Доклади на Българската академия на науките
Comptes rendus de l'Académie bulgare des Sciences

Tome 71, No 12, 2018

ENGINEERING SCIENCES

Systems theory

SOLVING AN EMOTIONAL MEMORY EQUATION

George Mengov, Trifon Trifonov*, Kalin Georgiev*

(*Submitted by Academician M. Hadjiiski on July 4, 2018*)

Abstract

To understand how the brain gives rise to emotional human behaviour, modern neuroscience extensively uses computer modelling. The differential equation for long-term memory (LTM) is an important element of a variety of cognitive-emotional models, including the Grossberg–Schmajuk recurrent gated dipole. Usually, such models are composed of coupled ordinary nonlinear differential equations with no analytical solution in the general case. Here we solve analytically a special case of the LTM equation as used in the gated dipole. In this version, the equation contains three Boolean variables, accounting for internally generated and externally induced emotions. We develop a software application showing the solution's usefulness.

Key words: Recurrent gated dipole, long-term memory differential equation, emotional memory

Introduction. Human emotional memory can be created by one's own experiences and by accounts of other people's experiences. A mathematical model of emotional memory should be able to accommodate both sources. Such a need arises when, for example, consumer behaviour must be understood and possibly predicted. A consumer might feel satisfaction with a good or service when dealing with it on their own, yet may also be influenced by other people's opinions about it. Putting these separate influences in one equation is a challenge, which can be addressed step-by-step by starting with controlled laboratory studies. Recently, such research was conducted in Sofia University's Faculty (School) of Economics

The first author is supported by the National Science Fund of the Republic of Bulgaria, Project DN 15/9: 12-11-2017, The Synergy between Process Philosophy and Elements of AI in Theory of Learning.

DOI:10.7546/CRABS.2018.12.12

and Business Administration. Participants in an experiment were given the task to accumulate a fictitious good *omnium bonum*, exchanged for real money after the game. In each round they had to choose one among four suppliers (A, B, C, and D) of the good behaving differently. For additional orientation, the participants received a Twitter-like stream of posts by other people who shared opinions about the suppliers. This situation involves an internal and an external source of consumer emotions, and can be modelled by the Grossberg–Schmajuk recurrent gated dipole [1]. The latter is composed of ordinary nonlinear differential equations for neural activations, neurotransmitters, and long-term memory. However, that model was initially designed to deal with a single source of emotion. In this paper we modify it to receive updates from two streams. To this end we develop a version of the LTM equation containing three Boolean variables, switched on and off in unison with the source of emotional signal. Modifying the dipole LTM equation to do multitasking and finding its analytical solution is the main contribution of this paper.

Technical issues in neurocomputational modelling. Mathematical models of cognition such as the gated dipole are usually composed of stiff differential equations whose solving is problematic. Often, finding suitable simplified equations and solving them analytically is a preferred option. This approach is especially useful with complex models where time scales span over two or three orders of magnitude. In the past, gated dipoles and related neural networks have been solved by combinations of numerical methods [2–5], various simplifications [6], or combinations of simplifications and analytical solutions for special cases [7, 8]. Of particular interest is the long-term memory equation [9] describing the emotional memory in the recurrent gated dipole, used as the main model in the omnium bonum experiment [10]. In the next sections we give a derivation of the solution and show a numerical example.

