A Small Model for Output Gap and Potential Growth Estimation: An Application to Bulgaria

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BEP 04-2015
Publication: April 2015
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April 9, 2015

Abstract

This paper presents the logic and structure of a small and parsimonious macroeconometric model designed for output gap and potential growth estimation in a data-poor environment. Such results can be useful in calculating the cyclically adjusted budget balances which are a key indicator for fiscal policies design in the framework of the EU Stability and Growth Pact. Empirical results using Bulgarian data are also included for illustrative purposes.

Keywords: business cycle, output gap, potential growth

JEL Classification: E32, E37

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1 Introduction

The concepts of ‘potential output’ and ‘output gap’ are among the key ones related to business cycle analysis. This seems natural since aggregate output is one of the most relevant indicators as far as economic growth and macroeconomic fluctuations are concerned. Also, it relates to the fact that output growth is among the most important goals of economic policies.

By saying this, an explicit note should be made that the output gap as a concept specifically is far from being uncontroversial. Real business cycle theory, for example, claims that it simply does not exist. This is so because in its constructs all aggregate economic fluctuations are Pareto-optimal, i.e. they cause no inefficiency. Therefore, from the policy perspective, there is nothing to stabilize or correct. As far as potential output is concerned, all actual developments correspond to a full-employment state, i.e. the macroeconomic potential is automatically achieved.

At the same time, New Keynesian theory takes seriously the notion of the existence of the output gap. From its perspective, aggregate demand shocks do cause inefficient short-term macroeconomic fluctuations (e.g. due to the presence of nominal and real rigidities in the economy). The output gap is the manifestation of those fluctuations, i.e. it is representative of the phenomenon of the business cycle.

The output gap can be both positive and negative, respectively when aggregate demand exceeds or falls short of aggregate supply. When it is positive, there is involuntary unemployment and/or idle production capacity; when it is negative, unemployment is lower than its ‘natural’ level, and production capacity is overloaded. In either case there is inefficiency as the economy deviates from the Pareto-optimal distribution of resources obtainable only at full employment. Inefficiency calls for government policy responses and justifies fiscal and monetary action aiming stabilization around the full-employment long-term equilibrium path which is associated with potential output.

In the EU context, monetary policy is, as a rule, in the prerogatives of the ECB and the ESCB. Countries outside the EMU like Bulgaria have committed to join it and adopt

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1 The simple fact that the present paper considers the output gap as existing therefore implies the implicit adoption the New Keynesian ideas.

2 Broadly speaking, potential output is the quantity that the economy can produce while utilizing optimally all production factors. The achievement of potential output means that there is no involuntary unemployment or over-employment and there is no idle of overloaded production capacity.
the Euro so no independent monetary action is foreseen for the future.\textsuperscript{3} In the Bulgarian case, even at present, the monetary policy reactions are almost impossible due to the currency board arrangement, so only fiscal actions are feasible. As far as fiscal policy is concerned, it can have three possible stances with respect to the business cycle: expenditure and tax policies have the potential to decrease, increase or be neutral to the inefficiencies related to the presence of the output gap. It is straightforward, though, that from the normative perspective fiscal policy should be either neutral or counter-cyclical so that inefficiencies are not augmented by policy action and are possibly reduced.

Following the requirements of the Stability and Growth Pact, all EU Member States are required to develop analytical capacity and toolkits for estimating potential GDP growth and the output gap. The requirement relates directly to the elaboration of the short- and medium-term fiscal policies, which is formalized in the respective stability or convergence programmes – the former prepared by EMU countries, and the latter by countries which have not yet adopted the euro.

In some New Member States, such as Bulgaria, analytical capacity in this respect was underdeveloped before accession. Therefore, estimates of the output gap and potential GDP were either not reported in the so-called Pre-accession Economic Programmes (which were the forerunners of the Convergence Programme), or the output gap was calculated somewhat superficially, e. g. by means of simply removing an HP trend from the real GDP series which does not allow to identify the factors contributing to potential growth.

This paper documents the features of a basic modelling tool developed by the present author in 2006-2007 at the Bulgarian Agency for Economic Analysis and Forecasting.\textsuperscript{4} This model is still being used by government administration and some independent economists, including for policy analysis, for defining the targeted budget balances over the medium term, and in the elaboration of the annual updates of the Bulgarian Convergence Programme.

\textsuperscript{3}The notable exceptions are the UK and Denmark which have their opt-outs.

