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Why is the American Workplace Racially Segregated by Occupation?

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WHY IS THE AMERICAN WORKPLACE RACIALLY SEGREGATED BY OCCUPATION?

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Abstract

Arrow (1998) asks, “What has economics to say about racial discrimination?” He replies – entirely correctly – that racial “segregation within an industry – that is, firms with either all black or all white labor forces” – may be explained by economic theory, but “the hypothesis of employer discrimination does not at all explain segregation by occupation, [and] discriminatory tastes of other employees ... may explain segregation [by firms] within industries but not segregation by occupation[s]” that are filled by racially distinct persons within firms. Becker (1957) and Akerlof and Kranton (2000 and 2010) offer economic theories that deal with social identity differentiation, but these lack rational choice theory foundations, insofar as they *impose* a utility indicator function as a primitive concept via *persuasion*, rather than such a function being *entailed by derivation* from a preference ranking relation defined on a set of outcomes, with restrictions imposed both on the set and the relation. This is a methodological weakness of their work relative to that of Arrow and Debreu (1954).

A more serious difficulty with these contributions is that they ascribe a utility function to each individual in an economy, but I prove that assigning to individuals binary preferences, with or without their numerical representation as utility indicator functions, entails the impossibility of interpersonal social-identity diversification, rendering all persons in society indistinguishable by identity. The information necessary to identify a person’s social identity is stripped off the model by the binariness restriction. A person in a binariness-salient model would simply not know against whom to discriminate. Economic theory is, therefore, endogenously color-blind, race-blind, gender-blind, ethnicity-blind, and in general, social-identity-blind. Everybody in the economy is White, or all persons are Black, or all female, or all Hispanics, and so on, but no two persons can endogenously have distinct social identities. This is also true of every player in a game, as in Nash (1951).

However, if preferences are *non-binary*, interpersonal social identity diversification is possible, though their real-valued utility function representation is impossible. This begs the question as to what exact form preferences must take to support the specific utility function of Akerlof and Kranton, which also is non-traditional relative to the utility indicator function in Arrow and Debreu.

As it happens, to exhibit diversity of persons by social identity, ascribing a utility function to a person is conceptually too restrictive. By substituting non-binary for binary preferences in the model of Arrow and Debreu, I extend their economic theory. The more general model I thus formulate has the following features: (i) there exists a social state in which all persons maximize their preferences on their feasible sets, (ii) endogenous interpersonal *social-identity diversification* characterizes this state of the economy, (iii) it is a free-market equilibrium without any state intervention, (iv) it is a *Pareto optimal* social state, and (v) a sizable proportion of Black workers are segregated into low-rank, low income jobs, whereas White workers in the *same observable proficiency domain* are placed in high-ranking, high-income jobs, thereby explaining *occupational segregation within firms along a racial divide*, which entails that (vi) income and wealth distributions vary by social identity. Thus free markets deliver a Pareto optimal state but it is fraught with remediable injustices. Further, my explanation meets standards Arrow sets for such a theory (see p. 20). (542 words)

Keywords: justice, social identity, discrimination, race, gender, non-binariness, maximization, rational choice theory, social choice theory, general equilibrium, game theory, asymmetric information, social norms

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1. Introduction

Economics deals with commodities and people. In investigating phenomena pertaining to distinct social identities of persons, how does an economist tell the social identity of one person apart from that of another? That is, in economic models in the game theory and general equilibrium traditions of Nash (1951) and Arrow and Debreu (1954) respectively, how is a male distinguished from a female, black from white, Christian from Hindu, Muslim, Sikh, and Buddhist, and so on. Well-defined boundaries that separate easily discernable groups of persons, where each distinct *community* is a partition of the human race, do, in fact, exist, at least in the minds of people. Moreover, all persons are affiliated with multiple communities, some by choice (economist versus a surgeon), though others by inexorable association (national origin). A person's affiliation with a community gives that person a mark of *community or group identity*, and a collection of his or her community markers defines a person's *social identity*.

Partition by gender does, of course, intersect with that by any other partitioning criterion such as religion or race. In buying and selling of toothpaste, it may not matter what the genders of the buyer and seller are. But in serving a client at a restaurant or in hiring an employee, the genders of the interacting persons does carry epistemic value for the agents. To deny that would be a severe empirical violation. *It is in the role of race in the pattern of employment, and with the race-dependent income distribution implications thereof in contemporary United States, with which this paper is concerned.* The purpose of this paper is to explain, in a formal general equilibrium model of an economy, in which every person is a personal preference maximizer, and every employer is a simple profit maximizers, that there is, in fact, racial segregation by occupation.

Telling apart the social identities of two persons in economic theory is a matter not typically discussed. This is because Nash deals with "players" rather than persons and Arrow and Debreu deal with "consumption units" rather than persons. Each consumption unit is defined exclusively by *four* characteristics: (a) an exogenously specified endowment vector of commodities, (b) an exogenously specified vector of fractional contractual claims over corporate profits of a finite number of "production units," (c) a utility function that such a decision maker is assumed to maximize, and (d) a set of real vectors on which the utility function is defined. Each player is defined by a (i) a utility function defined on the pay-offs or outcomes of a game, (ii) a strategy set from which the player chooses a maximal strategy (not dominated by any strategy), and (iii) a

correspondence that maps the player's strategy set to the set of pay-offs, which the player evaluates using the utility function, given all possible strategies that other players in the game can play.

The utility function is, in turn, axiomatically derived from the logically primitive concept of an individual's preference ranking relation, with restrictions imposed both on the relation and on the set on which it is defined.¹ Thus, a salient feature of this economic theory is that it has solid rational choice theory foundations, insofar as the utility function is derived from the logically more-primitive concept of a preference ranking relation.

If a consumption unit in Arrow and Debreu is interpreted as a person, then the i^{th} person is defined by $\langle \zeta_i, \alpha_i X_i, R_i \rangle$, where X_i is the closed convex set of commodity-consumption real vectors bounded from below, and on this set is defined R_i a ranking relation of weak preference that stands for "at least as good as", ζ_i is the person's commodity endowment vector that is bounded from below, and α_i is the vector the person's fractional claims to profits in some of the n firms in the economy. If a consumption unit is construed to be a person, then two persons can be distinct *only* in terms of these four characteristics, and *nothing* else whatever, precisely because Arrow and Debreu specify no *other* characteristics of an individual. Since race, ethnicity or gender do not form any part of the four characteristics that define a person, it should not come as a surprise that economic theory cannot distinguish among persons on the basis of their distinct races, ethnicities or genders, or more broadly by their distinct social identities. If there are any interpersonal social identity distinctions, they are exogenously imposed by the theorist on persons who are otherwise *endogenously homogenous* in terms of social identity. Economic theory is, therefore, endogenously color-blind, race-blind, gender-blind, ethnicity-blind, and in general, social-

¹Arrow and Debreu (1954, p. 268-269) write, "We assume the existence of a number of *consumption units*, typically families or individuals but including also institutional consumers. ... The set of consumption vectors X_i available to individual i ($=1, \dots, m$) is a closed convex subset of \mathcal{R}^l which is bounded from below; i.e., there is a vector $\xi_i \leq x_i$ for all $x_i \in X_i$ The choice by the consumer from a given set of alternative consumption vectors is supposed to be made in accordance with a preference scale for which there is a *utility indicator function* $u_i(x_i)$ such that $u_i(x_i) \geq u_i(x'_i)$ if and only if x_i is preferred to indifferent to x'_i by individual i ."

Further, they assume that $u_i(x_i)$ is a continuous, real-valued numerical representation of a preference relation R_i of individual i that stands for "at least as good as". They also assume that (i) R_i is a binary ranking relation defined on a compact and convex set X_i , and it is (ii) reflexive, (iii) transitive, (iv) complete, (v) continuous, (vii) convex, (vi) and satisfies non-satiation.

Arrow and Debreu (p. 270) go on to say, "We also assume that the i^{th} consumption unit is endowed with a vector ζ_i of initial holdings of different types of commodities available and a contractual claim to the share α_{ij} of the profit of the j^{th} production unit for each j ." They add, $\zeta_i \in \mathcal{R}^l; \exists x_i \in X_i, x_i < \zeta_i; \forall i, j, \alpha_{ij} \geq 0; \forall j, \sum_{i=1}^m \alpha_{ij} = 1$.

identity-blind. Everybody in the economy is White, or all persons are Black, or all female, or all Hispanics, or everyone has identical social status, and so on, but no two persons can endogenously have distinct social identities.² Such narrowness in the very definition of a player characterizes game-theoretic economic models as well.³

In fact, in the context of sum-ranking utilitarianism, Sen and Williams (1982, p.4) claim that “persons do not count as individuals in this any more than individual petrol tanks do in an analysis of the national consumption of petroleum.” Their metaphor of “individual petrol tanks” is intended to highlight how very narrowly a person is defined in economic *theory*, contrasted with the *reality* of a person’s varied, multiple, communally-shared marks of identification, as well as individually-unique distinctive traits.

If my argument is valid, it should be impossible for theories based on Arrow and Debreu or on Nash to explain social-identity based discrimination of any sort, including occupational segregation by race or gender. This is simply because a person would not know against whom to discriminate, when every person is a race-less, gender-less replica of every other person, each utterly indistinguishable from every other on the basis of distinct social identities. I shall argue that this *lack of distinguishability by social identity* between two persons is an *entailment* of the *binariness* restriction that is almost always imposed on each person’s ranking relation of weak preference in game theory, social choice theory, general equilibrium theory, and the theory of game-forms.

In the context of racial discrimination in the United States, Arrow (1998, p. 94-95) directly asks, “What has economics to say about racial discrimination?” He restrains himself from offering a theory, and though his objections are distinct from the one I have just outlined, his critique is most helpful in that it gets to the heart of the matter of what landmines to avoid in developing a *credible* theory of discrimination,

Most analysts, following Becker (1957), add to the usual list of commodities some special disutility which Whites attach to contact with Blacks, taste-based discrimination. ... The trouble with these explanations is that they contradict in a direct way the usual view of employers as simple profit-maximizers. While they do not contradict rational choice theory, they undermine it by introducing an additional variable.

There are at least two objections to this line of analysis. One is that introducing new variables easily risks turning the "explanation" into a tautology. ...and it certainly would be a parody of economics to multiply entities in this anti-Occamian fashion. Perhaps more serious is the neglect of Darwinian principles.

² In the next section, I present a formal argument to support this elementary claim.

³ The game called Battle of the Sexes is also, by this reasoning, a misnomer.

Arrow's objections to existing theories of discrimination thus are that: (i) "they contradict in a direct way the usual view of employers as simple profit-maximizers," (ii) "while they do not contradict rational choice theory, they undermine it," (iii) "introducing new variables easily risks turning the "explanation" into a tautology," and (iv) "the neglect of Darwinian principles." Specifically, if Darwinian principles are respected by a theory, according to Arrow (p. 95),

the hypothesis of employer discrimination does not at all explain segregation by occupation, [and] discriminatory tastes of other employees ... may explain segregation [by firms] within industries but not segregation by occupation[s]

that are filled by persons of distinct social identities within firms.⁴ Arrow argues that "segregation within an industry – that is, firms with either all black or all white labor forces," rather than racially segregated occupations within a firm, is all that these theories can explain.⁵

Discrimination, which stands in sharp contrast to meritocracy, is utterly repugnant to those who lead examined lives, in addition, of course, to being hideously painful to those who are at the receiving end of it; it is seen as an injustice.⁶ Despite its shortcomings,⁷ the moral principle that people should be rewarded for the instrumental value of their merit in achieving pre-specified, transparent goals trumps the alternative that persons should instead be rewarded based on their religion, ethnicity, gender, age, race, tribal affiliation, nationality, not being autistic, or being the relative of the president of a country, and the like. And the claim that insofar as markets exhibit *impersonal* transactions, they foster *meritocracy in practice*, constitutes an ideological basis of jettisoning state intervention in favour of unrestrained operation of markets. But reality is different. Arrow (1998, p. 93) also points out that,

We have clear evidence that Blacks were in the past excluded from a significant range of jobs and from purchase of housing and restaurant services. We have very strong evidence that these practices persist in some important measure. I ... suggest ... that market-based explanations will tend to predict that racial discrimination will be eliminated. Since they are not, we must seek elsewhere for *non-market factors* influencing economic behavior. The concepts of *direct social interaction* and *networks* seem to be good places to start.⁸

Thus if unregulated markets have supported in the past, for over two centuries, and continue to support today, a persistent stationary state characterized by the unambiguous injustice

⁴ My goal in this paper is to provide a theory of discrimination that is bereft of all of Arrow's fundamental objections but one that I claim is necessary.

