (1-\lambda)\left[-\frac{\phi C_t}{1-N_t} - b_0 S_t^{\eta}\right] \tag{2.23}$$

The Hosios (1990) condition in static context, and extended by Merz (1995) to dynamic settinggs, $\gamma = \lambda$, produces perfect insurance markets, and efficiency in the outcome of the

[19] However, this is just one way to solve the indeterminacy when it comes to the wage rate determination in a search and matching framework.

[20] This is just one way of decentralizing the wage-determination process. We do not discuss other ways to set wages.

[21] For detailed derivations, the interested reader is referred to Merz (1995).

wage-employment contracts. By setting the bargaing weights equal to the corresponding elasticities in the matching function, the Hosios condition internalizes the search externalities.[22]

$$w_t = \gamma\left[(1-\alpha)\frac{Y_t}{N_t} + a\frac{V_t}{1-N_t}\right] + (1-\gamma)\left[-\frac{\phi C_t}{1-N_t} - b_0 S_t^{\eta}\right] \quad (2.24)$$

The expression above is also referred to as a wage schedule, or a "wage curve," as documented in Blachflower and Oswald (1994).

A job is an asset owned by the firm, hence the optimality condition for vacancy is akin to an aset price equation. More specifically, a vacant job costs aV and changes state according to a process. Given the perfectly-competitive capital markets there will not be any capital gains/losses from expected changes in the valuation of the jobs/match. The firm compares expected profit from an occupied job versus the firm's expected profit from a vacant job.[23]

The wage rate is the weighted average of the marginal product of labor and the marginal rate of substitution between consumption and hours, where the latter can be regarded as the worker's outside opportunity.[24] The weights correspond to the relative bargaining power in the wage negotiation process. With endogenous search effort, we also have a weighted average of the marginal benefit from searching and the marginal cost of searching.[25] If the worker is employed, s/he can save on searching, as there will not be need to re-engage in search.

As suggested in Merz (1995), we can think of the wage expression as representing the two

[22]Note that this result is true only for matching functions which feature constant returns to scale, which is also the case in this paper. Also, in the presence of exogenous fiscal policy, allocations are no longer Pareto optimal.

[23]The number of jobs is endogenous: it is determined by the firm's profit maximization problem.

[24]More specifically, the benefit from not working is an additional hour of leisure (hence the marginal utility of labor), weighted by the "price" of consumption (marginal utility of consumption).

[25]The downside of this procedure is that the threat is the dissolution of the match. In reality, however, there is usually a stage of c ounteroffer being put on the table. Also wages could be subject to periodic renegotiations especially because workers and employers cannot fully predict all the dynamic effects resulting from the match.

"threat points" in the wage negotiations.[26] On the one hand, the household asks for the value of its marginal product less the cost of advertizing born by the firm. The firm, however, would be only willing to pay the worker's revervation wage, which equals the marginal disutility of work less the search cost incurred. Thus the equilibrium wage rate is a weighted average of the two, with the elasticity of the matching function with respect to the households' total search effort $S_t(1-N_t)$ could be regarded also as the households' bargaining strength.[27]

2.5 Stochastic process

It will be assumed that total factor productivity (TFP) process $\{A_t\}_{t=0}^{\infty}$ is stochastic, and follows an AR(1) dynamics

$$A_{t+1} = (1-\rho_a)A_0 + \rho A_t + \epsilon_t^a, \tag{2.25}$$

where $A_0 = A$ is the steady-state level of TFP, parameter ρ_a measures the persistence of the process, and $\epsilon_t \sim N(0, \sigma_a^2)$ are the unexpected innovations to the TFP, which are i.i.d. normal with zero mean and standard deviation σ_a.

2.6 Government

The government levies taxes on both capital and labor income to finance the wasteful government consumption and the lump-sum transfer. The budget constraint is balanced in every period.

$$\tau^k r_t K_t + \tau^l w_t N_t = G_t^c + G_t^{tr}, \tag{2.26}$$

where G_t^c denotes the wasteful government spending. The spending-to-output ratio willbe set equal to its data average, so that the level of spending varies with output. Government

[26]We could regard the process as a bilateral monopoly case - where both workers and firms have market power as the sole producer and buyer of labor. As a result, there will be generally a range for the wage, or the "zone of potential agreement," and the indeterminacy of the wage has to be brocken by some additional criteria, such as the Nash baragining criterion.

[27]Note that despite the fact that only contemporaneous variables enter the expression for the wage rate, both parties have fully considered all the dynamic implications that come into effect as a result of the realized match.

transfers will be residually determined, as they will be allowed to vary so that the government budget constraint is balanced in every period.

2.7 Decentralized Competitive Equilibrium (DCE) with Search Externalities

Given the total factor productivity (TFP) process $\{A_t\}_{t=0}^{\infty}$, the two tax rates $\{\tau^l, \tau^k\}$, the initial conditions for the (endogenous and exogenous) state variables k_0, A_0, a Decentralized Competitive Equilibrium (DCE) with search is defined to be a sequence of prices $\{r_t, w_t^m\}_{t=0}^{\infty}$, allocations $\{c_t, i_t, k_t, n_t, u_t, g_t, g_t^{tr}\}_{t=0}^{\infty}$, such that (i) expected utility is maximized; (ii) the stand-in firm maximizes dynamic profit; (iii) the wage rate is determined as an outcome from Nash bargaining between the households and the firm; (iv) government budget is balanced in each time period; (iv) all markets clear.

3 Data and model calibration

The model is calibrated to Bulgarian data at quarterly frequency. The period under investigation is 2000-2012. Quarterly data on the output, household consumption, private fixed investment shares in output, employment rate, the average wage rate, and the minimum wage rate was obtained from the National Statistical Institute (NSI). Following Vasilev (2015), capital income share is set to its average value $\alpha = 0.429$, and the labor income share is $1 - \alpha = 0.571$. Next, using Vasilev's (2015) estimate that the annual depreciation rate on physical capital is 5 %, in our quarterly model that corresponds to $\delta = 0.0125$.[28] Ganev's (2005) annual estimates of the average capital stock to output over the 1992-2007 are then converted to quarterly ones, thus obtaining that $K/Y = 13.964$. This gives us sufficient information to calibrate the discount factor from the steady-state Euler equation:

$$\beta = \frac{1}{1 + \alpha \frac{y}{k} - \delta} = 0.982.$$

The relative weight on leisure in the household's utility function, parameter $\phi = 1.803$, will be set to match the steady-state employment rate in Bulgaria over the period $n = 0.533$

[28]Similar values have been reported for the other EU member states in Goertzig (2007), such as Greece and Italy.

(NSI). Steady-state output will be normalized to unity, which produces $A = 0.605$. Burda (1997) estimates $m/n = 0.009$ for Bulgaria, which yields $\psi = 0.009$.

Scale parameter of the search cost function $b_0 = 0.001$, which is of the magnitude chosen in Merz (1995).[29] Similarly, due to lack of information, we will assume linear search costs and set $\eta = 1$. Again, the curvature of the search cost function does not affect our results quantitatively. Following Aldolfatto (1996), for the advertising costs, we set per vacancy cost a such that in steady state the share of advertising costs in output is 1%.[30] Since the shares of the search and recruiting costs in output will be shown in the next section to be minute, the size of the scale parameters is of little importance when it comes to the model dynamics over the business cycle. Next, the elasticity of job matches with respect to search effort, usually is estimated from matching function. However, given the short series available for Bulgaria, $\lambda = \gamma = 0.4$ will be adopted from Blanchard and Diamond (1990) and Petrolongo and Pissarides (2001).

Finally, the parameters for the total factor productivity process will be estimated by obtaining the Solow residuals from the Cobb-Douglas production function using data on output, capital and employment, and the estimated capital share. The Solow residuals are then deterended using the Hodrick-Prescott (1980) filter. Using the made-stationary series, an AR(1) model was estimated using Ordinary Least Squares. That produced $\hat{\rho}_a = 0.7$ with $s.e.(\hat{\rho}_a) = 0.117$, and $\hat{\sigma}_a = 0.044$. Table 1 on the next page summarizes the values of model parameters used in this paper.