Emotional memory equation. This is the equation as used to model the experimental data:

$$\frac{dz_{ik}(t)}{dt} = x_k(t)\left[-h_1 z_{ik}(t) + u_k(t) h_2 o_i(t) + \bar{u}_k(t) h_2 \tilde{o}_{ik}(t)\right]. \tag{1}$$

Here, $z_{ik}(t)$ is long-term memory, storing in two synapses the individual's positive and negative emotions. Index $k = A, B, C, D$ denotes the source of the signal. Index $i = 1, 2$ denotes satisfaction or disappointment, respectively. This human reaction may be a response to an event with the participant, in which case it is denoted by $o_i(t)$, or it may be communicated by other people in the experiment, in which case it is denoted as $\tilde{o}_{ik}(t)$. Variable $x_k(t)$ is Boolean with value 1 when a particular $z_{ik}(t)$ is active and zero otherwise. Similarly, $u_k(t)$ is a Boolean variable equal to 1 whenever the k-th source of activity is associated with internal emotion $\tilde{o}_{ik}(t)$, and is zero when the emotion comes from other people. Variable $\bar{u}_k(t)$ is the negation of $u_k(t)$. Quantities h_1, h_2 are constants.

Due to its nonlinearities, Eq. (1) does not admit analytical solution in the general case. Simplifications must be introduced to solve it in a meaningful special case. Eq. (1) is an extension of Eq. (2) below, accounting only for internally generated emotions:

$$\frac{dz_{ik}(t)}{dt} = x_k(t)\left[-h_1 z_{ik}(t) + h_2 o_i(t)\right]. \tag{2}$$

A solution to Eq. (2) can be directly extended to a solution of Eq. (1). While the entire recurrent gated dipole is composed of three kinds of differential equations, a substantial number of them are of the Eq. (1) kind. Here we deal with the latter only.

Every two seconds a new message from other participants arrives through the Twitter-like network and is suitably highlighted for 400 ms. That timing is designed to ensure adequate attention from the participant. While there is no guarantee that each message is attended, this is a simple way to reconcile reality with modelling constrains. Term $\bar{u}_k(t)h_2\tilde{o}_{ik}(t)$ in Eq. (1) accounts for the 400 ms window of attention to the stream of messages.

First we solve Eq. (2) under certain simplifying assumptions. Then we solve Eq. (1). Under the assumption $o_i =$ const, Eq. (2) has this solution:

$$z_{ik}(t) = \frac{h_2}{h_1} o_i + C_1 \exp[-t h_1 x_k(t)]. \tag{3}$$

When $t \to \infty$ the value of $z_{ik}(t)$ asymptotically approaches $o_i h_2/h_1$, which depends only on the constant signal o_i. Let at $t = t_0$ signal o_i jump and remain constant. The solution Eq. (3) can be rewritten as

$$z_{ik}(t) = \frac{h_2}{h_1} o_i^{\text{new}} + C_1 \exp[-(t - t_0) h_1 x_k(t)]. \tag{4}$$

By o_i^{old} we denote the signal value before the jump, and by o_i^{new} its value after the jump. Consider a very small time interval τ, which is split in half by the jump. Here, $0 < \tau \ll 1$. Just before the change, memory $z_{ik}(t)$ was

$$z_{ik}(t_0 - \tau/2) = \frac{h_2}{h_1} o_i^{\text{old}}. \tag{5}$$

From Eq. (4), at moment $t = t_0 + \tau/2$, memory $z_{ik}(t)$ becomes

$$z_{ik}(t_0 + \tau/2) = \frac{h_2}{h_1} o_i^{\text{new}} + C_1 \exp[-(t_0 + \tau/2 - t_o) h_1 x_k(t)] \approx \frac{h_2}{h_1} o_i^{\text{new}} + C_1. \tag{6}$$

To find C_1, we recall that changing $z_{ik}(t)$ is a slow process and

$$z_{ik}(t_0 - \tau/2) = z_{ik}(t_0 + \tau/2). \tag{7}$$

In Eq. (7) we substitute both sides for the rhs-s of Eq. (5) and Eq. (6) to obtain

$$\frac{h_2}{h_1} o_i^{\text{old}} = \frac{h_2}{h_1} o_i^{\text{new}} + C_1. \tag{8}$$