⁵ See Arrow (1998, p.95).

⁶ My objection to the discrimination literature in economics is distinct from Arrow's, insofar as for discrimination to be possible at least two social identities must exist – along lines that are racial, ethnic, gender, or whatever, which is impossible, I argue, in the model of Arrow and Debreu. However, Arrows suggestions for a credible theory of discrimination are still relevant independently of my argument.

⁷ On shortcomings of meritocracy, see Sen (1999), in a delightful set of essays by Arrow et al. (1999).

⁸ Italics added.

of racial discrimination in the United States that Arrow points to, some shine wears off the claim that free markets support meritocracy in practice, unless the indefensible claim is made that any such injustice is driven by state intervention itself.⁹ This diagnosis shakes up the ideological foundations against state intervention. Perhaps that is why some of the finest minds in economics have dwelt on the pernicious phenomenon of discrimination by social-identity distinctions, though unfortunately unsuccessfully, both for the reasons Arrow gives *and* because of my claim of interpersonal homogeneity of social identity in economic theory.¹⁰

Recent work of Akerlof and Kranton (2000, 2002, 2005, 2010) on *Identity Economics* contains extremely valuable insights. In their theory, social identity diversification characterises the population, so that they can explain phenomena involving operationally significant identity differentiation. Indeed, Akerlof and Kranton (2005, p. 13) argue – entirely correctly – that,

The combination of *identity*, *social category*, *norm* and *ideal* allows parsimonious modeling of how utility functions change as people adopt different mental frames of themselves – that is, as they take on different possible identities. Economists have recently adapted from psychology the idea that utility depends upon how a situation is framed (Kahneman and Tversky, 1979). Identity describes one special way in which people *frame* their situation.

Their theory helps explain a variety of phenomena involving the self-perception of a person’s identity, and, more importantly, phenomena that had thus far defied coherent economic explanation, such as the consequences of bringing about a change in that identity – in a classroom, or on joining a military academy, and the like.

Parsimony is undoubtedly a feature of their methodological approach. However, in the theory of Akerlof and Kranton, a person’s maximization of a non-traditional “utility function”¹¹ as a logically primitive conception is *imposed* via *persuasion*, rather than *entailed* by *derivation* from a preference ranking relation defined on a set of outcomes, with restrictions imposed both on the set and the relation. This clearly constitutes a weakness of their theory contrasted with that of

⁹ This was, in fact, the case before the abolition of slavery, but after the Civil Rights Act of 1964 became the law of the land, it would be false to claim that state intervention supports racial or gender discrimination. Instead it combats such practices by imposing hefty fines. However, the banking firm of Well Fargo settled a dispute, by paying \$175 million in 2012, in action brought against it by the Department of Justice of the U.S. government, for charging African Americans and Latinos higher interest rates or mortgages despite their credit-risk ratings comparable to their White clients. This is one persistent symptom of discrimination in the post-1964 United States.

¹⁰ See Akerlof (1976), Becker (1957), Phelps (1972), Spence (1974), and Stiglitz (1974), among others.

¹¹ Their utility function is nontraditional compared with one the standardly employed in economic theory, as in Arrow and Debreu. Akerlof and Kranton (2005, p. 14) specify $U(y, e; c) = \ln y - e + I_c - t_c|e^*(c) - e|, \forall c = N, O$, as the utility function to be maximized by a worker as a primitive concept, “where U is the worker’s utility, y is her income, e is her actual effort, c is her social category, I_c is her identity utility from being in category c , and $t_c|e^*(c) - e|$ is the disutility from diverging from the ideal effort level for category c , denoted $e^*(c)$.” See Footnote 2 for the specification of a utility function by Arrow and Debreu, to wit, $u_i(x_i)$.

Arrow and Debreu, if judged by Arrowian standards.¹² Thus Arrow's critique that "while they do not contradict rational choice theory, they undermine it" still applies to the contribution of Akerlof and Kranton.

If they could derive their utility function from underlying preferences, they would meet this specific standard of rigor in economic theory that Arrow demands. I claim, however, that they cannot do so, because the necessary underlying preference ranking relations that can support their claims about identity distinctions of persons *cannot* be binary in character, and thus cannot be given any compatible numerical representation, let alone the one Akerlof and Kranton themselves specify as the utility function ascribed to a person.

To provide rational choice theory foundations that are missing in Akerlof and Kranton's contributions, and to remove some deficiencies Arrow points to in the work of Becker and others, who also ascribe a utility function to each person in their explanations of discrimination, I suggest a non-binary personal preference approach in this paper. By substituting non-binary for binary preferences in the model of Arrow and Debreu, I extend their economic theory. The more general model I thus formulate has the following features: (i) there exists a social state in which all persons maximize their preferences on their feasible sets, (ii) endogenous interpersonal *social-identity diversification* characterizes this state of the economy, (iii) it is a free-market equilibrium without any state intervention, (iv) it is a *Pareto optimal* social state, and (v) a sizable proportion of Black workers are segregated into low-rank, low income jobs, whereas White workers in the *same observable proficiency* domain are placed in high-ranking, high-income jobs, thereby explaining *occupational segregation within firms along a racial divide*, explaining thereby such a glaringly visible, rampant and persistent, though undoubtedly removable, injustice.

Section 2 deals with the impossibility of endogenous interpersonal social-identity heterogeneity under binary personal preferences. Section 3 introduces a special kind of non-binary personal preference relation. Section 4 is devoted to the extension of a few results in social choice theory in Sen (1970) to existence and Pareto optimality of a socially diverse social state with personal non-binary preference maximization of a quasi-ordered set. Section 5 uses the Basu (2006) approach to proving that in such a social state, although *ex ante* persons in a group are not dissimilar in the observable distribution of their profit-generating capability, they can nevertheless

¹² Arrow's standards for judging the credibility of a theory appear to endorse reasoning, preferably formal, in favor of persuasion or rhetoric.

get segregated in the workplace *ex post* into distinct occupations based on their distinct social identities, even in profit-maximizing organizations. White Americans thus have high-rank, higher-paying jobs and African Americans in the *same observable proficiency domain* have low-rank, lower-paying jobs in the workplace. This arises if employers have racist *conjectures* about which race is more talented in producing a larger profit, and workers' talent is supermodular in shifting from one level to the next higher level. Section 6 contains some concluding remarks, particularly of a conceptual nature, but also the startling conclusion that over one-and-a-half million African American Males are missing – they would have been alive today in the counterfactual circumstance that African American males are as numerous relative to African American females as white males are relative to white females in the United States.

2. Impossibility of Endogenous Social Identity Diversification under Binariness

The stage is now set to prove the first main result of this paper.

Theorem 2.1: *Binariness of personal preference ranking relations implies endogenous interpersonal social-identity heterogeneity is impossible.*

Proof: Recall that the i^{th} person is defined by $\langle \zeta_i, \alpha_i X_i, R_i \rangle$ in the model of Arrow and Debreu. Add the assumption that R_i is a binary ranking relation.

Next consider two such distinct persons: John $\leftrightarrow \langle \zeta, \alpha, X, R_J \rangle$ and Katarina $\leftrightarrow \langle \zeta, \alpha, X, R_K \rangle$. Then, John *cannot be distinguished* from Katarina $\leftrightarrow (R_J = R_K)$. And, $(R_J = R_K) \leftrightarrow (\forall x \in X \ \& \ \forall y \in X \ \& \ [xR_J y \leftrightarrow xR_K y])$, which is true if, and only if, John and Katarina both identically rank every pair of alternatives in X . Thus 'John and Katarina cannot be distinguished' implies, and is implied by, they 'both rank every pair of alternatives identically.'

It also follows from the definition of a person that John *can be distinguished* from Katarina $\leftrightarrow (R_J \neq R_K)$. But, $(R_J \neq R_K) \leftrightarrow (\exists x \in X \ \& \ \exists y \in X \ \& \ xR_J y \ \& \ \sim xR_K y)$, which is true if, and only if, there exists at least one pair of alternatives in X that is ranked differently under R_J than under R_K . Thus, 'John can be distinguished from Katarina' implies, and is implied by 'at least one pair of alternatives is ranked differently by John and Katarina.'

It follows that the solitary basis of distinguishing between two persons is that they rank at least one pair of alternatives differently. John and Katarina are thus *not* distinguishable by social identity, because interpersonal distinctions are limited to being only in terms of distinct interpersonal rankings of at least a pair of alternatives, and nothing else. The concept of two socially distinct persons is itself not well-defined in a model of an economy with binary personal preferences. Hence endogenously every person in a binariness salient model of an economy has a single, solitary social identity. ■

Suppose John declares a red shirt to be at least as good as a blue shirt, and Katarina strictly prefers a blue shirt to red or is indecisive on the matter. Then, it is not altogether unreasonable to claim that they are distinguishable persons. However, solely on the basis of different rankings of

blue and red shirts it would be patently absurd to conclude that one person is male and the other female, or one Black and the other White, or they have distinct social identities.¹³

In essence, while in reality each person may have multiple dimensions, once a person is identified with a preference ranking relation *and* the property of binariness is imposed on this relation, all personal dimensions other than the specific order of ranking of alternatives are rendered inadmissible information in the very conception of a person in such a model. Aside from distinct rankings of at least one pair of alternatives, interpersonal differences are rendered invariant to *all other* conceivable dimensions on which any two persons could have differed, including gender, race, ethnicity and the like. Two persons who differ *exclusively* in terms of ranking at least a pair of alternatives are certainly distinct and distinguishable, but they are distinguishable *solely* in this regard, not by any means in terms of having distinct social identities!

3. Non-Binary Preferences

Thus, on the basis of Theorem 2.1, I argue that the binary preference ranking relation Arrow and Debreu ascribe to each consumption unit must be replaced by a *non-binary* personal preference ranking relation if the objective is to model social identity diversification in an economy.¹⁴ A ranking relation is non-binary in the Stig Kanger sense if, with a subscript referring to a person, R_i is a binary ranking relation of weak preference defined on a set S_i of alternatives, and V_i is the background set of the i^{th} person or player upon which R_i is dependent, so that $R_i(V_i)$ defined on S_i is that person's non-binary ranking relation of weak preference, $\forall i = 1, \dots, m$ & $m \geq 2$.

Note also that the personal feasible set S_i of mutually exclusive alternatives or strategies is very generally construed. However, in particular, it can also be taken to be the same set as a consumption unit's feasible set in Arrow and Debreu, *without redefining* the elements of a personal feasible set only of culmination outcomes, as in the set of payoffs in choice theory, game theory and in general equilibrium theory.¹⁵ Redefining the elements of the feasible set also 'easily risks

¹³ If a person is observed to be wearing a blue and red striped shirt as a matter of declared strict preference over either red or blue, would we conclude, solely on this basis, that it is a transgender person? In a binariness-salient model, exogenously imposing identity distinctions on persons who are, in fact, identity-wise homogenous is inadmissible.