[29] Merz (1995) sets this scale parameter to match unemployment duration, which is obtained as the ration of the stock of unemployment over the new job matches. We do not have this data for Bulgaria, so the choice for b_0 is to a certain degree arbitrary. However, since the search cost is quantitatively very small relative to output, this parameter is not a driving force in the model.

[30] Alternatively, the scale parameter for advertising a could be set to match average vacancy duration, which equals the ratio of posted vacancies/newly-created job matches (v/m).

Table 1: Model Parameters

Parameter	Value	Description	Method
β	0.982	Discount factor	Calibrated
α	0.429	Capital share in output	Data Avg.
δ	0.013	Depreciation rate	Set
ϕ	1.803	Weight attached to utility of leisure	Calibrated
η	1.000	Curvature of the search cost function	Calibrated
γ	0.400	Elasticity of job matches with respect to search effort	Calibrated
$1-\gamma$	0.600	Elasticity of job matches with respect to vacancies	Calibrated
ψ	0.009	Transition rate from employment to unemployment	Data Avg.
a	0.100	Per-unit advertising costs	Calibrated
b_0	0.001	scale parameter, search cost function	Calibrated
A	0.604	steady-state value of TFP	Set/Calibrated
ρ_a	0.701	AR(1) persistence coefficient, TFP process	Estimated
σ_a	0.044	st. error, TFP process	Estimated

4 Steady-State

Once model parameters were obtained, the steady-state ratios for the model calibrated to Bulgarian data were obtained. The results are reported in Table 2 on the next page.

Overall, the long-run behavior of data is well-matched by the steady-state values of the model. The great ratios - consumption and investment shares - are well-approximated, as well as the after-tax return to capital, where $\tilde{r} = (1-\tau^k)r - \delta$. Advertising and search costs are quite small relative to the size of the economy. Thus, despite the presence of search externalities the labor share is essentially identical to wn/y, which is the expression in the case with perfectly-competitive labor markets.

Table 2: Data Averages and Long-run solution

	Description	BG Data	Model
c/y	Consumption-to-output ratio	0.674	0.642
i/y	Fixed investment-to-output ratio	0.201	0.181
k/y	Physical capital-to-output ratio	13.96	13.96
g/y	Government cons-to-output ratio	0.176	0.176
wn/y	Labor share in output	0.571	0.571
rk/y	Capital share in output	0.429	0.429
$b_0 s^\eta/y$	Search cost-to-output per unemployed	N/A	0.001
av/y	Advertising vacancies cost-to-output	N/A	0.002
n	Employment rate	0.533	0.533
u	Unemployment rate	0.467	0.467
m	New matches	0.005	0.005
v	Vacancy rate	0.004	0.004
$\tilde{r}$	After-tax net return to physical capital	0.010	0.018

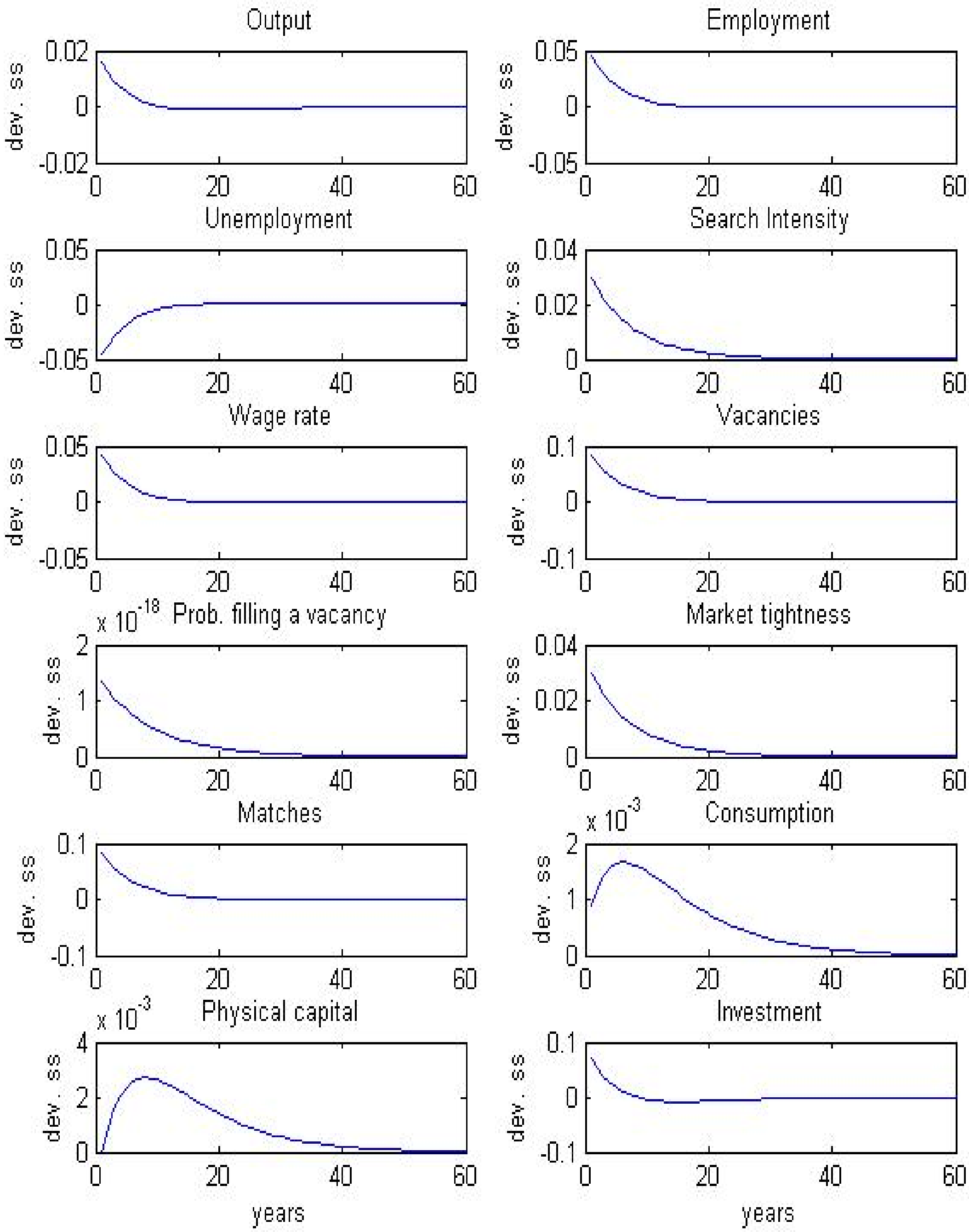
5 Out of steady-state model dynamics

Since the model does not have an analytical solution for the equilibrium behavior of variables, we need to solve the model numerically. This is done by log-linearizing the original equilibrium (non-linear) system of equations around the steady-state. This tranforms the approximate dynamics of the model into one that is represented as a first-order system of stochastic difference equations. First, we study the dyanmic behavior of model variables to an isolated shock to the total factor productivity process, and then we fully simulate the model to compare how the second moments of the model perform when compared against their empirical counterparts.

5.1 Impulse Response Analysis

This subsection documents the impulse responses of model variables to a 1% surprise innovation to technology. The impulse response function (IRFs) are presented in Fig. 2 on the next page. As a result of the one-time unexpected positive shock to total factor productivity,

Figure 2: Impulse Responses to a 1% surprise to a technology shock



output increases. This expands the availability of resources in the economy, so consumption, investment and government consumption also increase upon impact. At the same time, the increase in productivity increases the after-tax return on the two factors of production, labor and capital. Households respond to the incentives and start accumulating capital. In turn, the increase in capital input feeds back in output and adds to the effect of the technolofy shock. In the labor market, which is characterized by trade frictions, households increase the search effort, as the value of being employed is now higher, which in turn increases the probability of a match. On the firm side, the increase in the marginal product of labor also makes the value of a filled vacancy higher, so firms start advertizing positions.[31] Market tightness decreases, which increases the probability of employment, and decreases the congestion externalities in the labor market. Probability of becoming unemployed decreases. As a result, employment increases, and unemployment decreases. The number of matches being realized also increases. In turn, the increase in the labor input employed in the production further augments the increase in output.