The integration constant, therefore, is

$$C_1 = \frac{h_2}{h_1}(o_i^{\text{old}} - o_i^{\text{new}}). \tag{9}$$

Substituting the rhs of (9) in (4) gives

$$z_{ik}(t) = \frac{h_2}{h_1}o_i^{\text{new}} + \frac{h_2}{h_1}(o_i^{\text{old}} - o_i^{\text{new}})\exp[-(t-t_0)h_1x_k(t)], \tag{10}$$

which can be rewritten as

$$z_{ik}(t) = \frac{h_2}{h_1}o_i^{\text{old}}\exp[-(t-t_0)h_1x_k(t)] + \frac{h_2}{h_1}o_i^{\text{new}}\{1-\exp[-(t-t_0)h_1x_k(t)]\}. \tag{11}$$

Eq. (11) has an intuitive interpretation. As $t \to \infty$, there is a gradual shift from memory equilibrium around the old emotion o_i^{old}, given by term $(h_2/h_1)o_i^{\text{old}}\exp[-(t-t_0)h_1x_k(t)]$, towards a new equilibrium around o_i^{new}, given by the second term $(h_2/h_1)o_i^{\text{new}}\{1-\exp[-(t-t_0)h_1x_k(t)]\}$.

Eq. (11) was derived assuming $o_i^{\text{old}} = \text{const}$, which is inaccurate in the general case because o_i changes two or three orders of magnitude faster than $z_{ik}(t)$. We now show how Eq. (11) can be modified to resolve the issue.

For convenience, discrete time notation is introduced. We substitute $o_i^{\text{new}} = o_i(t)$ and set $t_0 = 0$ as well as $t = 1, 2, \ldots$ to account for the discrete time. Now, Eq. (11) cannot take o_i^{old} for moment $t-1$ because z_{ik} had not had enough time to adapt. To deal with this fact, we introduce $\hat{o}_i^{\text{old}}$. This is a hypothetical signal which, had it been maintained for sufficiently long time, would have put z_{ik} in its current value $z_{ik}(t-1)$. Formally assuming that this current value was an equilibrium, we can write:

$$z_{ik}(t-1) = \frac{h_2}{h_1}\hat{o}_i^{\text{old}}, \tag{12}$$

which gives

$$\hat{o}_i^{\text{old}} = z_{ik}(t-1)\frac{h_1}{h_2}. \tag{13}$$

Substituting o_i^{old} for $\hat{o}_i^{\text{old}}$ in Eq. (11) obtains

$$z_{ik}(t) = z_{ik}(t-1)\exp[-h_1x_k(t)] + \frac{h_2}{h_1}o_i(t)\{1-\exp[-h_1x_k(t)]\}. \tag{14}$$

In the case of two sources of emotion – internal o_i, and external $\tilde{o}_{ik}$, as in the experiment, the solution to Eq. (1) becomes:

$$z_{ik}(t) = z_{ik}(t-1)\exp[-h_1x_k(t)] + \frac{h_2}{h_1}\{1-\exp[-h_1x_k(t)]\}[u_k(t)o_i(t) + \bar{u}_k(t)\tilde{o}_{ik}(t)]. \tag{15}$$

Eq. (15) states that emotional memory $z_{ik}(t)$ can be updated by the participant's own experience with stimulus k (when $u_k = 1$), or by someone else's experience with the same stimulus (when $\bar{u}_k = 1$), but not by both events at the same time.

Implementation and validation. The obtained solution is used to model data from the omnium bonum economic experiment. The latter was conducted in