¹⁴ See Sen (1994b) on his completion of the 1970s unfinished work "Choice based on Preference" of the late Swedish philosopher Stig Kanger on non-binary ranking relations and their corresponding maximal sets.

¹⁵ While I do not need it for the present exercise, for a richer formulation that would provide a formalization of culture, of which shared beliefs are a part, I could take the liberty of assuming that S_i is, in some cases, a set of alternative, mutually exclusive *comprehensive outcomes* (action-process-consequence strings) that in individual preferences attach positive constitutive value *also* to the process by which the choice of action leads to the final consequence, rather than ascribing direct, constitutive value only to the *culmination outcomes*, which is the case with *game theory*

turning the “explanation” into a tautology’ according to Arrow (1998, p.95), and is thus an approach to be avoided.¹⁶

In the case of a *non-binary* relation of weak personal preference $R_i(\mathbf{V}_i)$ defined on a set S_i , a variation in \mathbf{V}_i (say, from \mathbf{V}_i^1 to \mathbf{V}_i^2) can, in general, alter a *single person’s* pair-wise ranking R_i of a pair of alternative actions $x, y \in S_i$, rendering $R_i(\mathbf{V}_i)$ a non-binary relation. This is because R_i , which is by assumption a binary relation, is *not* invariant to *tertiary* considerations entailed by differences in the personal background set of any given person, such that $\exists \mathbf{V}_i^1 \neq \mathbf{V}_i^2$ & $x R_i(\mathbf{V}_i^1)y$ & $\sim x R_i(\mathbf{V}_i^2)y$ are both admissible.¹⁷

Also, differences in the personal backgrounds of *distinct persons* can be accommodated by such a non-binary preference ranking relation. This is pure Kanger territory, as Sen (1994b) remarks. For example, $R_i(\mathbf{V}_{AM})$ could refer to the preference of the i^{th} person who is an African American male, and $R_i(\mathbf{V}_{CM})$ can represent a Caucasian male’s preference, defined on S_{AM} and S_{CM} , respectively. While the gender of both persons is the same, a shared trait that they are both male, in such a non-binary model the persons are of different races. Thus, both shared and distinctive social identities of persons can also be accommodated in a model of a society with individual Kanger-Sen non-binary preferences.

This is *not* the case in the Akerlof and Kranton theory of Identity economics. Instead, they deal with changing the mindset of a person i from the existing \mathbf{V}_i^1 before joining a military academy at West Point, to \mathbf{V}_i^2 after joining it and undergoing a change in self-perception, which changes the person’s preference and consequently individual behavior. A change in a single person’s belief of self-perception is a very different matter compared to two different persons having distinct

in Nash (1951) and in *general equilibrium theory* in Arrow and Debreu (1954) and Arrow and Hahn (1971), among others.

¹⁶ However, this is only one possible interpretation. There are *other* useful and important interpretations of both the elements of the feasible set as well as of the background set of which the ranking relation is a function. They could be utilized to explain intra-family interactions, for example. See Sen (1989).

¹⁷ It is perfectly legitimate to interpret $\mathbf{V}_i^1 \neq \mathbf{V}_i^2$ as two distinct *menus* faced by the same person. It is also legitimate to interpret them as distinct *identities of the chooser* (the host or I), which is crucial in the case of fiduciary responsibility, as Sen (1997) points out. Another interpretation is that different background sets constitutes a gateway that lets in information on the distinct social identities of different persons. The Akerlof and Kranton phenomenon can be interpreted as each member in a group of recruits into West Point has one back ground set of beliefs about oneself. However, training at the Academy changes their self-perception to a distinct characterization of oneself as a distinct background set. There can be many more interpretations. However, a numerical representation of such preferences is impossible in all cases. Also, it would *not* be legitimate to construe such preferences as state-contingent-probabilistic preferences, as, for example, in Karni (2005), in the theory of decision making under uncertainty, because there is no incompleteness of information in such a non-binary preference model.

identities. Such a crucial conceptual distinction gets lost in the utility function specification of Akerlof and Kranton, but this substantive difference is sharply brought out by the Kanger-Sen non-binary preference formulation that I deal with here.

Binary preferences are a special case of non-binary preferences, as $R_i(\mathbf{V}_i^j) = R_i(\bar{\mathbf{V}}) \equiv R_i, \forall i, j$. A utility function representation of preferences is, however, a casualty of switching from binary to more general non-binary preferences.

To see this, assume that a person's preferences are represented by a non-binary relation $R(\mathbf{V}^j)$ defined on the set S of mutually exclusive alternatives, where R is a binary relation defined on S and \mathbf{V}^j is the background set on which R is dependent, with $j = 1, \dots, k$. Suppose also that this person is identified with a utility function $f: S \mapsto \mathcal{R}$, where S is the set of mutually exclusive alternatives and \mathcal{R} is the set of real numbers. Then, by definition of a utility function, we have $\forall x, y \in S: [xR(\mathbf{V}^j)y \leftrightarrow f(x) \geq f(y)]$.¹⁸ Since $R(\mathbf{V}^j)$ is non-binary, $(\exists \mathbf{V}^1 \neq \mathbf{V}^2 \ \& \ \exists x, y \in S: xR(\mathbf{V}^1)y \ \& \ \sim xR(\mathbf{V}^2)y)$ are both admissible. Equivalently, $([f(x) \geq f(y)] \ \& \ \sim [f(x) \geq f(y)]) \leftrightarrow [f(x) \geq f(y)] \ \& \ [f(x) < f(y)]$, which is false. It follows that a non-binary preference relation cannot have a utility function representation.

This exposes the fact that a non-binary ranking relation, which permits interpersonal social identity diversification, happens to be one that cannot possibly be given numerical representation that can legitimately be called a utility function. Since utility functions in Becker and in Akerlof and Kranton are presumably numerical representations of personal preferences, their methodological approaches are deeply problematic, though their conclusions need not be.

Such a non-binary preference approach as outlined above can provide the rational choice theory foundation that Akerlof and Kranton's *Identity Economics* lacks.

4. Non-Binary Personal Preference Relations based Society

My purpose is to prove the existence of a non-empty set of social states at which all persons have achieved maximization of their respective personal non-binary preferences, which are required to be a quasi-ordering (reflexive and transitive, though not necessarily complete) of their respective personal feasible sets S_i .

4.1 Model

To develop the requisite theory, I make the following three assumptions.

¹⁸ See Footnote 2.

Axiom N (Non-binariness Axiom): Each individual is characterized by a personal non-binary ranking relation of weak preference that is defined on a finite set of alternatives. This preference relation is $R_i(\mathbf{V}^i)$, which is defined on a finite set S_i of the person's feasible set of alternatives, and \mathbf{V}^i is that person's *background set* upon which the *binary* ranking relation R_i is dependent, for all persons i from 1 to a finite m .

Notice that non-binariness of personal preferences is defined in a very specific sense.¹⁹

Axiom Q (Quasi-Ordering Axiom): Every person's non-binary relation of weak preference is reflexive and transitive but not necessarily complete.

Axiom M (Maximization Axiom): Every person engages in a volitional act of choice by maximizing one's own personal preferences defined on the personal feasible set of actions.

I shall prove that these three axioms collectively entail the following two claims:

Condition S (Social Identity Diversification): There exists of a social state with endogenous social differentiation of persons, with each person being a social creature with multiple social identities deriving from affiliation with distinct, though overlapping communities (or subsets) of persons in society.

Condition P (Pareto optimality): The social state identified in Condition *S* is Pareto optimal.

The groundwork for an examination of a non-binary society with social diversification is now laid. Some additional formalization is necessary, however, for the demonstration of the next principal conclusion of this paper.

4.2. Formalization

For a given set \mathbf{V}_i^j , let $R_i(\mathbf{V}_i^j)$ be person i 's binary relation of weak preference that stands for "at least as good as", which is defined on a finite set S_i of alternatives social states, and \mathbf{V}_i^j is a background set on which the binary relation R_i is dependent, with person $i = 1, \dots, m$, and $j = 1, \dots, k_i$ specifying the possible parametric variations, \mathbf{V}_i^j , of person i 's background set. Here, $m \geq 2$ is finite, $k_i \geq 1$ is finite, and S_i also finite with at least three elements.

For $R_i(\mathbf{V}_i^j)$, we can define the asymmetric part $P_i(\mathbf{V}_i^j)$ that stands for "strict preference", and the symmetric part $I_i(\mathbf{V}_i^j)$ that stands for "indifference" as follows.

Definition 1 STRICT PREFERENCE: $(\forall i, \forall j \ \& \ \forall x, y \in S_i): [[xR_i(\mathbf{V}_i^j)y] \ \& \ \sim [yR_i(\mathbf{V}_i^j)x]] \leftrightarrow [xP_i(\mathbf{V}_i^j)y]$

¹⁹ The precise content of the concept of non-binariness of preferences taken here, in *Axiom N*, is exactly the one formulated by Stig Kanger on which additional work was done by Sen (1994b), to complete Kanger's unfinished work, to bring it to the domain of mainstream choice theory. This is precisely the formulation of personal preference in Sen's (1997) individual choice theory.

Definition 2 INDIFFERENCE: $(\forall i, \forall j \ \& \ \forall x, y \in S_i): [[xR_i(\mathbf{V}_i^j)y] \ \& \ [yR_i(\mathbf{V}_i^j)x]] \leftrightarrow [xI_i(\mathbf{V}_i^j)y]$.

In this context, it is important to note that a variation in a *tertiary* consideration, viz., a parametric variation in the background set, can, in general, alter the order of personal preference insofar as $(\forall i, \exists x, y \in S_i \ \& \ \exists l \neq k): [xR_i(\mathbf{V}_i^l)y] \ \& \ \sim [xR_i(\mathbf{V}_i^k)y]$, are both admissible, thereby rendering $R_i(\mathbf{V}_i^j)$ a non-binary relation.

4.3. Existence of a Pareto Optimal State under Non-Binariness

To achieve these objectives, I utilize three lemmas in Sen (1970), and with relatively minor generalizations, prove an existence theorem below. First, however, some additional definitions are in order.

Definition 3: $R_i(\mathbf{V}_i^j)$ is *reflexive* over S_i if and only if $(\forall i, \forall j \ \& \ \forall x \in S_i): [xR_i(\mathbf{V}_i^j)x]$.

Definition 4: $R_i(\mathbf{V}_i^j)$ is *transitive* over S_i if and only if the following holds:
 $(\forall i, \forall j \ \& \ \exists x, y, z \in S_i): [[xR_i(\mathbf{V}_i^j)y] \ \& \ [yR_i(\mathbf{V}_i^j)z]] \rightarrow [xR_i(\mathbf{V}_i^j)z]$.²⁰

Definition 5: A ranking relation that is reflexive and transitive is called a *quasi-ordering*.²¹

Let $\mathbf{V} = \cup_{i=1}^m (\cup_{j=1}^{k_i} \mathbf{V}_i^j)$, and $S = \cap_{i=1}^m S_i \neq \emptyset$, and assume that S has at least three elements.

Definition 6: A *Non-Binary Collective Choice Rule* (NB-CCR) is a functional relation f that assigns exactly one social ranking $R(S, \mathbf{V})$ of S to an inter-personal non-binary preference profile, $(R_1(\mathbf{V}_1^j), \dots, R_m(\mathbf{V}_m^j))$ such that $R(S, \mathbf{V}) = f(R_1(\mathbf{V}_1^j), \dots, R_m(\mathbf{V}_m^j))$, where $\forall i, j: R_i(\mathbf{V}_i^j)$ is a quasi-ordering of S_i .²²

²⁰ Weaker forms of transitivity can be identified. $(\forall i, \exists j \ \& \ \exists x, y, z \in S_i): [[xR_i(\mathbf{V}_i^j)y] \ \& \ [yR_i(\mathbf{V}_i^j)z]] \rightarrow [xR_i(\mathbf{V}_i^j)z]$ is less demanding because it requires of each person that transitivity hold over some variations, $\exists j$, of a person's background set, though not necessarily over all variations. The first existence result I intend to prove also goes through, it is easy to see, on this weaker transitivity restriction on the social preference relation, and it does so also under acyclicity (of the strong kind), as in Berdillima and Naqvi (2011).