Over time, as capital is being accumulated, its marginal product starts to decrease, which lowers the households' incentives to save. As a result, capital returns to its steady-state forllowing a hump-shaped dynamics. Consumption also exhibits the same shape in its dynamic pattern. The rest of the variables return to their old steady-states in a monotone fashion as the effect of the one-time surprise innovation in technology dies out. Most of the labor variables return to their long-run equilibrium values within 20 quarters, or 5 model years, while capital and consumption take 8 years to return to their old steady-state values.

5.2 Simulation and moment-matching

We will now simulate the model 10,000 times for the length of the data horizon. Both empirical and model simulated data is detrented using the Hodrick-Prescott (1980) filter. Table 3 summarizes the second moments of data (relative volatilities to output, and contemporaneous correlations with output) versus the same moments computed from the model-simulated data. To minimize the sample error, the simulated moments are averaged out over the

[31]Interestingly, with a variable search effort in the model, the vacancy rate has to counteract the effect of two forces, search and unemployment.

computer-generated draws. The model-predicted 95 % confidence intervals are reported in square brackets next to the mean estimate from the model. The moments are presented in Table 3 below

Table 3: Business Cycle Moments

	Data	Model
σ_y	0.05	0.07 (0.01)
σ_c/σ_y	0.55	0.10 (0.02)
σ_i/σ_y	1.77	4.38 (0.02)
σ_g/σ_y	1.21	1.00 (0.00)
σ_n/σ_y	0.63	0.72 (0.02)
σ_{LS}/σ_y	0.43	0.31 (0.02)
σ_w/σ_y	0.83	2.38 (0.02)
$\sigma_{y/n}/\sigma_y$	0.86	1.72 (0.02)
σ_u/σ_y	3.22	0.86 (0.12)
σ_v/σ_y	2.54	4.52 (0.07)
σ_θ/σ_y	4.42	1.88 (0.04)
σ_w/σ_n	1.32	3.31 (0.02)
$corr(c, y)$	0.85	0.53 (0.06)
$corr(i, y)$	0.61	1.00 (0.00)
$corr(g, y)$	0.31	1.00 (0.00)
$corr(n, y)$	0.49	0.96 (0.01)
$corr(w, y)$	-0.01	-1.00 (0.00)
$corr(LS, y)$	0.48	0.42 (0.01)
$corr(\theta, y)$	-0.98	-0.95 (0.02)
$corr(u, y)$	-0.47	-0.95 (0.02)
$corr(v, y)$	0.49	0.99 (0.01)
$corr(n, y/n)$	-0.14	-0.97 (0.00)
$corr(u, v)$	-0.63	-0.98 (0.01)

The model matches quite well the absolute volatility of output, the empirical estimate is within the confidence band produced by the model. However, the model underestimates the variability in consumption, which could be due to the presence of government consumption, which overestimates the variability in data. The model also overestimates the variability in investment. This shortcoming of the model could be explained by the structural transformation of government property in private hands through voucher privatization, direct sales, and worker-management privatization. Public investment in infrastructure has been also substantial in the last few years. Still, the model is qualitatively consistent with the stylized fact that consumption generally varies less than output, while investment is more volatile than output. By construction, government spending in the model varies as much a in data.[32]

With respect to the labor market variables, the variability of employment predicted by the model is about the same as the one in data, but the variability of vacancies is not. The latter might be driven again by structural issues and structural transformation of the economy over the period. Nevertheless, the model is able to generate the Beveridge curve, the strong negative correlation between unemployment and vacancies documented in Fig. 1, despite the presence of a variable search effocrt, which, according to the Merz (1995) would cause the Beveridge curve to shift and generate a zero correlation in the model.[33] This negative co-movement between vacancies and unemployment is a stylized macroeconomic facts of the labor markets in other developed countries, *e.g.* US, as documented in Krause and Lubik (2014), and at the heart of Shimer's (2005) puzzle, as model fails to match the order of volatility of unemployment and vacancies.[34] Whatismore, the wage rate in the model is too volatile. As pointed out in Merz (1995), any incentives for firms to advertize a vacancy (due to increase in productivity, and thus an increase in the value of the filled vacancy) are quickly offset by the increase in wages.[35] Thus, the model fails to reporduce the variability of both unemployment and vacancies. Vacancies vary more than in data, so tightness varies

[32]Government consumption could have been matched better had we assumed that it follows a stochastic AR(1) process. However, such a match would have been by design.

[33]Variable search in the model was introduced to enrich the labor supply mechanism.

[34]One interpretation that has been proposed in the search and matching real business cycle literature is that firms do not want to hire during recessions.

[35]Note that since $\theta = v/u$, when θ increases (tightness), probability q of filling a avacancy (match) decreases, while p the job finding rate increases.

less than in data. The reason behing this mismatch could be driven by several posiible explanatory factors: (i) the fact that the model misses the out of the labor force (the discouraged workers) segment, which is significant in Bulgaria; (ii) the structural mismatch in the economy moving from agrigulture and heavy manufacturing to services, and/or (iii) the significant emigration to Western Europe, US and Canada.[36,37]

As in Andolfatto (1996), the wage rate behaves like average labor productivity. One reason for that is that the wage rule arizing from the Nash bargaining leads to Pareto optimal allocations. The other explanation is that the worker's outside option moves in the same directions as productivity in response to the technology shocks. In addition, the volatility of wages is higher than the variability of labor productivity, which means that the labor share is pro-cyclical in Bulgaria. This is what we see from Table 3 above as well. This is typical for recession periods, which is a good description for Bulgaria's transitional experience. In a recession, capital absorbs most of the negative effect and falls more than proportionally, while labor falls less than proportionally. The latter is due to the presence of employment insurance, firing costs and long-term contracts. Again, the standard RBC model cannot explain this.

Next, in terms of contemporaneous correlations, the model slightly overpredicts the procyclicality of the main macroeconomic variables - consumption, investment, and government consumption. However, this is a common limitation of the whole class of RBC models. How-

[36] Alternatively, Hangedorn and Monovskii (2008) introduce a very high replacement rate, which is above the normal unemployment benefits generosity and includes an opportunity cost component. Together with decreasing drastically workers' baragining power to $\lambda = 0.05$, that helps the model impove on the volatility in unemployment and vacancies. However, their calibration destroys the Beveridge curve, as it drives the correlation between unemployment and vacancies to zero. Also, the values chosen by Hangedorn and Monovskii does not provide by themselves a good explanation why the model fit is so good.

[37] In another strand of the literature, Lubik (2009) uses maximum likelihood estimation to evaluate the importance of search and matching frictions. He augments the framework with monopolistically-competitive firm, mark-up shocks, and variable vacancy posting costs. Krause and Lubik (2014) argue that adding shocks to the matching function (i.e., matching efficiency) causes the Beveridge curve to shift. In particular, matching shocks affect more unemployment than vacancies. On the other hand, as pointed out in Pissarides (2000), the wage equation is qualitatively the same.

ever, along the labor market dimension, the contemporaneous correlation of market tightness with output is well-matched. With the other variables, the signs are correct, but the model predicts a stronger co-movement than the one observed in data. Overall, the model with search and matching provides a richer framework that is able to capture well more aspects of labor markets in Bulgaria.