\textsuperscript{4}The Agency for Economic Analysis and Forecasting (AEAF) was the institution which was responsible, amongst other duties, for the elaboration of the Pre-accession Economic Programmes and the Convergence Programme of the Bulgarian government until 2009. It was closed by a decision of the Council of Ministers in the summer of 2010, and a part of its functions and staff were transferred to the Ministry of Finance.
2 A brief review of the basic approach adopted

While practically all modern monetary business cycle models can produce results on potential GDP growth and the output gap, in many occasions their development is not feasible due to staff and time limitations in government administration. For example, dynamic stochastic general equilibrium (DSGE) tools for business cycle analysis require devoting a considerable part of the limited amount of staff for a non-negligible period of time, and also a high level of expertise in designing, maintaining, and running them so that meaningful results are produced. These constraints lead to allowing some compromise in the approach used. In order to have only the selected indicators estimated while foregoing the option of having a fully-fledged model capable of policy analysis and simulation, small-scale specialized but accessible models have to be used.

The model presented here broadly follows the framework commonly adopted at the EU level,\(^5\) thus it is based upon the so-called production function approach.\(^6\) The functional form is Cobb-Douglas, and production factors are employment, physical capital, and technological development. The model considers only the supply side, therefore it is an incomplete representation of the economy. As a consequence, it is only suitable for positive analysis, and should be avoided as a tool to simulate policy or to make forecasts.

The idea behind estimating the output gap and potential growth via this approach is that the same production technology delivers different results depending on the rate of utilization of the production factors: efficient or ‘normal’ utilization is only assumed when GDP is at its potential level, while any other rate is considered inefficient. Also, it also assumes that all unexplained influences are represented as technology shocks, therefore long-term technological development is the one associated with full employment.

The concept of ‘normal’ is quite broad for empirical usage. Therefore, a practical measure of it is necessary. Deriving such a measure draws on the idea that over the business cycle the values of the macroeconomic variables fluctuate around their long-term (equilibrium) paths. In particular, GDP fluctuates around its potential level (which

\(^5\) Described in detail in Denis, Mc Morrow, and Röger (2002), and further extended in Denis, Grenouill-leau, Mc Morrow, and Röger (2006). The purpose of having a common method addresses both feasibility of development and inter-country comparability of results.

\(^6\) For a review and application of other approaches to the economy of Bulgaria, see for example Ganev (2004).
is non-constant but has its own dynamics manifested in potential growth), and thus
the output gap turns out to be the difference between actual real output and poten-
tial output. The same logic pertains to variables such as labour supply, labour demand,
consumption, investment, etc.

The main modelling task then boils down to ‘splitting’ actual time series into their
long-term (trend) and short-term (cycle) components. In the current case it is assumed
that extracting one of the components (the trend) allows to automatically compute the
other (the cycle) as a residual.\footnote{Thus the cyclical component contains also the purely random disturbances.}

3 Structure of the model

3.1 Main equations

The supply side of the economy is described with the following macroeconomic produc-
tion function which is Cobb-Douglas with Hicks-neutral technological change:

$$Y_t = A_t (K_t)^\alpha (L_t)^\beta,$$

where $Y_t$ is real output (real GDP) at time $t$, $L_t$ is labour employed in production (aggregate employment), $K_t$ is physical capital, $A_t$ is the level of technological development, and $\alpha$ and $\beta$ are parameters signifying the respective output elasticities. The function complies with the neoclassical assumptions, in particular it is characterized with constant returns to scale (i.e. $\alpha + \beta = 1$).

Physical capital $K_t$ is assumed to develop according to the following relationship:

$$K_t = I_t + (1 - \delta)K_{t-1}$$

where $I_t$ is gross fixed capital investment, and $\delta$ is the depreciation rate.

The level of technology $A_t$ (known also as total factor productivity, TFP) is an un-
observable quantity. One of the most frequently used empirical approaches is to obtain
it from GDP data as a residual.\footnote{This approach was proposed by R. Solow therefore the measure is also known as ‘the Solow residual’ (Solow 1957).} It is calculated using the so-called growth accounting technique, and essentially reduces to the following equation:

\[
\frac{\Delta A_t}{A_t} = \frac{\Delta Y_t}{Y_t} - \alpha \frac{\Delta K_t}{K_t} - \beta \frac{\Delta L_t}{L_t}
\]

(3)

What this equation tells is that TFP growth is equal to the unexplained (by capital and labour growth) part of output growth.