²¹ Reflexivity of $R_i(\mathbf{V}_i^j)$ is not a very demanding restriction. To see this, consider two persons, indexed m and f , where m is male and f is female. Then for all persons i , $\forall j: xR_i(\mathbf{V}_i^j)x$ and $\forall j: xR_i(\mathbf{V}_i^j)x$ essentially means that both men and women regard an element of their feasible set at least as good as itself under all possible variations of their respective background sets, whether it rains or shines, for example. Similarly, for the transitivity property, $\forall j: xR_i(\mathbf{V}_i^j)y \ \& \ yR_i(\mathbf{V}_i^j)z \rightarrow xR_i(\mathbf{V}_i^j)z$ and $\forall j: xR_i(\mathbf{V}_i^j)y \ \& \ yR_i(\mathbf{V}_i^j)z \rightarrow xR_i(\mathbf{V}_i^j)z$ means that both men and women have transitive preferences for all variations of their respective background sets. For additional detail, see the previous foot note.

²² Sen (1970, p.28) defines, *A collective choice rule is a functional relation f such that for any set of n individual orderings R_1, \dots, R_n (one ordering for each individual), one and only one social preference relation R is determined, $R = f(R_1, \dots, R_n)$. Italics in original. Note that if R is restricted to being an ordering, this CCR is Arrow's (1951) social welfare function, the impossibility of existence Arrow proved, in one of the greatest insights of the 20th Century. However, here in Sen's definition, he does not even require R to be binary. Although Sen (1993) does prove Arrow's impossibility theorem by requiring R to be only binary and reflexive, dropping altogether both of Arrow's additional restrictions, that R is complete and transitive. This theorem is considerably more general than Arrow's impossibility theorem, because it shows the impossibility over a wider range of CCRs, though over the same domain as Arrow's.*

By $P(S, \mathbf{V})$ we denote the asymmetric part of $R(S, \mathbf{V})$. We next turn to *unanimity over a pair of alternatives* under all possible variations of the background set to define Pareto preference.

Definition 7A: $\forall j \ \& \ \exists x, y \in S: [\forall i: x R_i(\mathbf{V}_i^j) y] \leftrightarrow x \hat{R}(S, \mathbf{V}) y$.

Definition 7B (weak Pareto rule): $\forall j \ \& \ \exists x, y \in S: [\forall i: x P_i(\mathbf{V}_i^j) y] \leftrightarrow x \hat{P}(S, \mathbf{V}) y$.

Remark: Definition 7A is a generalization of the Pareto ‘preference or indifference’ rule denoted by $\hat{R}(S, \mathbf{V})$ to accommodate non-binary personal preferences over the set S of alternative social states, and similarly, Definition 7B is a generalization of the *weak Pareto rule*, denoted by $\hat{P}(S, \mathbf{V})$.

The *weak Pareto rule* in Definition 7B is an ethical principle.

Definition 8: A social state x in S is *Pareto optimal* if and only if it is *not* Pareto dominated by any state y in S in accordance with Definition 7B.

Finally, using Definitions 6, 7A and 7B, and by requiring that $[x R(S, \mathbf{V}) y] \leftrightarrow [x \hat{R}(S, \mathbf{V}) y]$ and $[x P(S, \mathbf{V}) y] \leftrightarrow [x \hat{P}(S, \mathbf{V}) y]$, we can obtain a maximal social interaction outcome by using the following two lemmas.

LEMMA 4.1. $\hat{R}(S, \mathbf{V})$ is a quasi-ordering of S .

Proof: Following Sen (1970, Lemma 2*a, p.29),

$\forall j \ \& \ \forall x, y \in S$, since by Definition 7A, for $y = x$, $\forall i: x R_i(\mathbf{V}_i^j) x$, it follows that $\hat{R}(S, \mathbf{V})$ is reflexive.

Also,

$$\begin{aligned} \exists x, y, z \in S: [x \hat{R}(S, \mathbf{V}) y \ \& \ y \hat{R}(S, \mathbf{V}) z] &\rightarrow [\forall j \ \& \ \{\forall i: x R_i(\mathbf{V}_i^j) y \ \& \ y R_i(\mathbf{V}_i^j) z\}] \\ &\rightarrow \forall j \ [\forall i: x R_i(\mathbf{V}_i^j) z] \\ &\rightarrow x \hat{R}(S, \mathbf{V}) z. \quad \blacksquare \end{aligned}$$

Next, consider

DEFINITION 9: $M(\hat{R}, S, \mathbf{V}) = \{x | x \in S \ \& \ \sim[\exists y \in S: y \hat{P}(S, \mathbf{V}) x]\}$ is a *Social Maximal Set*.

Remark: The social interaction maximal set of socially undominated elements of S with respect to the social quasi-ordering relation \hat{R} is fully captured by Definition 9 with respect to the weak Pareto rule $\hat{P}(S, \mathbf{V})$, which is the asymmetric part stated in Definition 7B.

LEMMA 4.2. *The maximal set is non-empty for every finite set quasi-ordered by a non-binary preference ranking relation.*

Proof: Again, following Sen (1970, Lemma 1*b, p.11, and Sen (1997)), let $S = \{x_1, \dots, x_h\}$. Assign a real number $a_1 = x_1$, and follow the recursive rule $x_{q+1} \hat{P}(S, \mathbf{V}) x_q \rightarrow a_{q+1} = x_{q+1}$, and $a_{q+1} = a_q$ otherwise, so that by construction, x_h is a maximal element. \blacksquare

This is also the most general formulation of Arrow’s impossibility theorem to date. However, all I require R to do is to generate a non-empty social maximal set, which I define presently.

Remark: Note that since $(\forall i, \exists x, y \in S_i \ \& \ \exists l \neq k): [xR_i(\mathbf{V}_i^l)y] \ \& \ \sim [xR_i(\mathbf{V}_i^k)y]$ are both admissible, personal preferences are non-binary, but this poses no problem for obtaining a nonempty *social maximal* set since the personal non-comparability of a pair of alternatives in S_i is rendered irrelevant for defining the maximal set. This, of course, is not true of the *social optimal* set of best elements that is defined as $C(\hat{R}, S, \mathbf{V}) = \{x | x \in S \ \& \ \forall y \in S: x \hat{R}(S, \mathbf{V})y\}$, which is rendered empty by $(\forall i, \exists x, y \in S_i \ \& \ \exists l \neq k): [xR_i(\mathbf{V}_i^l)y] \ \& \ \sim [xR_i(\mathbf{V}_i^k)y]$ being both admissible.

Thus, requiring maximizing behavior as an act of volitional personal choice, instead of the more demanding optimization, does have an advantage in the case of non-comparability arising from non-binariness of personal preferences. In fact, it should not come as a surprise that once there is a social quasi-ordering which ranks at least one pair of alternatives, though not necessarily all such pairs, if and only if these two alternatives are comparable over all individuals and over all background sets, there must exist an element which is Pareto undominated and thus Pareto optimal. This follows from Zorn's lemma.

In the case of personal choice theory, Sen (1997) exploits precisely this combination of non-binariness of preferences (and the entailed incomplete ranking), *and* maximizing behavior that precipitates the existence of a maximal element despite incompleteness. He thus obtains more general results than can be deduced from binary personal preferences that constitute a complete ordering *and* optimizing behavior that necessitates completeness for the existence of an optimal element – this is the formulation of standard rational choice theory. Sen's (1997) formalization is of a decision maker who has non-binary preferences. To obtain a nonempty *social* interactional set, rather than a nonempty *personal* maximal set, a group of finite n persons needs to be considered. For this case, based on *Lemma 4.1* and *Lemma 4.2*, the following result is immediate.

Theorem 4.1 (Existence of Socially Diverse Pareto Optimal State): *Axioms N, Q and M are sufficient for the existence of a social state that is characterized by Conditions S and P.*

Proof:²³ It is to be proved that for every set of non-binary personal preferences $(R_1(\mathbf{V}_1^j), \dots, R_m(\mathbf{V}_m^j))$ over a finite set S of alternative social states, where $\forall i, j: R_i(\mathbf{V}_i^j)$ is a quasi-ordering, there exists a nonempty maximal social interaction set $M(\hat{R}, S, \mathbf{V})$ that contains at least one Pareto optimal state.

By *Lemma 4.1*, the Pareto preference-or-indifference relation $\hat{R}(S, \mathbf{V})$ is a quasi-ordering of the set S of alternative social states. And the Pareto-optimal subset of S is identical to the social maximal set $M(\hat{R}, S, \mathbf{V})$. Further, since S is finite, and $\hat{R}(S, \mathbf{V})$ quasi-orders it, by *Lemma 4.2*, $M(\hat{R}, S, \mathbf{V})$ is

²³ The proof of *Lemma 4.1*, and therefore of *Theorem 4.1*, goes through if transitivity is weakened to *Acyclicity*: $(\forall i, \forall j \ \& \ \forall x_1, x_2, \dots, x_l \in S): [(x_1 P_i(\mathbf{V}_i^j) x_2 \ \& \ x_2 P_i(\mathbf{V}_i^j) x_3, \dots \ \& \ x_{l-1} P_i(\mathbf{V}_i^j) x_l)] \rightarrow x_1 R(\mathbf{V}_i^j) x_l$, provided “quasi-ordering” is replaced by “reflexive and acyclic ranking.” These two may be called *Lemma 4.1A* and *Theorem 4.1A*, respectively. For further details, see Berdellima and Naqvi (2011).

nonempty. Hence a non-binary personal preferences based social interaction outcome exists, and it is Pareto optimal. ■

Notice that, unlike the case of the existence of equilibrium in an Arrow-Debreu exchange economy, and unlike the case of existence of Nash equilibrium, for my existence result I do not impose the requirement of completeness; in fact, I cannot impose it because incompleteness is *entailed* by non-binariness, if it is to have any cutting power at all. Nor indeed do I require preferences to be binary, as in Kelsey (1995). Moreover, moving from *form* to alternative *interpretations* of the background set, and by considering parametric variations of this set, many of the inadequacies in explanations of social and economic phenomena entailed by binariness are entirely jettisoned, replaced instead by a much more comprehensive and richer informational conceptual structure in which the unanimity property over a pair of alternatives, inherent in the weak Pareto rule, can still be defined. Also, the scope and reach of this non-binary model is so substantial that it also capable of formalizing some of the criticism of the game theory literature contained in Sen (1985).

5. Existence of a Free Market Pareto Optimal State with Discrimination

Having demonstrated the existence of a Pareto optimal social state in an economy with non-binary personal preferences, a state that exhibits persons with endogenously diversified social identities, the next step is to show that injustice in the form of racial (or gender, caste, religious, and so on) discrimination that expresses itself as occupational segregation by social identity is consistent with this state. To show that such is indeed the case, I consider a special case of the production side of the model of Arrow and Debreu.