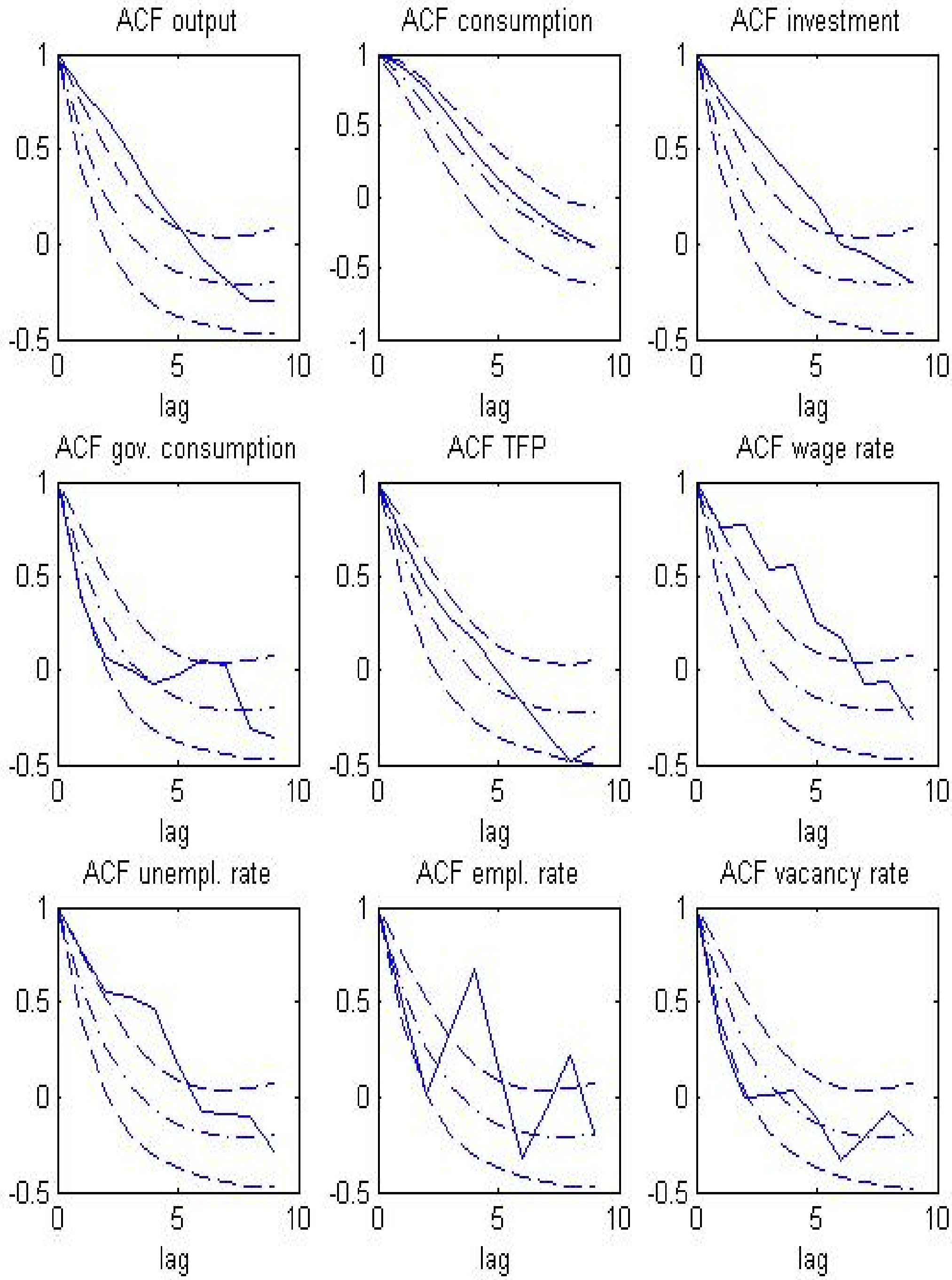
In the next subsection, we take the analysis one step further. Instead of reporting only the contemporaneous correlation, we investigate the correlation between labor market variables at leads and lags, thus evaluating how well the model matches the phase dynamics among variables. In addition, the autocorrelation functions of empirical data, obtained from an unrestricted VAR(1) are put under scrutiny and compared and contrasted to the simulated counterparts generated from the model.[38]

5.3 Auto- and cross-correlation

This subsection discusses the auto-(ACFs) and cross-correlation functions (CCFs) of the major model variables. The empirical ACFs and CCFs (solid line) are plotted in Fig. 3

[38]Note that after the log-linearization, the model could be viewed as a structural VAR model, with the only source of disturbance being the innovations to the total factor productivity.

Figure 3: Empirical vs. simulated ACFs



against the average simulated AFCs and CCFs, and the 95% confidence band (dashed line). Following Canova (2007), this comparison is used as a goodness-of-fit measure. For better comparison, the autoregressive coefficients are also documented in Table 4 below.

Table 4: Autocorrelations for Bulgarian data and the model economy

		k			
Method	Statistic	0	1	2	3
Data	$corr(u_t, u_{t-k})$	1.000	0.765	0.552	0.553
Model	$corr(u_t, u_{t-k})$	1.000	0.613	0.324	0.118
	(s.e.)	(0.000)	(0.106)	(0.150)	(0.157)
Data	$corr(n_t, n_{t-k})$	1.000	0.484	0.009	0.352
Model	$corr(n_t, n_{t-k})$	1.000	0.569	0.262	0.057
	(s.e.)	(0.000)	(0.107)	(0.147)	(0.149)
Data	$corr(y_t, y_{t-k})$	1.000	0.810	0.663	0.479
Model	$corr(y_t, y_{t-k})$	1.000	0.574	0.269	0.063
	(s.e.)	(0.000)	(0.106)	(0.147)	(0.150)
Data	$corr(a_t, a_{t-k})$	1.000	0.702	0.449	0.277
Model	$corr(a_t, a_{t-k})$	1.000	0.618	0.329	0.122
	(s.e.)	(0.000)	(0.103)	(0.147)	(0.155)

As seen from both Fig. 3 and Table 4 above, the model compares well vis-a-vis data. Empirical ACFs for output and investment are slightly outside the confidence band predicted by the model, while the ACFs for total factor productivity, household consumption and government consumption are well-approximated by the model. Labor market variables are also well-described by the model dynamics: ACFs for vacancies, employment and unemployment are close to predicted ones until the third lag. The ACF for the wage rate is well-captured only until the first lag. However, this is a common shortcoming of this class of models; a wage rate determined within a Nash bargaining framework has been demonstrated to feature such limitations (e.g. Shimer 2010).[39] Overall, the search and matching model, and the trade frictions in particular, generates persistence in output and both employment and

[39] Alternative modeling of wage determination is proposed by Moen (1997), where wage rates are posted in advance.

unemployment, and is able to respond to the criticism in Nelson and Plosser (1992), Cogley and Nason (1995) and Rotemberg and Woodford (1996), who argue that the RBC model does not have a strong internal propagation mechanism besides the strong persistence in the TFP process.[40] Whatismore, the search and matching approach dominates the setup with invisible hours, developed by Rogerson (1988), and incorporated in the RBC setup by Hansen (1985). In those models, labor market is modelled in the Walrasian market-clearing spirit, and output and unemployment persistence is low. In contrast, the model with search and matching frictions is able to generate high persistence in lags, due to the history dependence arising from the employment status.[41]

Table 5: Dynamic correlations for Bulgarian data and the model economy

		k						
Method	Statistic	-3	-2	-1	0	1	2	3
Data	$corr(n_t, (y/n)_{t-k})$	-0.342	-0.363	-0.187	-0.144	0.475	0.470	0.346
Model	$corr(n_t, (y/n)_{t-k})$	0.000	0.000	0.000	0.002	0.000	0.000	0.000
	(s.e.)	(0.007)	(0.006)	(0.006)	(0.031)	(0.004)	(0.001)	(0.003)
Data	$corr(n_t, w_{t-k})$	0.335	0.452	0.447	0.328	-0.04	-0.39	-0.57
Model	$corr(n_t, w_{t-k})$	0.000	0.000	0.000	0.002	0.002	0.000	0.000
	(s.e.)	(0.002)	(0.006)	(0.002)	(0.044)	(0.005)	(0.001)	(0.003)
Data	$corr(v_t, u_{t-k})$	0.171	-0.314	-0.308	-0.630	-0.010	0.240	0.220
Model	$corr(v_t, u_{t-k})$	0.181	0.069	-0.166	-0.983	-0.042	0.182	0.257
	(s.e.)	(0.158)	(0.159)	(0.166)	(0.010)	(0.148)	(0.138)	(0.150)

Next, as seen from Table 5 above, over the business cycle, labor productivity leads employment. This is consistent with the finding in Merz (1995) for the case of the US as well.