In order to be able to use equation (1) to portray both potential and actual output, it is useful to modify it somewhat by introducing rates of utilization of the production factors. It does not make much sense to think that in some periods technology is either partially or excessively used so such utilization rates are logical to introduce only for capital and labour. The modification would look like:

\[
Y^a_t = A_t(\theta_1 K_t)^\alpha(\theta_2 L_t)^\beta,
\]

(4)

where the utilization rates $\theta_1$ and $\theta_2$ are positive numbers. When they are both equal to 1, then potential output, $Y_f^t$, is achieved. As it is implied from the practical viewpoint, those rates are restricted to a finite interval much narrower around 1 than $(0, +\infty)$ but it can be only empirically established as theory alone does not give a normative prescription.

### 3.2 Practical adjustments of the common EU approach

The capital stock in an economy is an observable quantity but it is rarely observed in practice, if at all, by national statistical offices. The Bulgarian National Statistical Institute (NSI), in particular, does not produce such figures. Therefore, the entire series concerning this variable needs to be additionally constructed using the available information from the national accounts. Gross fixed capital investment figures are there, but the recursive nature of equation (2) requires a starting value to base the calculation on. Usually, the approach used in this respect, i.e. to calculate the initial capital stock $K_0$,
is that of Hall and Jones (1999), which, formally stated, is:

\[ K_0 = \frac{I_0}{\delta + g} \]  \hspace{1cm} (5)

where \( g \) is the average growth rate of investment over a longer period of time, i.e. it approximates the equilibrium rate of investment growth, and \( I_0 \) is the amount of investment in the initial period. In the Bulgarian case \( g \) is highly volatile ranging from -20% to about +69% over the period 1991-2014, thus it is not straightforward to approximate the long-term rate with the simple average. On the other hand, the breaks in the (not so long) series due to changes in the data compilation methodology are not in favour of applying a more sophisticated method to find the long-term investment rate. Therefore, a variant of (5) assuming \( g = 0 \) is adopted, i.e.:

\[ K_0 = \frac{I_0}{\delta} \]  \hspace{1cm} (6)

suggesting equality between \( K_0 \) and \( K_{-1} \). Such an assumption cannot be duly assessed with respect to its realism, as 1990 is the first year in the data series, and there are no preceding observations compiled using the national accounts methodology.\(^9\) Nevertheless, as far as only current and future periods are of importance, this assumption is far from being crucial as its influence gradually vanishes over time.

Concerning equation (2), as noted in Ganev (2005), using simple recursion to obtain the consecutive values of the capital stock might result in an infinitely-lived capital. To illustrate the issue, take \( K_0 \) to equal 100 units, and assume no investment thereafter. At a depreciation rate of \( \delta \in (0, 1) \), after \( t \) periods of time \( K_t \) will equal \( (1-\delta)^t K_0 \). When \( t \to +\infty \) it will become smaller and smaller but will never depreciate fully. Abstracting from its decreasing volume, this leads to the illogical result that capital can stay eternally productive. Therefore, here we also apply the following adjustment in order to allow capital fully depreciate over a specified number of periods:

\[ K_t = \sum_{i=0}^{n-1} (1-i\delta) I_{t-i} + (1-n\delta)K_{t-n}, \quad n = 1, \ldots, t \]  \hspace{1cm} (7)

\(^9\)The preceding years’ data was produced using the Material Product System of the former Eastern Bloc.
For \( n > t \) it is assumed that \((1 - n\delta)K_{t-n} \equiv 0\); for \( n > t + 1 \) both \( K_{t-n} \equiv 0 \) and \([1 - (n - 1)\delta]I_{t-n+1} \equiv 0\); etc. In other words, initial capital and investments cannot become negative after they fully depreciate.

### 3.3 Trend-cycle decomposition

Denis, Grenouilleau, Mc Morrow, and Röger (2006) apply Kalman filter estimation to an unobserved-components model to extract the NAIRU, and the Hodrick-Prescott (HP) filter to extract trend TFP and trend labour force. Correspondingly, the cyclical components are then found directly as residuals.

The unobserved-components models tend to pose a challenge for government administration in countries like Bulgaria, for mainly two reasons. First, the quantity and quality of data do not provide sufficient comfort to perform the estimation and obtain trustworthy results. Second, competitive disadvantages in the labour market pose the risk of not being able to attract and retain in government service the necessary qualified staff. Therefore, the presently used approach relies entirely on the HP filter, i.e. the NAIRU is also estimated using it. Thus, greater accessibility of the model is provided.

The HP filter (Hodrick and Prescott 1997) is readily programmed in most of the econometric packages and its application is straightforward. The trend extraction is performed via minimization of the following expression with respect to \( \tau \):

\[
\sum_{t=1}^{T-1} (x_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} (\tau_{t+1} - 2\tau_t + \tau_{t-1})
\]

where \( x_t \) is the original series from which the HP trend is extracted, \( \tau_t \) is the trend itself, and \( T \) is the sample size. Usually \( \lambda \), the smoothing parameter, is set to equal 100 when annual data are used. This is also the value used in the present study.