Recall that Arrow and Debreu $\forall i = 1, \dots, m, \langle \alpha_i, \zeta_i, R_i(V_i^j) \rangle, \forall j = 1, \dots, k_i$ defines the i^{th} person. Let $m \geq 4$. Partition the m persons in the economy into two disjoint groups, $1, \dots, m_1$, who belong to the set \mathbf{B} , and are called Black, and have darker-color skin, and m_1+1, \dots, m who belong to the set \mathbf{W} and are called White, and have lighter-color skin. Thus there are two groups of persons in the economy: $\forall i = 1, \dots, m_1, \langle \alpha_i, \zeta_i, R_i(B_i^j) \rangle$ and $\forall i = m_1 + 1, \dots, m, \langle \alpha_i, \zeta_i, R_i(W_i^j) \rangle$.

Further partition \mathbf{B} and \mathbf{W} into two disjoint sets each, \mathbf{B}_L and \mathbf{B}_E and \mathbf{W}_L and \mathbf{W}_E such that $\forall i: (\alpha_i = 0 \rightarrow i \in \mathbf{L} = \mathbf{B}_L \cup \mathbf{W}_L) \& (\alpha_i > 0 \rightarrow i \in \mathbf{E} = \mathbf{B}_E \cup \mathbf{W}_E)$. I shall call a person who belongs to \mathbf{E} an *employer* (because such persons own corporate shares) and a person who belongs

to L , a potential employee or *worker* (since such persons have no income derived from corporate profits). Thus there are employers and workers of both races.

Consider the special case of $\zeta_i \in \mathcal{R}^3$. Thus $\forall i: \zeta_i = (\zeta_{ia}, \zeta_{id}, \bar{l}_i)$, where ζ_{ia} and ζ_{id} are non-negative endowments of commodities a , architectural design and d , gourmet dinner, and $\bar{l}_i > 0$ is the endowment of labor of the i^{th} person.²⁴ These quantities are flows per unit time. Every person engages in the production of architectural designs or gourmet dinners, and consumes architectural designs, gourmet dinners and leisure, $\bar{l}_i - h$, where h is the number of hours per period that the person works in a firm, either in the architectural designs industry or the gourmet dinners industry. Since leisure is unspent labor, and a person cannot consume more than 24 hours of leisure per day (nor, for that matter, provide more than 24 hours of labor per day), the i^{th} person's consumption vector $x_i = (x_{i1}, x_{i2}, x_{i3})$, is bounded from below by $\xi_i = (\xi_{i1}, \xi_{i2}, \xi_{i3})$, so that $\xi_i \leq x_i$.²⁵ Only architectural designs and gourmet dinners are produced, and labor is a primary factor of production, as in Theorem 2 of Arrow and Debreu.

Employers consume all their leisure time (by assumption, their corporate income is sufficient for them to not work at all, and still consume all the architectural designs and gourmet dinners they want, as part of their respective maximal consumption sets, and only workers supply labor, again as part of their respective maximal consumption sets).

In every firm in every industry, there are two employment categories: *low-rank jobs* that pay the economy-wide *wage rate*, and *high-rank jobs* for which employers pay a *talent premium* over and above the wage rate that they pay low-rank workers. The argument developed in the rest of this section is based entirely on Basu (2006). Following Basu (2006), I assume that the i^{th} worker's production function is

$$y_i^j = (1 + e_i) g^j(h; \theta), j = a, d \quad (1)$$

if the worker produces commodity j , where a worker produces either architectural designs or graphic designs or gourmet dinners, but not both.²⁶ Also, $\forall j = a, d$, $g^j(h; \theta)$ is the output of commodity j per worker who puts in h hours of labor per period and is characterized by a parameter $\theta \in [1, q]$, which is a positive integer with a finite upper bound q , where θ is a worker's

²⁴ For concreteness, we can consider $\bar{l}_i = 24$ hours per day.

²⁵ In particular, $\forall i: -24 \leq x_{i1} = \bar{l}_i - h \leq 24$.

²⁶ I make the usual assumption that $g^j(h; \theta)$, seen as a production function of labor h , satisfies the Inada conditions, is homogenous of degree one in h , and exhibits positive but diminishing marginal product of labor hours h .

characteristic that is an *indicator of talent* of a worker (in terms of creativity or entrepreneurial spark to sign more contracts), with a higher value standing for greater talent, and which varies from one worker to another, but which is *not* observable by any employer.

In (1), e_i is the *proficiency* of the i^{th} worker, such that $0 < e_i < 1$. Here, e_i is the second characteristic of a worker, though unlike talent, it is observable by employers, perhaps by the worker's educational attainment. The third characteristic of a worker, which is also observable by employers is z_i , which the worker's racial identity. Assume that $z_i = B, W$, where B means that the worker is Black and W means the worker is White.

Since z_i does not figure in the production function (1), one might be tempted to conclude that racial distinctions will be inconsequential to the complexion of the equilibrium, but that would be an error. Race matters, however, as we shall see presently, because of the role played by θ , in spite of the fact that talent and racial identity of a worker are, in fact, *not* linked. Thus,

$$\forall h > 0: (\theta_2 > \theta_1) \rightarrow g^j(h; \theta_2) > g^j(h; \theta_1), \forall j = a, d. \quad (2)$$

It is important to note the fact that the talent of a worker is independent of the worker's race, as is the worker's proficiency. I next specify four axioms to capture these ideas.

Axiom PD: Proficiency distribution among Black and White workers is the same, but for a linear transform of population, and is observable to employers.

Axiom TD: Talent distribution of Black and White workers is the same, but for a linear transform of population, but is unobservable to employers.

Axiom C: In hiring a worker, all employers who compare profit-wise inconclusive Black and White workers, behave on the basis of the conjecture that White workers are more talented than Black workers.

Axiom SM (Supermodularity): The higher the talent level θ of a worker, the greater is worker's talent-induced productivity differential from one level to the next. That is, $\forall j = a, d \ \& \ \forall h > 0$

$$\forall k \in [3, q] \rightarrow g^j(h; k) - g^j(h; k - 1) > g^j(h; k - 1) - g^j(h; k - 2). \quad (3)$$

Next I specify a characteristic of a social state that refers to an unjust state as **Condition J**, and then prove the main result of the paper as Theorem 5.1

Condition J (Violations of Justice): The social state is consistent with flagrant violation of justice that takes the form of segregation of most, though not all, Black persons in lower-ranked occupations with which are associated lower incomes, with simultaneous placement of White persons in higher-ranked occupations with higher incomes, regardless of the firms or the sectors in which they work.

Theorem 5.1: *Axioms N, Q, M, PD, TD, C and SM are sufficient for the existence of a social state that is characterized by Conditions S, P and J.*

Proof: From Theorem 6.1, **N**, **Q** and **M** \rightarrow **S** and **P**. Thus it remains to prove that, in addition, **PD**, **TD**, **C** and **SM** \rightarrow **J** also.

The profit derived by a firm from employing the i^{th} worker in industry $j = a, d$ is

$$\pi_i = p_j y_i^j - \omega h - \rho \theta \geq 0, \quad (4)$$

where employers are only interested in workers who do not earn a negative profit. Also, in (4), the employer is a price-taker in both commodity and factor markets. Therefore, the commodity price p_j , the hourly wage ω , and the *premium* ρ that employers pay *per unit of additional talent level* (or contract signed with a new client) per period, to get a worker of higher talent θ to work for them in higher-ranked occupations; these are all parametric for the firm.

Substituting for the productions function (1) and (2) in (4), we have $\forall j = a, d$,

$$\pi_i = p_j(1 + e_i) g^j(h; \theta) - \omega h - \rho \theta \geq 0. \quad (5)$$

Since the number of hours for which a worker is hired is a decision variable for each employer, the first order conditions for profit maximization are $\forall h > 0$,

$$p_j(1 + e_i) g_h^j(h; \theta) = \omega, \quad \forall j = a, d \quad (6)$$

In (6), $(1 + e_i) g_h^j(h; \theta)$ is the marginal product of labor time of the i^{th} worker in producing the j^{th} commodity, and from this it is possible to solve for \hat{h} as a function of parameters p_j , e_i , ω and θ . I assume that $\hat{h} > 0$. I shall also assume, for simplicity, that every worker is hired for the same number of hours per day in both sectors, say eight hours per day, though nothing of consequence rests on this assumption.

A limited liability clause in the employment contract would protect a worker from earning a negative profit, but not it will fail to protect the employer from incurring a loss, if the worker's talent level is sufficiently low. I assume this, as does Basu (2006).

Next, using (6), $\forall j = a, d$, define e^* from the zero profit condition as

$$(1 + e^*) g^j(\hat{h}; 1) = \frac{\rho}{p_j} + \frac{\omega}{p_j} \hat{h}.$$

Also from (6), zero profit implies

$$(1 + e^*) g^j(\hat{h}; 2) = \frac{2\rho}{p_j} + \frac{\omega}{p_j} \hat{h}$$

from these two relationships it follows that

$$(1 + e^*) [g^j(\hat{h}; 2) - g^j(\hat{h}; 1)] = \frac{\rho}{p_j}, \quad (7)$$

Similarly, define e^0 as

$$(1 + e^0) [g^j(\hat{h}; q) - g^j(\hat{h}; q - 1)] = \frac{\rho}{p_j}. \quad (8)$$

The RHS of (7) and (8) are equal, but, due to the supermodularity property in *Axiom SM*, on the LHS we know that,

$$g^j(\hat{h}; q) - g^j(\hat{h}; q - 1) > g^j(\hat{h}; 2) - g^j(\hat{h}; 1).$$

This implies that $e^0 < e^*$.

Every worker with efficiency greater than e^* – whether Black or White – will get a high-rank employment contract irrespective of the associated value of θ , and earn $\omega\hat{h} + \rho\theta$, because such a worker is guaranteed to turn a profit for the employer. However, no worker with efficiency less than e^0 will get any high-rank employment contract, regardless of the associated value of θ , because such a worker, if paid a talent premium, is bound to incur a loss for the employer.²⁷

The interesting case is of a worker whose efficiency is $e_i \in (e^0, e^*)$. In this case, *for a given wage rate*, depending upon the talent level of the worker, high-rank employment, which pays a talent premium, could lead to profit or loss for the employer – under a limited liability clause that applies to the worker, which I have assumed is the case. Since employers cannot observe the innate talent of a worker, in such a case they clearly have a dilemma. They must form a conjecture about a worker’s talent. If employers – both Black and White – were to base their conjectures of a worker’s talent on the racial identity of the worker, specifically on *Axiom C*, then they will offer high-rank employment contract to a worker *if, and only if*, the worker is White ($z_i = W$), who will then earn $\omega\hat{h} + \rho\theta$.

This implies that Black workers with efficiency $e_i \in (e^0, e^*)$ would not get high-rank employment, but it does not follow that they will be unemployed. Reconsider (5), but for $\theta = 0$, which is the category of workers who are conjectured by employers to have no talent at all. Intersectoral labor mobility ensures that values of marginal product of such workers are equal in all industries, and their common value is the single, economy-wide wage rate of $\theta = 0$ workers, who get the low-rank employment contracts in general equilibrium. This wage rate $\hat{\omega}$ is determined endogenously by²⁸

$$p_a(1 + e)g_h^a(H^a; 0) = p_d(1 + e)g_h^d(H^d; 0) = \hat{\omega},$$

where e is average efficiency of workers and H^j is aggregate employment of labor in industry $j = a, d$.