[40]Furthermore, Den Haan *et al.* (2000) endogeneize the job separation rate and introduce a costly capital mobility into an RBC model with search and matching to show that the augmented framework can produce an amplified propagation mechanism for technology shocks.

[41]More specifically, the employment status depends on the household's status last period. The two transition probabilities are not the same: the probability of losing the job is fixed, while the probability of a match is endogenous.

In contrast to the search and matching model, the standard market clearing (Walrasian) setting cannot account for this fact, as in the standard RBC model a technology shock can be regarded as a factor shifting the labor demand curve, while holding the labor supply curve constant. Therefore, the effect between employment and labor productivity is only a contemporaneous one. In contrast, the search model breaks the one-to-one relationship between productivity and employment, Productivity leads employment, as matching takes time.

The very low dynamic correlation between wages and employment in Bulgaria is well-approximated in the model. Besides the fact that the wage rate is determined through Nash-bargaining procedure, the presence of fiscal policy also helps to move the correlation in the right direction: Taxes decrease the return to both labor and capital, while the presence of government spending diverts some of the resources available, as wasteful government consumption rule is modelled as a fixed share of output. Finally, the dynamic correlation between vacancies and unemployment in Bulgaria is also well-captured by the model. Increase in vacancies leads to decrease in unemployment, and that is what we see in data. That is also a dimentison that the standard RBC model (Vasilev 2009) cannot capture, since vacancies are not featured there.

6 Conclusions

In this paper we investigate the quantitative importance of search and matching frictions in Bulgarian labor markets. This is done by augmenting an otherwise standard real business cycle model a la Long and Plosser (1983) with two-sided costly search and fiscal policy. The model is consisten with data along the labor market dimension, and dominated setups rooted in the Walrasian paradigm, e.g Vasilev (2009), as well as the indivisible labor extension used in Hansen (1985). The search-and-matching setup introduces history-dependence in employment status, which raises the persistence in both employment and unemployment, something that Hansen's (1985) and Rogerson's (1988) setups cannot capture, since running an unemployment lottery over time erases all history-dependence.

References

Andolfatto, D. (1996) ”Business cycles and labor market search,” *American Economic Review* 86(1): 112-132.

Beveridge, W. (1944) *Full Employment in a Free Society*. Allen and Urwin: London, UK.

Blanchard, O.J. and P. Diamond (1989) ”The Beveridge Curve,” Brookings Papers on Economic Activity, 1-60.

Blanchard, O.J. and P. Diamond (1990) ”The aggregate matching function,” 159-201, in Peter Diamond, ed., *Productivity/growth, unemployment*. MIT Press, Cambridge: Mass.

Blanchflower, D. G. and A. J. Oswald (1994) *The Wage Curve*. MIT Press: Cambridge, UK.

Burda, M. (1997). ”Unemployment, labor markets and structural change in Eastern Europe,” in: *Transition to the market Economy: Critical Perspectives on the World Economy*, by Paul hare and Junior R. Davis(eds.), Routledge: Taylor and Francis, US.

Burnside, C., E. Martin and S. Rebelo (1993) ”Labor Hoarding and the Business Cycle,” *Journal of Political Economy*, 101(2): 245-73.

Canova, F. (2007) *Methods for Applied Macroeconomic Research*. Princeton University Press: Princeton, NJ.

Christiano, L. and M. Eichenbaum (1992) ”Current real business cycle theories and aggregate labor market fluctuations,” *American Economic Review*, 86: 430-450.

Cogley, T. and J.M. Nason (1995) ”Output dynamics in Real-Business-Cycles,” *American Economic Review* 85(3): 492-511.

Den Haan, W. G. Ramey and J. Watson (2000) "Job Destruction and the Propagation of Shocks," *American Economic Review*, 90: 482-498.

Diamond, P.A (1982) "Wage determination and Efficiency in Search Equilibrium," *Review of Economic Studies*, 49: 217-227.

Hagedorn, M. and I. Manovskii (2008) "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review*, 98: 1692-1706.

Hansen, G.D. (1985) "Indivisible labor and the business cycle," *Journal of Monetary Economics* 16, 309-327.

Hansen, G. D. and Sargent, T. J. (1988) "Straight time and overtime in equilibrium," *Journal of Monetary Economics*, 21(2-3): 281-308.

Hodrick, R.J. and E.C. Prescott (1980) "Post-war US business cycles: An empirical investigation. Unpublished manuscript (Carnegie-Mellon University, Pittsburgh, PA).

Howitt, P. (1988) "Business Cycles with Costly Search and Recruiting," *Quarterly Journal of Economics*, 103(1): 147-65.

Hosios, A.J. (1990) "On the efficiency of matching and related models of search and unemployment," *Review of Economic Studies* 57, 279-298.

Jackman, R., R. Layard, and S. Nickel (1991) *Unemployment, macroeconomic performance and the labor market*. Oxford University Press: Oxford, UK.

Kennan, J. (2006) "Private Information,Wage Bargaining and Employment Fluctuations," *NBER Working Paper* 11967.

Krause, M.U. and T.A. Lubik (2014) "Modeling Labor Markets in Macroeconomics: Search and Matching," *Federal Reserve bank of Richmond Working Paper No. 14-19*.

Kydland, F. and E. Prescott (1982) "Time to build and aggregate fluctuations," *Econometrica*, 50(6): 1345-70.

Layard, R., S. Nickell, and R. Jackman (2005) *Unemployment: Macroeconomic Performance and the Labor Market*, 2nd ed. Oxford University Press: Oxford, UK.

Long, J. and C. Plosser (1983) "Real Business Cycles," *Journal of Political Economy*, 91(1): 39-69.

Lubik, T. (2009) "Estimating a Search and Matching Model of the Aggregate Labor Market," *Federal Reserve Bank of Richmong Economics Quarterly*, 95(2): 101-20.

Lucas, R.E. (1987)*Models of business cycles.* Basil Blackwell: Oxford, UK.

Merz, M. (1995) "Search in the labor market and the real business cycle," *Journal of Monetary Economics* 36, 269-300.

Moen, E.R. (1997) "Competitive Search Equilibrium," *Journal of Political Economy* 105: 385-411.

Mortensen, D. T. (1982) "The matching process as a non-cooperative/bargaining game." In: *Economics of Information and Uncertainty*, by J.J. McCall (ed.). University of Chicago Press: Chicago, IL.

Mortensen, D.T. (1994) "The cyclical behavior of job and worker flows," *Journal of Economic Dynamics and Control* 18, 1121-1142.

Mortensen, D.T. and C. Pissarides (1993) "The cyclical behavior of job creation and job destruction," in J.C. van Ours, G.A. Pfannn, and G. Ridder (eds.), *Labour demand and* *equilibrium wage formation*, 201-22, North-Holland: Amsterdam, Holland.

Mortensen, D. and C.A. Pissarides (1994) "Job Creation and Job Destruction in the Theory of Unemployment," *The Review of Economic Studies*, 61: 397-415.

Mortensen, D.T. and Nagypal, E. (2007) "More on Unemployment and Vacancy Fluctuations," *Journal of Economic Dynamics and Control* 10: 327-347.

Nelson, C. R. and C.I. Plosser (1982) "Trends and Random Walks in Macroeconomic Time Series," *Journal of Monetary Economics*, 10(2): 139-62.

van Ours, J.C. and G. Ridder (1992) "Vacancies and the recruitment of new employees," *Journal of Labor Economics* 10, 138-155.

Petrongolo, B. and C.A. Pissarides (2001) "Looking into the black box: A survey of the matching function," *Journal of Economic Literature*, 39: 390-431.

Phelps, E.S.(1970) *Microeconomic foundations of employment and inflation theory* (ed). Norton: New York, NY.

Pissarides, C.A. (1985) "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages," *American Economic Review*, 75(4): 676-90.