### 4 Data and their transformations

The historical span of the dataset utilized in the empirical application of the model covers the period 2003–2014. It is determined by data availability and consistency. The
frequency of the data is annual, and the source for all series used is the National Statistical Institute of Bulgaria. GDP and investment are taken at 2010 constant prices.

4.1 Labour market data

Labour supply in the present model is identified with the labour force (denoted by $L_s^t$). Data on this variable are directly available from NSI, specifically from its Labour Force Survey (LFS). While these are the actual realizations of labour supply, in order to calculate the potential GDP, the labour force level which corresponds to it (potential labour supply) is needed. This is accomplished by making use of the participation rate concept which is represented by the following ratio:

$$\psi_t = \frac{L_s^t}{N_w^t}, \quad (9)$$

where $\psi_t$ is the participation rate, and $N_w^t$ is the working-age population at time $t$. The full-employment level of the participation rate, $\psi_f^t$, is obtained after HP-filtering the original $\psi_t$ series. Then the full-employment level of the labour force, $L_s^{s,f^t}$, is found through:

$$L_s^{s,f^t} = \psi_f^t \cdot N_w^t \quad (10)$$

Unemployment figures are also available from the LFS. Their HP-filtered counterparts give estimates of the NAIRU. If the latter is denoted by $u_f^t$, then effective potential employment (labour demand), $L_l^{f}$, equals:

$$L_l^{f} = (1 - u_f^t) \cdot L_s^{s,f^t} \quad (11)$$

All labour market figures used in the empirical calculations relate to the population at the age of 15-64 years.
4.2 Capital stock

The capital stock is calculated using equation (7). Investment is assumed to equal gross fixed capital formation, available from the national accounts. In order to eliminate the influence of the initial capital stock in the most recent periods of the data sample, its value is calculated for 1990. The depreciation rate is assumed to remain constant at 5% annually for the whole period 1990-2014.

The obtained capital stock series turns out to be a smooth one so there is no need to extract a long-term component. This comes from the fact that the formula ‘compounds’ investment; also, viewed from a different perspective, the capital stock is a weighted average of all the available past data on investment, so it is already a ‘long-term’ variable.

Figure 1: Capital stock in the Bulgarian economy (BGN mn)

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10 There is a structural break in the gross fixed capital formation series in 1996 due to data revisions and changes of methodology but with the available information no estimate of its influence on calculation outcomes can be provided.

11 Filtered capital series are also difficult to interpret.
4.3 Total factor productivity

For the purposes of the model it is not really necessary to utilize formula (3) to obtain TFP growth. Instead, as TFP is only needed as an index, the latter is calculated after taking natural logs of both sides of equation (1) and rearranging terms:

\[
\ln A_t = \ln Y_t - \alpha \ln K_t - (1 - \alpha) \ln L_t
\]  

(12)

where \( \alpha \) (the capital share) is taken to equal 0.35.\(^{12}\)

As already mentioned, the natural log of trend TFP (denoted by \( \zeta_t = \ln A^f_t \)) equals the HP value of the original series obtained in (12).

4.4 Forecasts of variables

Instead of using the approaches outlined in the reviewed literature which prescribe purely econometric tools of forecasting (such as ARIMA techniques), here we turn to the macroeconomic forecasts as generated by the main models of the government administration. The Bulgarian government forecasting model is, generally speaking, a VBA-powered spreadsheet realization of IMF’s financial programming approach. To solve the model, Excel’s built-in solver is utilized. A version of this spreadsheet model has been used here, too, to generate forecast figures for the current exercise. External assumptions are taken from IMF’s latest World Economic Outlook database (October 2014). In this way, the current output gap model would act as a satellite model to the main forecasting one, and thus consistency of the results would be achieved.

Since the HP filter is a symmetric one (the future and the past enter with equal weights in the calculation of the trend), it tends to perform relatively poorly at sample endpoints. Therefore, it is a good idea to extend the sample at least one period beyond the period of interest (e. g. by generating a forecast value for one step further). In the present context, the forecast period extends to 2020.