Thus, Black workers with efficiency $e_i \in (e^0, e^*)$ would *not* be unemployed in general equilibrium. In fact, employers will offer a low-rank employment contract to a worker *if, and only if*, the worker is Black ($z_i = B$), who will then earn $\hat{\omega}\hat{h}$. So, why will employers *not* decline offering contracts to Black workers with efficiency $e_i \in (e^0, e^*)$? Simply because it is a simple profit-maximizing act for them to offer such workers a low-rank employment contract, with no talent premium, i.e., $\rho = 0$. Employers would be violating the simple profit maximization rule if they did not employ Black workers with efficiency $e_i \in (e^0, e^*)$ in low-rank, low-income jobs. Hence the worst that can be done to such Black workers is to offer them low-rank contracts, which carry an income of $\hat{\omega}\hat{h}$, with $\hat{\omega}$, as noted above, being the labor *market-clearing wage rate* that is determined endogenously in general equilibrium. Thus an employer’s dilemma is not whether or not to offer a contract to such Black workers. Rather, who should be offered a higher-rank job contract and who should be offered a lower-rank one is the employers’ dilemma. And this, they solve, along racial lines, so that the talent of White workers gets rewarded, but that of Black workers does not, if any of these workers efficiency falls between e^0 and e^* .

²⁷ Thus, since this economy lacks a safety net, only workers with efficiency greater than e^0 will survive. Alternatively, if there is a state-sponsored safety net, then all persons with efficiency less than e^0 will be dependent on the disability-welfare program, regardless of race.

²⁸ While for a firm, ω is a parameter, for the economy it is an endogenous variable.

Hence, employers – both Black and White – will sign contracts with White workers for high-rank occupations, in all firms in all industries. Similarly, employers – both Black and White – will sign contracts with Black workers for low-rank occupations, in all firms in all industries. There will thus emerge income inequality along racial lines, with White workers earning higher incomes, $\widehat{w}\widehat{h} + \rho\theta$, $\exists\rho > 0$, than Black workers, who will earn income, $\widehat{w}\widehat{h}$.²⁹

Therefore, such a conjecture as *Axiom C*, along with the supermodularity axiom, will, in fact, become a self-fulfilling conjecture with support in general equilibrium, despite no actual link between talent and race or proficiency and race. An unjust occupational segregation by racial identity will hence be the stationary state outcome, and thereby the Pareto optimal social state will satisfy *Condition J*. ■

This theorem is unquestionably derisive of Pareto optimality as a principle worthy of pursuit on ethical grounds. Moreover, since such an unjust Pareto optimal outcome is the result of unrestrained, unregulated markets, the free-market ideology suffers a fatal blow, or loses its neocon luster, unless, of course, one favors the moral position that a person should be placed at a higher rank and paid a higher salary because of his or her gender, race, religion, ethnicity and the like, rather than rank and remuneration being based on merit.

More profound is Basu’s (2006) insight that there are actually multiple equilibria – some discriminatory, others not so – and it is possible to nudge or jolt an economy from a “bad” to a “good” equilibrium in some circumstances, but there are other circumstances, particularly if the nature of work performed is *habit forming*, in which more *prolonged* intervention that is *external* to the economy may be required for the removal of this injustice. It is not my purpose here to dwell on this matter, but the importance of Basu’s (2006, 2010) examination of this issue must not be underestimated.

6. Concluding Remarks

As we can see, the Arrow Debreu model, modified by adding *Axioms N* and *A*, is a very rich and powerful model that is capable of precipitating a general equilibrium outcome that is *endogenously* characterized by differences in wealth holdings, which are also associated with socially distinct persons. Here, in summary form, I describe how I meet the *attainable* Arrowian standards of a theory of racial segregation, and the manner in which I rely on the work of Basu. In addition, I address some issues of conceptual and empirical nature.

6.1 Arrow, Basu and the Proposed Theory

²⁹ This, of course, is the case with workers whose efficiency $e \in (e^0, e^*)$. All workers, both Black and White, will get high-rank jobs and earn higher incomes if their efficiency is greater than e^* .

Different employment categories within corporations were and are predominantly populated by persons of distinct races in the United States, but a credible theory that would explain this fact remains elusive despite the best efforts of economic theorists. This gap in economic theory was noticed and highlighted by Arrow (1998), who also set down four criteria and one specific suggestion, among other suggestions, for a credible theory of racial segregation by occupation within firms. These are:

1. Employers should be treated as simple profit-maximizers
2. Rational choice theory should characterize decision making by individuals
3. Introduction of new variables should be avoided so keep it from becoming tautology
4. Darwinian principles should *not* be neglected
5. Direct social interaction and networks appear to be useful concepts.

I provide in this paper what I believe to be a credible theory of racial discrimination that takes the form of segregation by occupation within firms. My theory meets Arrow's standards (1), (2) and (4) and (5), but it violates (3). I claim that (3) must be violated, otherwise all persons, without exception, would be perfect replicas of one another in terms of having the same social identity, and consequently no one would be able to know against whom to discriminate. This endogenous interpersonal homogeneity in an economy is an entailment of the restriction of binariness imposed on personal preference ranking relations. For the mere possibility of discrimination to arise as a phenomenon, it is necessary for the model of an economy to exhibit endogenous interpersonal heterogeneity, not homogeneity, of social identities. One way to achieve this is by the introduction of a new variable, to wit, a person's background set, on which the person's binary preference ranking relation is dependent, thereby rendering personal preferences (defined on the set of alternatives) non-binary in the Kanger-Sen sense.³⁰

Taking the lead from Basu (2006), I assume that workers have three characteristics, of which two can be observed by employers but the third cannot. Proficiency and race of a worker are observable, but the talent for creativity in signing more contracts with clients is not, though it is bounded from above. If a worker's proficiency is at least as high as a critical value that even with no talent the worker would generate a positive profit if hired, then employers offer the worker, regardless of the worker's race, a high-rank job that pays a talent premium in addition to the general equilibrium economy-wide wage rate of those who are presumed to have no talent. On the other hand, if a worker's proficiency is so low that even the highest possible talent level would result in

³⁰ This is one kind of binariness, but by no means every kind.

the employer's incurring a loss, the worker receives no job offer whatever.³¹ In the *non-trivial* intermediate-proficiency interval between the lower and higher critical values, employers face a dilemma because they have no way of knowing whether employing a particular worker is a profit-making or a loss-incurring proposition, arising from their incapability to discern a worker's talent level. Employers therefore do not know which worker in this intermediate-proficiency domain to offer a low-rank job that pays the equilibrium wage rate only and which worker to offer a high-rank job that also carries a talent premium.

Employers seek resolution of this dilemma by basing their conjecture of a worker turning a profit on the worker's race, which is an observable characteristic. Specifically, if they conjecture that White workers are more talented than Black workers, there exists a general equilibrium in which White workers receive high-rank job offers while Black workers receive low-rank offers, provided workers' talent levels are characterized by the supermodularity property. This occurs despite both Black and White workers actually exhibiting the same distribution of both proficiency and talent. That's my complete theory.

The force of Darwinian principles is muted because the combination of talent-level and proficiency of a worker makes for non-identically productive workers *ex ante*, among both Blacks and Whites. It is just that employers cannot observe the talent-level of a worker in either race. They do observe the proficiency and race of every worker. And the *relative frequency* distributions in population of proficiency, of talent, and *inter alia* of overall productivity of a worker in adding to profit income of employers, are all *independent distributions*. There is thus *no* pre-existing linkage between race and talent-level or between race and proficiency among workers. In such a set of circumstances, Darwinian forces do weed out certain expressions of collective racism, but they are unable to rule out racially segregated occupations within firms and other organizations.

Employers face a work force in which workers actually vary by profit-generating capability, but they cannot tell a profit-making worker apart from a loss-incurring worker in either race. The goal of the employers is solely to maximize profit. Yet the employer must determine which worker to offer a higher-paid high-rank job and which one to offer the lower-paid low-rank job. They must then form a conjecture about which worker is which. Basu (2006) shows that under supermodularity of the talent *levels*, the conjecture of employers that Whites are more talented in signing profitable contracts with clients than are Blacks is a self-fulfilling conjecture, in some

³¹ Thus does not survive in the absence of a safety network.

general equilibria. *Ex post* results a disparity in the Black workers getting stuck in low-rank jobs, which pay less, and White workers place in high-rank jobs, which are more lucrative. Of course, the super-proficient workers get high-rank jobs with high salaries regardless of race. But that is a tiny minority.

Consequently, fractional claims to profit streams of different firms, as in owning corporate equity, would reflect a distribution in favor of Whites. Recalling that $\forall i = 1, \dots, m$, α_i is the i^{th} person's vector of claims to profits of the n firms in the Arrow-Debreu model,

$$1 - \frac{\text{Number}(\alpha_i > 0 \rightarrow i = \text{White})}{m} < \varepsilon \ \& \ \exists \varepsilon > 0$$

would be true *ex post* in a general equilibrium.³²

Further, rational choice as an act of non-binary preference maximization of a quasi-ordered set of resource-constraint feasible alternatives characterizes every person. The set of alternatives is modified, however, by certain conjectures that have no actual basis *ex ante* in social reality and yet are powerful enough to shape social reality *ex post*. By contrast, in Arrow and Debreu, individuals maximize a binary preference ranking relation that completely orders the set of alternatives.

Replacing binary by non-binary personal preferences in the model of Arrow and Debreu is sufficient to retain individual-level rational choice foundations and to exhibit endogenous interpersonal diversity of social identity. This merely permits the possibility of racially segregated occupations within corporations in general equilibrium. It does not prove that a general equilibrium exhibits such discrimination. For that I employ the (1) supermodularity axiom on an unobserved worker characteristic and a (2) racist conjecture adopted by employers to aid them in the act of simple profit maximization. Together with the rest of the conditions, these two assumptions drive the existence of a general equilibrium in which jobs within firms are racially segregated.

Communities are networks. There is direct social interaction in that employers cannot help but observe every person's social identity and they take this into account in distinguishing the profit-producing from loss-incurring workers. Recall that this is despite any actual link between race and profit-generating capability of a person.

6.2 Conceptual Departure: Non-Binary Preferences

³² Especially if such an equilibrium state persists period after period.

The upshot of *Axiom N* is to broaden the informational content of the definition of a person compared to that in contemporary economic theory, so that endogenous social identity differentiation of persons in an economy becomes possible as an *entailment*. Indeed, alternative approaches to characterizing a person (as a decision maker) in economic theory can be subjected to informational analysis, by examining each approach in terms of the types of information that it admits and the types it excludes. For example, Arrow and Debreu employ one definition of a person who acts as a decision maker in their economic theory. While the utility function they ascribe to a person is unique up to a positive monotonic transformation (ordinal measurability), Nash has a stricter characterization insofar as the utility function he assigns to a decision maker is unique up to a positive affine transformation (cardinal measurability). Naturally, the *stronger* the restrictions placed on the characterization of a person, the greater is the information rendered *inadmissible* in describing a person. In turn, narrower will be the variability of the characteristics of any given person, and thus greater the extent of homogeneity *across* distinct persons.

On the non-binary relation *I* I only impose the requirements of *reflexivity* and *transitivity*.³³ I retain the assumption of Arrow and Debreu that every person maximizes his or her own preference on the feasible set, which, as in their model, is a subset of the Euclidian space, except that I take this feasible set to be *finite*.³⁴

For a certain class of problems in economic theory, the variation of social identity of persons in a group is inconsequential. An example is the existence result of Arrow and Debreu. They seek a vector of positive commodity prices (in a unit simplex) at which aggregate demand equals aggregate supply for each commodity. For this aggregation, it is not material whether and how much demand for corn, say, comes from a Black male and how much from a White female, so that to identify each individual in a group by a binary preference ranking relation is quite legitimate.

³³ Actually, acyclicity is sufficient, instead of transitivity, but it is not my purpose to produce the most general possible model, but merely to develop a model that can form the basis of an economic theory characterized by (a) the possibility of endogenous interpersonal social identity diversification, alongside (b) personal preference maximization, for which reflexivity and transitivity are sufficient, if the feasible set is finite.

³⁴ This is also a simplifying assumption, made again because the purpose is not to achieve the greatest possible generality, but, instead, identity diversification and individual rational choice behavior. If this set is taken to be infinite, or indeed dense, then, in addition to reflexivity and transitivity, I would also have to impose the restriction of *foundedness*, but that would constitute a technical generalization, not one that leads to additional insight into the economic phenomenon under investigation here.