Pissarides, C.A. (1986) "Unemployment and vacancies in Britain," *Economic Policy* 3, 499-559.

Pissarides, C.A. (2000) *Equilibrium Unemployment Theory*, 2nd ed. The MIT Press: Cambridge, Mass.

Ramey, V.A. (2011) "A Brief History of the Real Business Cycle-Search& Matching Literature," Mimeo.

Rogerson, R. (1988) "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics*, 21(1):3-16.

Rotemberg, J. and M. Woodford (1996) "Real-Business-Cycle Models and the Forecastable Movements in Output, Hours, and Consumption," *American Economic Review*, 86: 71-89.

Seater, J.J. (1977) "A unified model of consumption, labor supply and job search," *Journal of Economic Theory* 14: 349-72.

Shimer, R. (2005) "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review* 95: 25-49.

Shimer, R. (2010) *Labor Markets and Business Cycles*, Princeton University Press: Princeton, US.

Vasilev, A.Z. (2009) "Business cycles in Bulgaria and the Baltic countries: an RBC approach," *International Journal of Computational Economics and Econometrics*, vol. 1(2), pages 148-170.

Wright, R. (1986) "Job Search and Cyclical Unemployment," *Journal of Political Economy*, 94(1):38-55.

Technical Appendix

6.1 Households' problem

The problem can be simplified as

$$
\begin{aligned}
\mathcal{L} &= E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \ln C_t + \phi \ln(1 - N_t) \\
&\quad -\lambda_t [C_t + K_{t+1} - (1-\delta)K_t + b_0 S_t^{\eta}(1 - N_t) - (1-\tau^l) r_t K_t - (1-\tau^l) w_t N_t - \Pi_t - G_t^{tr}] \\
&+ \mu_t [(1-\psi)N_t + p_t S_t (1 - N_t) - N_{t+1}] \Big\},
\end{aligned}
\tag{6.1}
$$

where λ_t, μ_t are the Lagrangean multiplier in front of the households' budget constraint and the law of motion for employment.

FOCs:

$$C_t : \frac{1}{C_t} = \lambda_t \tag{6.2}$$

$$K_{t+1} : \lambda_t = \beta E_t \lambda_{t+1} [(1-\tau^k)\alpha \frac{Y_{t+1}}{K_{t+1}} + (1-\delta)] \tag{6.3}$$

$$S_t : \lambda_t b_0 \eta S_t^{\eta-1}(1 - N_t) = \mu_t p_t (1 - N_t) \tag{6.4}$$

$$N_{t+1} : \beta E_t \lambda_{t+1} \left[(1-\tau^l) w_{t+1} + b_0 S_{t+1}^{\eta} - \frac{\phi}{1 - N_{t+1}} \right] + \beta E_t \mu_{t+1} [(1-\psi) - p_{t+1} S_{t+1}] = \mu_t \tag{6.5}$$

$$\lim_{t \to \infty} \frac{1}{C_t} K_{t+1} = 0 \tag{6.6}$$

Substitute out the Lagrangean multiplier attached to the budget constraint, as well as the probability of a match

$$\frac{1}{C_t} b_0 \eta S_t^{\eta-1} = \mu_t p_t \tag{6.7}$$

$$
\begin{aligned}
-\frac{\beta\phi}{1 - N_{t+1}} + \beta E_t \frac{1}{C_{t+1}} (1-\tau^l) w_{t+1} + \beta E_t \frac{1}{C_{t+1}} b_0 S_{t+1}^{\eta} \\
+ \beta E_t \mu_{t+1} (1-\psi) - \beta \mu_{t+1} p_{t+1} S_{t+1} = \mu_t
\end{aligned}
\tag{6.8}
$$

Note that

$$\mu_t = \frac{b_0 \eta S_t^{\eta-1}}{C_t p_t} \tag{6.9}$$

Rearrange terms

$$\beta E_t \frac{1}{C_{t+1}} \left[(1-\tau^l) w_{t+1} + b_0 S_{t+1}^{\eta} - \frac{\phi}{1-N_{t+1}} \right] + \beta E_t \mu_{t+1} [(1-\psi) - p_{t+1} S_{t+1}] = \mu_t \quad (6.10)$$

or

$$\beta E_t \frac{1}{C_{t+1}} \left[(1-\tau^l) w_{t+1} + b_0 S_{t+1}^{\eta} - \frac{\phi}{1-N_{t+1}} \right] + \beta E_t \frac{b_0 \eta S_{t+1}^{\eta-1}}{C_{t+1} p_{t+1}} [(1-\psi) - p_{t+1} S_{t+1}] = \frac{b_0 \eta S_t^{\eta-1}}{C_t p_t}$$

Multiply by p_t

$$p_t \beta E_t \frac{1}{C_{t+1}} \left[(1-\tau^l) w_{t+1} + b_0 S_{t+1}^{\eta} - \frac{\phi}{1-N_{t+1}} \right] + p_t \beta E_t \frac{b_0 \eta S_{t+1}^{\eta-1}}{C_{t+1} p_{t+1}} [(1-\psi) - p_{t+1} S_{t+1}] = \frac{b_0 \eta S_t^{\eta-1}}{C_t}$$

or

$$\frac{b_0 \eta S_t^{\eta-1}}{C_t} = p_t \beta E_t \left\{ \frac{1}{C_{t+1}} \left[(1-\tau^l) w_{t+1} + b_0 S_{t+1}^{\eta} \right] + \frac{\phi}{1-N_{t+1}} + \frac{b_0 \eta S_{t+1}^{\eta-1}}{C_{t+1} p_{t+1}} [(1-\psi) - p_{t+1} S_{t+1}] \right\}$$

6.2 Firm's problem

The problem of the firm is also one of constrained optimization nature:

$$\max_{\{K_t, V_t, N_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_t \left\{ A_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - r_t K_t - a V_t - \nu_t [(1-\psi) N_t + q_t V_t - N_{t+1}] \right\}, \quad (6.11)$$

where

$$\beta_t = \beta E_{t-1} \left(\frac{C_t}{C_{t-1}} \right) \quad (6.12)$$

is the stochastic discount factor.

FOCs:

$$K_t : \alpha \frac{Y_t}{K_t} = r_t \quad (6.13)$$

$$V_t : \nu_t q_t = -a \quad (6.14)$$

$$N_t : \beta_t \left[(1-\alpha) \frac{Y_{t+1}}{N_{t+1}} - w_{t+1} - \nu_{t+1} (1-\psi) \right] = -\nu_t \quad (6.15)$$

From the optimality condition for vacancies, express $\nu_t = -a/q_t$ and plug it into the FOC for employment to obtain

$$\beta_t \left[(1-\alpha) \frac{Y_{t+1}}{N_{t+1}} - w_{t+1} + \frac{a(1-\psi)}{q_{t+1}} \right] = \frac{a}{q_t} \quad (6.16)$$

or

$$\beta E_{t-1}\left(\frac{C_t}{C_{t-1}}\right)\left[(1-\alpha)\frac{Y_{t+1}}{N_{t+1}} - w_{t+1} + \frac{a(1-\psi)}{q_{t+1}}\right] = \frac{a}{q_t} \tag{6.17}$$

In the literature, this is also referred to as the job creation condition (JCC). In other words, this equation characterizes the firm's labor demand.