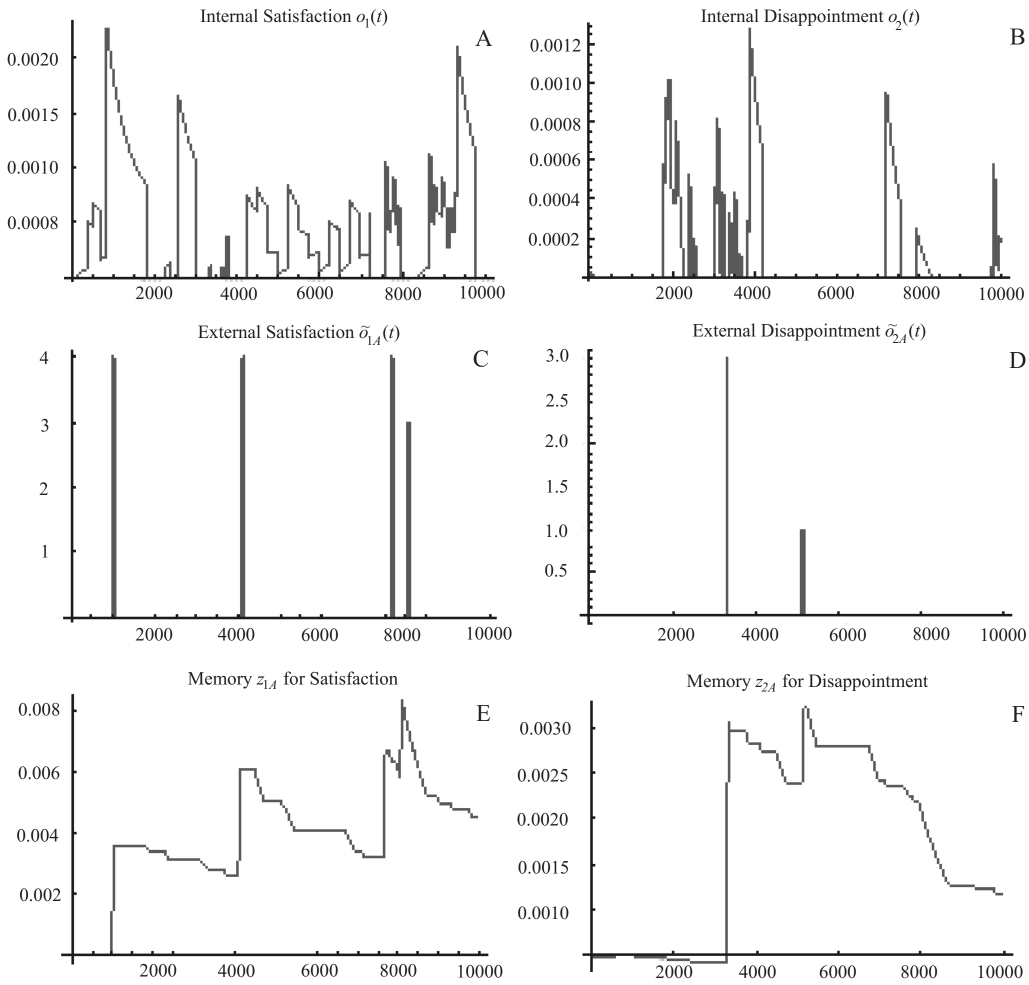

Fig. 1. Numerical simulation of the model. A. Model-generated internal satisfaction of a participant throughout the experiment. The emotion is caused by eye-balling of the four omnium bonum offers or by a successful deal with a supplier. B. Disappointment is caused by suppliers' underperformance. C. The participant receives posts by other players communicating their emotions, in this case satisfaction, in the form of high-powered 400 ms pulses. D. The other players are sometimes sending signals of disappointment. E. All experienced satisfactions with, e.g., Supplier A, are stored in emotional memory z_{1A}. F. Similarly, memory z_{2A} stores all the negative emotional experiences with that supplier. Memories for Suppliers B, C, and D are similar and are not shown here. Memories z_{1A}, z_{2A} are influenced by two kinds of satisfaction and disappointment – internal and external. All abscissas are time in centiseconds. All ordinates are unitless

a local computer network, releasing the Twitter-like posts among all participants about the satisfactions and disappointments that everyone experienced with each deal. A low level transmission control protocol (TCP) socket server written in Java was implemented to carry out high speed transmission of the messages. For the purposes of the economic experiment, communication happened essentially instantaneously.