\(^{12}\)This value is calibrated following estimates widely used in literature which attribute to capital a production elasticity of about 1/3.
4.5 Potential GDP growth and output gap calculation

Potential GDP is found after combining the relevant pieces of obtained results, i.e.:

\[ Y_f^t = \exp(\zeta_t)(K_t)^{\alpha}(L_t^f)^{1-\alpha} \] (13)

The calculation of potential GDP growth is then trivial. Finally, the output gap \( \gamma_t \) is found via:

\[ \gamma_t = \frac{Y_t - Y_f^t}{Y_f^t} \times 100 \] (14)

This is a magnitude expressed as a percentage, therefore it is interpreted as the percentage deviation from full-employment GDP. When it is positive, it relates to an overheating economy, and when it is negative, it relates to an economy in recession.

5 Estimation results for Bulgaria

The results from running the model\textsuperscript{13} correspond logically to the facts observed throughout the years. While it is far beyond the objective of this article to analyse in depth the developments of the Bulgarian economy, it is illustrative to mention some selected facts in order to broadly assess the usefulness of the model.

The initial years in the sample (2003 and 2004 in the present case) can be ignored due to a possible bias resulting from the HP filter, but the figures obtained for the next several years clearly match the dynamics of a booming economy.

The results show that in 2005 the economy is slightly below its potential (the output gap is negative at -0.6%) but, as Figure (3) indicates, the short-term growth of output is very high (the actual GDP growth rate for the same year was 6.0%). In the following year the output gap becomes positive (1.44%), increasing strongly to respectively 4.1% and 5.2% in 2007 and 2008. Actual GDP growth rates in 2006-2008 were 6.5%, 6.9%, and 5.8%.

\textsuperscript{13}The input dataset and the R code are provided on request.
That the economy was overheating was corroborated by several indicators. The HICP annual average inflation rate in 2006-2008 equalled correspondingly 7.4%, 7.6%, and 12.0%. The unemployment rate fell to record-low values in 2007 and 2008: 6.9% and 5.6%. In the same two years, the current account deficit reached almost a quarter of GDP.

The recession of 2009 is matched by a negative output gap in the same year and the following, including the last historical figures for 2014. Again, the values of unemployment rates, inflation, and foreign sector variables fully correspond to the estimation results. Moreover, the output gap is closing slowly over the forecast horizon which reflects the sluggish recovery and the weak prospects for the future.

Potential growth figures are also in line with the general observations on economic developments. As Figure 4 shows, rates are high before the recession (from 2005 to 2009 in excess of 4%), and they drop abruptly to levels slightly above 1%, tending to remain there for long. In broad terms, this is a reflection of investment activity: gross fixed capital formation had shares of up to 40% before the crisis, and those shares receded to fall
short of a quarter of GDP in the following years. At present, no significant acceleration in investment activity is projected until the forecast horizon.

6 Conclusion

It is beyond doubt that real-time output gap estimates based on techniques using symmetrical filters are rarely precise, if at all.\textsuperscript{14} This stems from the fact that to calculate them, forecast information is used, and therefore all forecast uncertainty is directly transferred to the output gap estimates. The present model and the results that it produces are hardly an exception. Nevertheless, at least in qualitative terms the estimates can serve as a guideline – e. g. to not take the wrong direction of government action.

The model presented here could be a useful tool, especially in a data-poor environment. It can be helpful in summarizing and formalizing short-run economic develop-

\textsuperscript{14}More or less the same thing can be said about any estimate of the output gap, no matter what approach is applied.
ments which, as already briefly discussed, are visible in many aspects in the raw data.

Of course, it can be only a transitory tool, and fully-fledged macroeconomic frameworks in which output gap and potential growth calculation is integrated are always preferred. The gradual data accumulation and the slow but definite process of the establishment of a new generation of economists having up-to-date training in economic methodology would hopefully bring about better tools to serve policy analysis in Bulgaria and countries like it.
References


Appendix

Table 1: Input data

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<th>Year $t$</th>
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<th>$Y_t$</th>
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<td>18511</td>
<td>10.4</td>
<td>73.3</td>
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</table>

$L_t$: Employment, 15-64 years (thousands)
$Y_t$: GDP at 2010 constant prices (BGN mn)
$I_t$: Gross fixed capital formation at 2010 constant prices (BGN mn)
$u_t$: Unemployment rate, 15-64 years (%)
$\psi_t$: Participation rate, 15-64 years (%)
$N^w_t$: Working-age population, 15-64 years (thousands)
Table 2: Estimation results

<table>
<thead>
<tr>
<th>Year</th>
<th>$Y_t^f$</th>
<th>$\gamma_t$</th>
<th>$g_t$</th>
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<td>2003</td>
<td>58262.25</td>
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<td>65570.79</td>
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<td>1.09</td>
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</table>

$Y_t^f$ Potential GDP at 2010 constant prices (BGN mn)
$\gamma_t$ Output gap (%)
$g_t$ Potential GDP growth (%)