7 Steady-State

$$1 = \beta[(1-\tau^k)\alpha\frac{y}{k} + 1 - \delta] \tag{7.1}$$

$$\frac{a}{c} = \mu(1-\gamma)\frac{m}{v} \tag{7.2}$$

$$\frac{b_0\eta s^{\eta-1}(1-n)}{c} = \mu\gamma\frac{m}{s} \tag{7.3}$$

$$m = \psi n \tag{7.4}$$

$$u = 1 - n \tag{7.5}$$

$$m = v^{1-\gamma}[su]^{\gamma} \tag{7.6}$$

$$c + \delta k + b_0 s^{\eta}(1-n) + av = y \tag{7.7}$$

$$y = Ak^{\alpha}n^{1-\alpha} \tag{7.8}$$

$$\frac{\beta}{c}[(1-\tau^l)(1-\alpha)\frac{y}{n} + b_0 s^{\eta}] + \frac{\beta\phi}{1-n} = \mu[1 - \beta(1-\psi-\gamma\frac{m}{1-n})] \tag{7.9}$$

$$p = \frac{m}{s(1-n)} = \left(\frac{\theta}{s}\right)^{1-\gamma} \tag{7.10}$$

$$\theta = \frac{m}{1-n} \tag{7.11}$$

$$q = \frac{m}{v} = \left(\frac{s}{\theta}\right)^{\gamma} \tag{7.12}$$

8 Log-linearization

8.1 Capital accumulation equation

$$k_{t+1} = i_t + (1-\delta)k_t \tag{8.1}$$

Take natural logarithms from both sides to obtain

$$\ln k_{t+1} = \ln[i_t + (1-\delta)k_t] \tag{8.2}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln k_{t+1}}{dt} = \frac{d\ln[i_t + (1-\delta)k_t]}{dt} \tag{8.3}$$

$$\hat{k}_{t+1} = \frac{d[i_t + (1-\delta)k_t]}{dt}\frac{1}{i + (1-\delta)k} \tag{8.4}$$

Using that in steady state $k = i + (1-\delta)k$, or $i = \delta k$

$$\hat{k}_{t+1} = \frac{di_t}{kdt} + (1-\delta)\frac{dk_t}{kdt} \tag{8.5}$$

$$\hat{k}_{t+1} = \delta\hat{i}_t + (1-\delta)\hat{k}_t \tag{8.6}$$

8.2 Employment dynamics equation

$$n_{t+1} = (1-\psi)n_t + m_t \tag{8.7}$$

Take natural logarithms from both sides to obtain

$$\ln n_{t+1} = \ln[(1-\psi)n_t + m_t] \tag{8.8}$$

Totally differentiate w.r.t. time to obtain

$$\frac{d\ln n_{t+1}}{dt} = \frac{d\ln[(1-\psi)n_t + m_t]}{dt} \tag{8.9}$$

$$\frac{dn_{t+1}}{dt}\frac{1}{n} = \frac{d[(1-\psi)n_t + m_t]}{dt}\frac{1}{(1-\psi)n + m} \tag{8.10}$$

Using that in steady state $n = (1-\psi)n + m$,

$$\frac{dn_{t+1}}{dt}\frac{1}{n} = (1-\psi)\frac{dn_t}{dt}\frac{1}{n} + \frac{dm_t}{dt}\frac{1}{n} \tag{8.11}$$

or

$$\frac{dn_{t+1}}{dt}\frac{1}{n} = (1-\psi)\frac{dn_t}{dt}\frac{1}{n} + \frac{dm_t}{dt}\frac{1}{n}\frac{m}{m} \tag{8.12}$$

Pass to log-deviations to obtain

$$n\hat{n}_{t+1} = (1-\psi)n\hat{n}_t + m\hat{m}_t \tag{8.13}$$

8.3 Matching function

$$m_t = v_t^{1-\gamma}[s_t(1-n_t)]^\gamma \tag{8.14}$$

Take natural logarithms from both sides to obtain

$$\ln m_t = (1-\gamma)\ln v_t + \gamma \ln s_t + \gamma \ln(1-n_t) \tag{8.15}$$

Totally differentiate w.r.t. time to obtain

$$\frac{d\ln m_t}{dt} = (1-\gamma)\frac{d\ln v_t}{dt} + \gamma\frac{d\ln s_t}{dt} + \gamma\frac{d\ln(1-n_t)}{dt} \tag{8.16}$$

Pass to log-deviations

$$\hat{m}_t = (1-\gamma)\hat{v}_t + \gamma\hat{s}_t - \frac{\gamma n}{1-n}\hat{n}_t \tag{8.17}$$

8.4 Log-linearizing transition probability from unemployment to employment

$$p_t = \frac{M_t}{S_t(1-N_t)} \tag{8.18}$$

$$\hat{p}_t = \hat{m}_t - \hat{s}_t + \frac{n}{1-n}\hat{n}_t \tag{8.19}$$

Alternatively

$$p_t = \left(\frac{\theta_t}{S_t}\right)^{1-\gamma} \tag{8.20}$$

or

$$\hat{p}_t = (1-\gamma)\hat{\theta}_t - (1-\gamma)\hat{s}_t \tag{8.21}$$

8.5 Log-linearizing market tightness

$$\theta_t = \frac{M_t}{1 - N_t} \tag{8.22}$$

$$\hat{\theta}_t = \hat{m}_t + \frac{n}{1-n}\hat{n}_t \tag{8.23}$$

8.6 Log-linearizing transition probability from an unfilled vacancy to a filled one

$$q_t = \frac{m_t}{v_t} \tag{8.24}$$

$$\hat{q}_t = \hat{m}_t - \hat{v}_t \tag{8.25}$$

Alternatively,

$$q_t = \left(\frac{S_t}{\theta_t}\right)^{\gamma} \tag{8.26}$$

or

$$\hat{q}_t = \gamma \hat{s}_t - \gamma \hat{\theta}_t \tag{8.27}$$

8.7 Production function

$$y_t = A_t k_t^{\alpha} n_t^{1-\alpha} \tag{8.28}$$

Take natural logarithms from both sides to obtain

$$\ln y_t = \ln A_t + \alpha \ln k_t + (1-\alpha)\ln n_t \tag{8.29}$$

Totally differentiate w.r.t. time to obtain

$$\frac{d \ln y_t}{dt} = \frac{d \ln A_t}{dt} + \alpha \frac{d \ln k_t}{dt} + (1-\alpha)\frac{d \ln n_t}{dt} \tag{8.30}$$

Pass to log-deviations to obtain

$$\hat{y}_t = \hat{A}_t + \alpha \hat{k}_t + (1-\alpha)\hat{n}_t \tag{8.31}$$

8.8 Euler equation

$$\frac{1}{c_t} = \beta E_t \frac{1}{c_{t+1}}[(1-\tau^k)\alpha\frac{y_{t+1}}{k_{t+1}} + (1-\delta)] \tag{8.32}$$

Take natural logarithms from both sides to obtain

$$\ln\frac{1}{c_t} = \ln\beta E_t \frac{1}{c_{t+1}}[\alpha\frac{y_{t+1}}{k_{t+1}} + (1-\delta)] \tag{8.33}$$

$$-\ln c_t = \ln\beta - \ln E_t c_{t+1} + \ln E_t[(1-\tau^k)\alpha\frac{y_{t+1}}{k_{t+1}} + (1-\delta)] \tag{8.34}$$

Totally differentiate w.r.t. time and simplify to obtain

$$-\frac{d\ln c_t}{dt} = -\frac{d\ln E_t c_{t+1}}{dt} + \frac{d\ln E_t[(1-\tau^k)\alpha\frac{y_{t+1}}{k_{t+1}} + (1-\delta)]}{dt} \tag{8.35}$$

or

$$-\hat{c}_t = -\hat{c}_{t+1} + \frac{dE_t[(1-\tau^k)\alpha\frac{y_{t+1}}{k_{t+1}} + (1-\delta)]}{dt}\frac{1}{\alpha\frac{y}{k} + (1-\delta)} \tag{8.36}$$

Noting that in equilibrium $\beta = (1-\tau^k)\alpha\frac{y}{k} + (1-\delta)$

$$-\hat{c}_t = -\hat{c}_{t+1} + \beta(1-\tau^k)\alpha\frac{dE_t\frac{y_{t+1}}{k_{t+1}}}{dt} \tag{8.37}$$

$$-\hat{c}_t = -\hat{c}_{t+1} + \beta(1-\tau^k)\alpha[\frac{dy_{t+1}k - dk_{t+1}y}{dtk^2}] \tag{8.38}$$

$$-\hat{c}_t = -\hat{c}_{t+1} + \beta(1-\tau^k)\alpha\frac{y}{k}\hat{y}_{t+1} - \beta(1-\tau^k)\alpha\frac{y}{k}\hat{k}_{t+1} \tag{8.39}$$