However, there are other problems addressed by economic theory, such as racial or gender discrimination, religious or tribal conflict, and equally important, ethnic or community cooperation, or variation in the nature of cooperation in organizations across countries, for which the social identity of a person, and thus the diversity of identities across persons in a groups, are intrinsically material to the phenomena under investigation. For examination of issues in which social identity diversity of persons is operationally significant, it follows from my impossibility result that the characterization of individuals by binary preferences must necessarily be abandoned in favor of non-binary preferences. In such a case, the practice of identifying an individual with a utility function representation of a binary preference relation must also be forsaken.

I wish to emphasize that in my approach, preference, though non-binary, is construed strictly as fulfillment of desire or yearning in the *well-being* sense, rather than taken in the *choice-behavioral* sense, and I treat the maximization of this preference as *one of* the motivations that drives individual conduct. Sen (1994b and 1997) provides several contexts in which a person rationally maximizes one's own preference, while simultaneously violating the property of binariness of preferences.³⁵

6.3 Conceptual Departures: Positive Instrumental Value of Action Norms

I depart from the implicit assumption in economic theory based on Nash and on Arrow and Debreu that the instrumental value of social norms is zero in influencing individual behavior.³⁶ I replace this by the explicit assumption that there is a *positive instrumental value* (over and above the direct, constitutive value) of *social norms* in deliberately *restricting* an individual's instrumental possibilities further, so that the set of socially acceptable actions or strategies becomes a *proper subset* of the set of norms-independent feasible set of alternatives.³⁷ This may be called *Action Norms Axiom*, or *Axiom A*. *Axiom C*, which pertains to employers' conjecture that White workers are more talented than Black workers, is a special case of *Axiom A*. Non-binary personal preferences, *Axiom N*, and positive instrumental value of action norms in restricting individual instrumental possibilities are, *Axiom A*, new to economic theory.

³⁵ Sen (1997, p. 477-478) mentions (i) reputation and indirect effects, (ii) social commitment and moral imperatives, (iii) direct welfare effects, and (iv) conventional rule following, as possible motivations for such behavior. And, in the same spirit, he also develops a Fruit Passing game with common knowledge of norms (p. 762).

³⁶ Most analysts, following Nash (1951), also adopt this as an implicit assumption in the game theory literature, although this has come under sharp criticism by Sen (1985), on which more presently.

³⁷ See Sen (1985 and 1994a)

Axiom C is a special case of *Axiom A*, insofar as it gives specific content to *one* set of collective beliefs that effectively influence the character of the equilibrium outcome that exhibits members of one social group as more talented *ex post* than members of a distinct group, despite the absence of any link between talent and social identity *ex ante*. However, the specific content of *Axiom A* would not be *Axiom C* if an explanation of a different phenomenon – for example, intra-family wellbeing distribution or public debate and reasoning in national governance – involving heterogeneous social identities were the objective. The content of *Axiom A* is thus completely context dependent.³⁸

Axiom A relates to the functional, rather than intrinsic, value of norms. Indeed action norms do just this. They are social norms of conduct, with their own history and evolutionary paths, to which individuals feel obligated to conform, as requirements of membership of a community. Sen (1994a, p. 387) argues that if people follow,

the Kantian requirements of action morality ... “Act always on such a maxim as thou canst at the same time will it to be universal law”, then “the people involved ... could all end up having more fulfillment of their *unmodified* [personal well-being salient] preferences. ... Obviously, confessing oneself is not such an act in the [unrepeated] prisoner’s dilemma, since neither prisoner wants that behavior to be “universal.” The Smithian-Kantian *self-imposed restraints* differ from constraints given from outside.

It is, of course, possible to translate the impact of instrumental constraints – even self-imposed ones – into *reconstructed* “objective functions” ... But then the interpretation of the “objectives” would have to be correspondingly different ... The crucial question is not whether the “maximizing format” can continue to work in the Smithian-Kantian case (it certainly can), but whether there are reasonable arguments that support self-imposition of action norms, with a deliberate restriction of instrumental possibilities.³⁹

Such norms can increase individual wellbeing (as in an escape from Prisoners’ Dilemma by members of one community but not another) or worsen it for some persons (as in the outcome of discrimination), by deliberately restricting individual alternatives or strategies further, thereby making the socially acceptable set a proper subset of the norms-independent feasible set. That is the meaning of permitting the possibility of *positive* instrumental value of action norms, which I entertain as *Axiom A*, but which is taken to be *zero* in game theory and in the bulk of contemporary economic theory.

6.4 Non-Binary Preferences and Influential Action Norms

³⁸ For example, its content would be *Axiom C'*, that refers to collective beliefs that influence the *ex post* inequality in the distribution of wellbeing and freedoms among members of a family in a model of intra-family relationships, or it would be *Axiom C''* that pertains to conjectures regarding cooperation in state governance by democracy via public discourse in some countries but not others, among other context-dependent beliefs or conjectures. For example, Britain, post-WWII Germany, and some Scandinavian countries, among others, regularly exhibit public debate and reasoning in governance, but this not regularly evident in some other countries such as the United States, India, Ukraine and the Russian Federation, to mention a few.

³⁹ Italics in original.

A Kanger-Sen non-binary preference ranking relation is sufficiently richer in terms of its information content that it can accommodate *positive* instrumental value of action norms (over and above their direct, constitutive value, which can form part of the goals themselves) in promoting or eroding unmodified individual wellbeing, in addition to its *zero* instrumental value, which is implicitly assumed in traditional economic theory. Therefore, *Axiom A* is not incompatible with *Axiom N*, and thus I engage in no logical hara-kiri in taking both axioms on board.

Sen (1997) actually demonstrates two equivalent ways of modeling an individual who abides by social norms. One is maximizing a person's *binary* wellbeing-salient preference subject to self-imposed restraints, and the other is maximizing a person's *non-binary* ranking relation of wellbeing-salient preference, which is formally a binary relation that is conditional on the person's background set, as in late Stig Kanger's departure from binariness.⁴⁰ Despite the equivalence that Sen establishes for a *single* individual, it is important to take note of the fact that for a collection of two or more persons this equivalence does *not* carry through. Thus, if the object is to gain a better understanding of racial discrimination, tribal conflict, and the like, the binary preference approach gets trumped by the non-binary one, because the former cannot endogenously distinguish conceptually between any two persons on the basis of their distinct social identities.

Moreover, in economic theory every person is concerned solely with parametrically given prices in the market, and thus personal decision-making expresses itself as a person's direct interaction only with the market, not with other persons.⁴¹ Direct social interaction, between persons with distinct social identities, is, therefore, completely absent in economic theory.⁴² It would also be helpful to have a formal theory of social interaction, which takes the form of

⁴⁰ See Sen (1994b).

⁴¹ Arrow (1998) is clearly headed in this direction when he says (on p. 94),

The theoretical picture of a market is one of *impersonal* exchange. ... There is no particular relation between a supplier and a demander; that is, a supplier is indifferent about supplying one demander or another, and vice versa. ... Certainly, employment of labor involves *direct personal relations* between employee and employer (or the latter's agents) as well as among employees. Similarly, credit relations ... have typically required *direct personal interaction* between debtor and financial institution. ... Let us ask whether a market-based model can broadly satisfy the[se] empirical constraints. ... On the usual interpretation, it cannot. If the members of the two races, after adjusting for observable differences in human capital and the like, received different wages or were charged different prices in commodity or credit markets, an arbitrage possibility would be created which would be wiped out by competition.

⁴² When the auctioneer announces a finite vector of prices, agents make offers of supplies and demands for goods. If there is positive (negative) excess demand for a commodity in the aggregate, the auctioneer raises (lowers) the price of that commodity. On this see Arrow and Hahn (1971). Once such an iterative process leads to zero excess demands for all commodities, a Walrasian equilibrium is achieved, and only then do agents engage in trades at such prices. Agents in such an economy never really have any form of direct interaction, social or otherwise.

interpersonal interaction among social beings with multiple community affiliations, *in addition to* each person's interaction with the Walrasian auctioneer, as in Arrow and Hahn (1971). *Axiom N* permits that, and *Axiom A* gives it teeth. *Axiom C*, which is a special case of *Axiom A*, induces talent differentiation among persons who are distinguished by social identity.

While the talent of a worker is *ex ante* independent of the worker's social identity. However, the Basu-inspired *Axiom C* accomplishes a talent-based *ex post* differentiation of workers in the economy, into more talented and less talented workers, which is important because talent-induced productivity is the sole concern of firms that demand labor, because employers are simple profit maximizers, as in Arrow and Debreu, and as Arrow reminds us that any departure from this assumption "easily risks turning the "explanation" into a tautology." Behavior based on such beliefs that have no innate basis in social reality can actually produce a social reality that vindicates those beliefs. This is the upshot of conjectural general equilibrium theory.⁴³

6.5 Community Membership and Obligation to not Violate Action Norms

A person's identifying with or associating with distinct, though overlapping communities of other persons in society is not inconsequential. The collectivity of these *others* with whom we associate, due to one cause or another, with some of our social identities taking priority over others on different occasions, cannot but influence our own personal beliefs of what we believe to be reality, including, and especially *social reality*. What is more, the expressed values by others in the varied communities to which we belong colors our own notions of what is deemed *sacred* and what *profane*. Similarly, values of right and wrong, or of good and bad, lower or higher status, or what is or is not one's duty or obligation to do in a given circumstance, are operationally influenced solely because we belong to different communities. Association begets obligation. Obligation influences motivation. There are therefore three motivations that serve to rule out some elements of a person's feasible set as *socially unacceptable* individual behavior: (i) personal preference maximization subject to feasibility constraints and (ii) selecting, to a certain extent, a set of personal action-behavior norms from alternative sets of such norms, as restrictive obligations we are willing to abide by due to community affiliations, and (iii) some instrumental possibilities

⁴³ Seen from this perspective, the Arrow-Debreu equilibrium arises if all agents have exclusively *competitive conjectures* viz., that market prices are parametrically external to their decisions, even though such conjectures are, in fact, false, simply because it is impossible to mathematically add up a finite number of demand curves of all the firms that produce a commodity to arrive at a downward sloping industry demand curve for that product. But, that poses no problem because, Hahn would argue, Arrow and Debreu show that there exists, under specified conditions, a general equilibrium in which such conjectures are vindicated.

restricted by inexorable social identities of a person. Interpersonal interaction of such social creatures is social interaction. This relates to Sen's (1985, p. 345-346) claim that,

If the recognition that we can all better pursue our respective goals by jointly departing from [our] goal priority makes us do exactly that, why should that departure change the nature of the [unmodified] goals that we are trying to pursue? ... If taking everything into account, every member of a group does better in terms of the[ir] respective goals by following one type of behavior pattern rather than another, then that *is* a justification for the first pattern of behavior.

This forms part of Sen's (1985) critique of the behavioral foundations of game theory. Neglect of this latter consideration of choice of behavioral norms by persons in society has impoverished economic theory and rendered it asocial.

6.6 *Forms of Expression of Racial Discrimination in the United States*

Were it not for utter existential intolerance of any forms of social heterogeneity under the preference structure circumscribed by the restriction of binariness to every decision maker, including to consumption units in Arrow and Debreu or players in Nash, there would be no need to introduce any new variable. But the binariness property of preferences in their models is so inimical to social identity diversification that, in principle, for racial discrimination, or of any other conflict or cooperation to exhibit itself, it is necessary to introduce a new variable. I use $R_i(\mathbf{V}_i)$, as a non-binary relation of personal preference dependent on that person's background set, instead of only R_i in Arrow-Debreu and in Nash, without the background set \mathbf{V}_i that I include.