Figure 1 shows a numerical simulation example with a real participant data. The top two plots present computed satisfaction and disappointment. In the middle are sequences of pulses appearing at the moments when messages from other people are flashed out on the screen. At the bottom, two plots show how the person's local emotional memories evolve when interacting with one of its game partners, Supplier A. Those memories are formed by internally generated satisfaction and disappointment, as well as by externally communicated ones.

A version of this experiment without social network [8, 10] already validated the use of Eq. (14). The entire model including that recurrent solution was shown to adequately describe and even predict the economic choices of those participants who made decisions emotionally. Figure 1 gives another kind of validation, now for Eq. (15) – the emotional memories z_{1A}, and z_{2A} are changing at the right moments and in the correct way due to both internal and external sources of emotion. We have every reason to expect that this solution, known to be mathematically correct, will make the entire computational model adequate for the complex behaviour in the social network.

Conclusion. In science it is natural that when addressing more daunting tasks, we face more difficult technical problems. In this paper we dealt with an element of a successful cognitive-emotional model, used as a guiding theory for a lab economic experiment. That element – a differential equation about emotional memory – had to be modified to be used in a more complex experiment. As the discussed equation had no solution in the general case it was analytically solved in a meaningful special case. The new solution was tested and shown to be adequate. Essentially, our achievement here is that we successfully circumvented the need for sophisticated numerical integration of stiff equations.

REFERENCES

[1] Grossberg S., N. Schmajuk (1987) Neural dynamics of attentionally-modulated Pavlovian conditioning: Conditioned reinforcement, inhibition, and opponent processing, Psychobiology, **15**(3), 195–240.

[2] Grossberg S., R. Raizada (2000) Contrast-sensitive perceptual grouping and object-based attention in the laminar circuits of primary visual cortex, Vision Research, **40**, 1413–1432.

[3] Grossberg S., J. R. Williamson (2001) A neural model of how horizontal and interlaminar connections of visual cortex develop into adult circuits that carry out perceptual groupings and learning, Cerebral Cortex, **11**, 37–58.

[4] Grossberg S., A. Seitz (2003) Laminar development of receptive fields, maps, and columns in visual cortex: The coordinating role of the subplate, Cerebral Cortex, **13**, 852–863.

[5] Raijmakers M. E. J., L. J. van der Maas, P. C. M. Molenaar (1996) Numerical Bifurcation Analysis of Distance-Dependent On-Center Off-Surround Shunting Neural networks, Biological Cybernetics, **75**, 495–507.

[6] Grossberg S., W. Gutowski (1987) Neural dynamics of decision making under risk: Affective balance and cognitive-emotional interactions, Psychological Review, **94**(3), 300–318.

[7] Mengov G., K. Georgiev, S. Pulov, T. Trifonov, K. Atanassov (2006) Fast computation of a gated dipole field, Neural Networks, **19**, 1636–1647.

[8] Mengov G. (2014) Person-by-person prediction of intuitive economic choice, Neural Networks, **60**, 232–245.

[9] Grossberg S. (1997) Birth of a Learning Law, Technical Report CAS/CNS-97-017, Boston University.

[10] Mengov G. (2015) Decision Science: A Human-Oriented Perspective, Berlin Heidelberg, Springer-Verlag.

Faculty of Economics
and Business Administration
Sofia University "St. Kliment Ohridski"
125, Tsarigradsko shosse Blvd, Bl. 3
1113 Sofia, Bulgaria
e-mail: g.mengov@feb.uni-sofia.bg

*Faculty of Mathematics and Informatics
Sofia University "St. Kliment Ohridski"
5, James Bourchier Blvd
1164 Sofia, Bulgaria
e-mail: triffon@fmi.uni-sofia.bg
kalin@fmi.uni-sofia.bg