8.9 FOC vacancy rate

$$\frac{a}{c_t} = \mu_t(1-\gamma)\frac{m_t}{v_t} \tag{8.40}$$

Take natural logarithms from both sides to obtain

$$\ln\frac{a}{c_t} = \ln\mu_t(1-\gamma)\frac{m_t}{v_t} \tag{8.41}$$

$$\ln a - \ln c_t = \ln\mu_t + \ln(1-\gamma) + \ln m_t - \ln v_t \tag{8.42}$$

Totally differentiate w.r.t. time to obtain

$$\frac{d\ln a}{dt} - \frac{d\ln c_t}{dt} = \frac{d\ln \mu_t}{dt} + \frac{d\ln(1-\gamma)}{dt} + \frac{d\ln m_t}{dt} - \frac{d\ln v_t}{dt} \tag{8.43}$$

Simplify to obtain

$$-\frac{d\ln c_t}{dt} = \frac{d\ln \mu_t}{dt} + \frac{d\ln m_t}{dt} - \frac{d\ln v_t}{dt} \tag{8.44}$$

Pass to log-deviations

$$-\hat{c}_t = \hat{\mu}_t + \hat{m}_t - \hat{v}_t \tag{8.45}$$

8.10 FOC search effort

$$\frac{b_0 \eta s_t^{\eta-1}(1-n_t)}{c_t} = \mu_t \gamma \frac{m_t}{s_t} \tag{8.46}$$

Take natural logarithms from both sides to obtain

$$\ln \frac{b_0 \eta s_t^{\eta-1}(1-n_t)}{c_t} = \ln \mu_t \gamma \frac{m_t}{s_t} \tag{8.47}$$

$$\ln b_0 + \ln \eta + (\eta - 1)\ln s_t + \ln(1-n_t) - \ln c_t = \ln \mu_t + \ln \gamma + \ln m_t - \ln s_t \tag{8.48}$$

Totally differentiate w.r.t time and simplify to obtain

$$(\eta-1)\frac{d\ln s_t}{dt} + \frac{d\ln(1-n_t)}{dt} - \frac{d\ln c_t}{dt} = \frac{d\ln \mu_t}{dt} + \frac{d\ln m_t}{dt} - \frac{d\ln s_t}{dt} \tag{8.49}$$

$$(\eta-1)\hat{s}_t - \frac{n}{1-n}\hat{n}_t - \hat{c}_t = \hat{\mu}_t + \hat{m}_t - \hat{s}_t \tag{8.50}$$

or

$$\eta \hat{s}_t - \frac{n}{1-n}\hat{n}_t - \hat{c}_t = \hat{\mu}_t + \hat{m}_t \tag{8.51}$$

8.11 Dynamic FOC employment

$$[E_t \frac{\beta}{c_{t+1}}[(1-\alpha)\frac{y_{t+1}}{n_{t+1}} + b_0 s_{t+1}^{\eta}] - \frac{\beta\phi}{1-n_{t+1}}] = \mu_t - \beta E_t \mu_{t+1}[1-\psi-\gamma\frac{m_{t+1}}{1-n_{t+1}}] \tag{8.52}$$

Take natural logarithms from both sides and totally differentiate w.r.t time to obtain

$$\frac{d[E_t \frac{\beta}{c_{t+1}}[(1-\alpha)\frac{y_{t+1}}{n_{t+1}} + b_0 s_{t+1}^{\eta}] - \frac{\beta\phi}{1-n_{t+1}}]}{dt} = \frac{d\mu_t}{dt} - \frac{d\beta E_t \mu_{t+1}[1-\psi-\gamma\frac{m_{t+1}}{1-n_{t+1}}]}{dt} \tag{8.53}$$

$$\begin{aligned} &\frac{\beta}{c}(1-\alpha)\frac{y}{n}E_t\hat{c}_{t+1} + \frac{\beta}{c}(1-\alpha)\frac{y}{n}E_t\hat{y}_{t+1} - \frac{\beta}{c}(1-\alpha)\frac{y}{n}E_t\hat{n}_{t+1} \\ &= \mu\hat{\mu}_t - \beta\mu E_t\hat{\mu}_{t+1}[1-\psi-\gamma\frac{m}{1-n}] + \beta\gamma\mu\frac{m}{1-n}E_t\hat{m}_{t+1} + \beta\gamma\mu\frac{mn}{(1-n)^2}E_t\hat{n}_{t+1} \end{aligned} \tag{8.54}$$

8.12 Market clearing

$$y_t = c_t + k_{t+1} - (1-\delta)k_t + g_t + b_0 s_t^\eta (1-n_t) + av_t \tag{8.55}$$

Take natural logarithms from both sides and totally differentiate w.r.t to time to obtain

$$\frac{d}{dt}\ln y_t = \frac{d}{dt}\Big[c_t + k_{t+1} - (1-\delta)k_t + g_t + b_0 s_t^\eta (1-n_t) + av_t\Big] \tag{8.56}$$

$$y\hat{y}_t = c\hat{c}_t + \hat{k}k_{t+1} - (1-\delta)\hat{k}k_t + g\hat{g}_t + \eta b_0 s^\eta (1-n)\hat{s}_t - b_0 s^\eta n\hat{n}_t + av\hat{v}_t \tag{8.57}$$

8.13 Nash bargaining rule

$$w_t = \gamma\Bigg((1-\alpha)\frac{y_t}{n_t} + \frac{av_t}{1-n_t}\Bigg) + (1-\gamma)\Bigg(-\frac{1-n_t}{\phi c_t} - b_0 s_t^\eta (1-n_t)\Bigg) \tag{8.58}$$

Take natural logarithms from both sides and totally differentiate w.r.t to time to obtain

$$\frac{d}{dt}\ln w_t = \frac{d}{dt}\ln\Bigg\{\gamma\Bigg((1-\alpha)\frac{y_t}{n_t} + \frac{av_t}{1-n_t}\Bigg) + (1-\gamma)\Bigg(-\frac{1-n_t}{\phi c_t} - b_0 s_t^\eta (1-n_t)\Bigg)\Bigg\} \tag{8.59}$$

$$w\hat{w}_t = \frac{d}{dt}\Bigg\{\gamma\Bigg((1-\alpha)\frac{y_t}{n_t} + \frac{av_t}{1-n_t}\Bigg) + (1-\gamma)\Bigg(-\frac{1-n_t}{\phi c_t} - b_0 s_t^\eta (1-n_t)\Bigg)\Bigg\} \tag{8.60}$$

$$w\hat{w}_t = \gamma(1-\alpha)\frac{y}{n}\hat{y}_t - \gamma(1-\alpha)\frac{y}{n}\hat{n}_t + \frac{d}{dt}\Bigg\{\Bigg(\frac{av_t}{1-n_t}\Bigg) + (1-\gamma)\Bigg(-\frac{1-n_t}{\phi c_t} - b_0 s_t^\eta (1-n_t)\Bigg)\Bigg\}$$

$$w\hat{w}_t = \gamma(1-\alpha)\frac{y}{n}\hat{y}_t - \gamma(1-\alpha)\frac{y}{n}\hat{n}_t + \frac{av}{1-n}\hat{v}_t - \frac{avn}{(1-n)^2}\hat{n}_t + \frac{d}{dt}\Bigg\{(1-\gamma)\Bigg(-\frac{1-n_t}{\phi c_t} - b_0 s_t^\eta (1-n_t)\Bigg)\Bigg\}$$

$$\begin{aligned} w\hat{w}_t = \gamma(1-\alpha)\frac{y}{n}\hat{y}_t - \gamma(1-\alpha)\frac{y}{n}\hat{n}_t + \frac{av}{1-n}\hat{v}_t - \frac{avn}{(1-n)^2}\hat{n}_t + (1-\gamma)\frac{1-n}{\phi c}\hat{c}_t + (1-\gamma)\frac{n}{\phi c}\hat{n}_t \\ -\eta b_0 s^\eta (1-n)\hat{s}_t + b_0 s^\eta n\hat{n}_t \end{aligned}$$