The operational issue turns not on whether a new variable should be introduced (it certainly does have to be for social differentiation), but it critically hinges on what exactly should be the properties of the set, \mathbf{V}_i called a person's background set in $R_i(\mathbf{V}_i)$. In Arrow and Debreu (1954) and in Nash (1951), for all persons or players, $\mathbf{V}_i = \bar{\mathbf{V}}$, turns out to be a special case of the Knager-Sen non-binary ranking relation $R_i(\mathbf{V}_i)$.

However, in the context of racial discrimination in the United States, for males, $\mathbf{V}_1 = \{\textit{African male}\}$ and $\mathbf{V}_2 = \{\textit{Caucasian male}\}$. This particular characterization of background sets would not apply at all if the interest were in examining gender discrimination, whether in the U.S., or elsewhere.⁴⁴ Individual conjectures would also have to be correspondingly different, in terms of substantive content, pertaining to genders, rather than relating to racial differences. Notice that the consequences for life expectancy differences would still be biased in the same direction,

⁴⁴ For instance, $\mathbf{V}_1 = \{\textit{male}\}$ and $\mathbf{V}_2 = \{\textit{female}\}$. The ratio of women to men in Sub-Saharan Africa is 1.01, USA, 1.03 whereas that in India is 0.92, for instance, in 2011.

in the distinct forms of racial and gender discrimination, caused by adverse social treatment of the aggrieved social group.

In fact, employers imbibe values from the collective beliefs that embody prejudice against Blacks in the formation of their conjectures. Unfortunately, however, upon examination of the imbibed beliefs, the simple-profit maximizing employers would find that, due to the force of such pernicious beliefs in society, the prejudicial beliefs are vindicated. This is because by reducing the incentives for African Americans to engage in education, training and self-improvement, they have *ex post* rendered vast numbers, though, of course, not all, of the African American community to actually exhibit lower productivity, thereby reinforcing the stereotype. After all, racial discrimination does manifest itself glaringly in terms of just shy of *one-and-a-half million missing African American males* in early 21st Century. It also expresses itself in terms of a 5 ½-year shorter life expectancy for African American males compared to White males in early 21st Century.⁴⁵ A higher incarceration rate of African Americans is another symptom of the prejudice against Blacks working through the social-economic-political system in the country, even today. Why would it not express itself also in terms of endogenous productivity differentials in the manner Basu (2006 and 2010) has suggested in a different context?

6.7 Shift in Approach to Identity-inclusive Economic Theory

The crucial point is that the ‘new variable’, which is a person’s background set, is dependent on the *context* of the issue under investigation. We ought not to look for some mathematical properties such as compactness or convexity, but rather treat background sets as context-dependent. In explaining a specific social-interaction or political-interaction phenomenon, the specification of background sets that are relevant to the community affiliations of the persons involved must bear a closer proximity to reality, as must the collective beliefs or conjectures material to relationships between the related groups, for the model to have greater predictive power. Thus on the one hand identity variation among persons opens up another degree of freedom in modeling behavior, on the other it comes with the burden for an economic theorist to show more responsibility as a careful social anthropologist or a political scientist. This is thus not a field for tweaking an axiom here or there to get additional results, for they may have little or no value in explaining social and political phenomena that have an underlying economic foundation in non-binary individual rationality.

⁴⁵ See Appendix.

Employers hire employees whose efficiency and talent are race-independent *ex ante*, but triggered by employers' conjectures, workers end up exhibiting *non-identical* race-based talent-induced productivity levels. Since workers end up becoming productivity-wise non-identical along a racial divide, the Darwinian principle Arrow refers to loses much of its cutting power.

I am actually advocating a move in the direction back from generality to particularity of modeling the specific phenomenon under investigation. In ascertaining aggregate demand for a commodity at any given price, it is irrelevant whether the demander is Black or White. Not so, however, if the object of investigation is racial discrimination versus ethnicity-based cooperation. The characterization of a person's background set is necessarily context dependent, and contexts happen to be particular and plural.⁴⁶

6.8 Identity and Violence

To summarize the entire discussion, existing collective beliefs that effectively mediate personal wellbeing maximization as a volitional act of choice by each of a finite number of players, by deliberately limiting individual instrumentalities, taken together with representation of the maximand by a Kanger-Sen non-binary ranking relation that quasi-orders a belief-independent feasible set of alternatives of choice, is the formal framework I present in this paper, whether taken as an exchange model or with production as in Arrow and Debreu. This model is considerably richer in terms of the information that it deems admissible in the characterization of an individual member of a group. The replacement of a binary by a non-binary ranking relation precipitates the possibility of social identity diversification of individuals in the group with overlapping community affiliations, which is disallowed by binariness.

Since each community affiliation obligates an individual to *not* violate specific types of action norms that the community deems unacceptable, individual instrumental possibilities become further restricted. By explicitly admitting the possibility that the instrumental value (over and above the constitutive value) of action norms is *not* zero – unlike the implicit assumption in

⁴⁶ This context-dependent background set in an analysis of racial discrimination in the United States is by no means the same as in the matter of gender discrimination in the world. The form it takes, the groups involved, the proportions of the population with distinct gender social identities, all vary greatly. Caste based conflict in India is another context completely distinct from both gender and racial discrimination, although all three, far too often, lead to the same disagreeable outcome, of making the lives of far too many persons considerably more morbid and shorter. In the pursuit of more and more general results in economic theory, we appear to have gone too far, in some respects, though not in all, in the direction of homogenizing persons by our assumptions to make our models incapable of addressing specific social injustices. Some back-tracking in the direction of context-dependent particularity seems to be necessary for addressing issues of specific social injustices and their remedies.

contemporary economic theory that it is zero – collective beliefs begin to have some cutting power in translating personal preference maximization into exhibited behavior. In some instances, the role of collective beliefs may be to engender cooperation and thereby enhance every player's own unmodified wellbeing, as in an escape from the Prisoners' dilemma. In other instances, collective beliefs can support conflicts that are abhorrently unjust social outcomes, such as discrimination, shorter life spans of some members of a group for reasons that are extra-biological and higher incarceration rates, to name a few.

Linkages between *wealth* and social *identity*, with concomitant socio-economic and political-economic inequalities, come to life *endogenously* in society, and thereby provide a simple explanation of the capture of differential rents by ethnically, racially, or other social-identity based distinct groups of persons in an economy.

Sen (2006) identifies *the illusion of destiny* of a person's unique identity that can breed hatred and violence, when fomenters of communal discord emphasize excessively such a solitary, divisive social identity of persons, *inter alia* devaluing a great many other social identities that the same persons actually share. Sen (2006) also argues that bloodshed need not, in fact, be the outcome if the spuriousness of this illusion of destined divisive identity is exposed through public discourse. He champions transparency and uninhibited public discourse in also serving to remove injustices of discrimination and killings inevitably inflicted by perpetrators of intolerance. I have tried in this paper to provide the outlines of a formal economic-theoretic framework in which these weighty matters can be discussed coherently. Much, of course, remains to be done, especially discovering how to make the removal of such injustices feasible.

Appendix

Racial Discrimination in the United States

According to the U.S. Census Bureau, given the populations in thousands in the last two columns of Table 1 below, it follows that in 2009 there were 917 African American males per one thousand African American females. However, per one thousand White females, there were 985 White males. Thus, there are 68 fewer African American males per 1,000 African American females than there are White males per 1,000 White females in the American population.

Table 1

1.5 million African American Males are Missing

	Female(1000s)	Male(1000s)	M/F Ratio
Black	21,808	19,996	917/1,000
White	125,391	123,528	985/1,000
B(M/F) - W(M/F) = -68/1,000		Black Males =21,808,000 x	
<i>Source: U.S. Census Bureau, 2009</i>		-0.068 = -1,483,000,000	

Table 2

Black males live 5 ½ years less than White males

L. E. (years)	Male	Female
Black	70.68	77.57
White	76.19	81.21
Life Expectancy: Black males – White males= -5.51		
<i>Source: Centers for Disease Control, Atlanta, USA, 2007</i>		

To determine exactly how many Black males would have been alive if Black women's *life expectancy achievement* were *also*, counterfactually, as high as that of White women, see

Table 3

Life Expectancy (years)	Female	Handicap Conversion Factor
Black	77.57	81.21/77.57=1.047
White	81.21	
Black Males Missing = -1,483,000,000x 1.047=		
-1.55 million		

As indicated in Table 1,⁴⁷ since there were in 2009, in terms of thousands, 21,808 African American females, multiplying this by 68 gives 1.483 million, or almost *1.5 million missing African American males* relative to the counterfactual case in which black males are as numerous relative to black females as white males are to white females.⁴⁸ That they are fewer than they should be, can also be confirmed by the life expectancy figures for 2007 in Table 2: African American males are expected to live, on average, to the age of 71, White males to 76, African American females to 78, and White females to 81. On average, in the U.S., “Black males” live 5 ½ years less than do “White males.” Why?

Anthropologists tell us that pure races are almost extinct: certainly in the U.S. in the first decade of the 21st Century, due to genetic mixing over more than two centuries, the answer must be positioned not in the biological domain but in the field of social treatment received by Blacks versus Whites.⁴⁹ This is an inescapable conclusion for this society at this time.

Notice that there are *three* distinct types of injustice here, and almost 1.5 million additional African American males would be alive today if only *one* of these injustices were removed, viz., the consequence of adverse treatment suffered by Black males relative to Black females when compared with the biologically-determined ratio of male-to-female populations in the U.S. For more on this see Sen (1992). While I do not pursue this important line of investigation here, notice that this figure of 1.483 million missing Black males would turn out to be an *underestimate* if the *second* injustice suffered by Black females relative to White females were also removed. One rough measure of this number would be to consider how many more Black males would have been alive if Black females, counterfactually, were to have life expectancy as high as that of White females: $(81.21/77.57) \times 1.483 = 1.55$ million, as in Table 3. The third adjustment would arise from accommodation of the lower life expectancy of American White females relative to the best

⁴⁷ For Life expectancy figures, see source: Table 103. U.S. National Center for Health Statistics, National Vital Statistics Reports (NVSR), Deaths: Final Data for 2007 Vol. 58, No. 19, May 2010, at <http://cdc.gov/NCHS/products/nvsr.htm#vol58/>. For population by race and gender, see <http://www.census.gov/compendia/statab/2012/tables/12s0006.pdf>.

⁴⁸ These are Census Bureau figures, and they include those persons – Black and White, male and female – who are incarcerated. Moreover, consider the following counterfactual question. In the U.S., relative to females of the same race, if Black males were accorded equally favorable treatment as are White males, how many additional Black males would there be? Answer: one million, four hundred and eighty-three thousand *additional* Black males should have been alive, but are not.

⁴⁹ The categories ‘Black’ and ‘White’ in reference to persons are human constructs, not innate in biological nature, so much so that the very definitions vary from country to country. In the U.S., a person is considered Black if she has one drop of ‘Black blood’ whereas in Brazil a person is deemed White if she has one drop of ‘White blood’. In South Africa there is a four-part classification in descending order of social valuation: Whites, Asian-Indians, Coloreds, and Blacks. Thus, most Whites in Brazil and all Coloreds in South Africa would be classified as Blacks in the U.S.

international standards of women's life expectancy, such as in Norway, which has consistently ranked among the very top countries in terms of the Human Development Index.

In the model I have proposed, there can be multiple equilibria supported by distinct rational conjectures in the manner of Basu (2010), thereby implying the existence of equilibria in which, *ex post*, African American males exhibit lower talent-induced productivity, lower life expectancy and lower peak-median income, not to mention a disgracefully higher incarceration rate that African Americans males suffer than do White males.

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