

# Person-by-person prediction of intuitive economic choice



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## ABSTRACT

Decision making is an interdisciplinary field, which is explored with methods spanning from economic experiments to brain scanning. Its dominant paradigms such as utility theory, prospect theory, and the modern dual-process theories all resort to formal algebraic models or non-mathematical postulates, and remain purely phenomenological. An approach introduced by Grossberg deployed differential equations describing neural networks and bridged the gap between decision science and the psychology of cognitive–emotional interactions. However, the limits within which neural models can explain data from real people’s actions are virtually untested and remain unknown. Here we show that a model built around a recurrent gated dipole can successfully forecast individual economic choices in a complex laboratory experiment. Unlike classical statistical and econometric techniques or machine learning algorithms, our method calibrates the equations for each individual separately, and carries out prediction person-by-person. It predicted very well the behaviour of 15%–20% of the participants in the experiment – half of them extremely well – and was overall useful for two thirds of all 211 subjects. The model succeeded with people who were guided by gut feelings and failed with those who had sophisticated strategies. One hypothesis is that this neural network is the biological substrate of the cognitive system for primitive–intuitive thinking, and so we believe that we have a model of how people choose economic options by a simple form of intuition. We anticipate our study to be useful for further studies of human intuitive thinking as well as for analyses of economic systems populated by heterogeneous agents.

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## 1. Introduction

General Charles de Gaulle of France once remarked that it was difficult to govern a nation that had 246 different kinds of cheese. Besides the obvious message about developed countries being sophisticated, these words hint that economic choice is not only important but also somewhat frustrating. Economists have studied its more traditional aspects extensively and have come to the understanding that the axioms used in economic and political theory need revision (Sen, 1997). To better explain and predict, they ought to account for the subtle rationality of seemingly irrational decisions as in Amartya Sen’s famous example of somebody taking a fruit from a basket with two fruits, but refusing to do so when only one is left. Behavioural economics has addressed the general issue by relaxing its axioms as well as by equipping them with more empirical knowledge about the human being’s cognitive characteristics.

In the meantime, psychology has gone a long way in understanding human decision processes. Kahneman and Tversky’s research programme enriched economic analysis with findings about the heuristic and emotional aspects of decision making (Kahneman, 2003, 2011; Tversky & Kahneman, 1971, 1981). In our time, it has been established that a decision is reached in the complex interaction of two cognitive systems. Different theories have labelled them in different ways, but in general it is believed that there is one system for “intuitive”, “experiential”, or “impulsive” reasoning, also called “System I”, and another for “logical”, “rational”, or “reflective” reasoning, also called “System II” (Epstein, 1994, 2003; Kahneman & Frederick, 2002; Schneider & Shiffrin, 1977; Stanovich & West, 2000; Strack & Deutsch, 2004). Recent reviews on the subject can be found in (Alós-Ferrer & Strack, 2014; Brocas & Carrillo, 2014; Dayan, 2009), while some of the recent modelling advances constitute (Andersen, Harrison, Lau, & Rutström, 2014; Fudenberg & Levine, 2006; Fudenberg, Levine, & Maniatis, 2014; Mukherjee, 2010). In this view, the intuitive system is automatic, effortless, emotion-driven, governed by habit, but difficult to change, while the logical system is effortful, controlled and slow, but flexible and able to adopt complex decision rules. Easy tasks are dealt with

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predominantly by the former, while complications prompt the intervention of the latter. Buying some cheese would demand mostly intuition – but being not entirely simple – would also need some input from the logical system, while governing a nation would ask for a lot more of it. Therefore, the cognitive load due to every decision can be regarded as a point on a linear segment, with the domains of the intuitive and logical systems located at its ends. Of course this dichotomy is somewhat schematic, as studies of the brain using modern scanning technologies have led to the understanding that the neural basis of emotion and cognition is highly integrated and hardly decomposable (Pessoa, 2008). It has been suggested (Evans & Stanovich, 2013) that it would be better not to speak of “System I” and “System II”, but of “Type I” and “Type II” processes instead. Hence, it would be more correct to refer to dual-process, rather than dual-system theories.

Somewhat apart from the above family of dual-process theories is fuzzy-trace theory (Reyna & Brainerd, 1991, 2008), which posits that intuition is “gist-based”, i.e., resorting to vague memories about the gist of information relevant in a decision situation. In this view, intuition is more advanced than logical reasoning and is more characteristic of experts rather than novices. This theory has successfully explained a variety of empirically established cognitive effects. This is perhaps the only dual-system theory that regards intuition as a sophisticated form of mature reasoning.

It proved worthwhile to approach the issues of intuitive–emotional vs. logical–rational choice in general, and with regard to economics and business in particular, with the tools of mathematical neuroscience. Its ideas have for long translated into a variety of models that look promising for understanding the complexities of decision making. Perhaps the earliest theoretical forays in this direction were made in the 1960–1970s by Stephen Grossberg, with most of his ideas summarized in Grossberg (1980). Not necessarily driven by the needs of economic analysis, further contributions have assisted in developing the field. Affective balance theory (Grossberg & Gutowski, 1987) extended and subsumed Kahneman and Tversky’s prospect theory. A cognitive–emotional model, CogEM, of reinforcement learning and motivated attention (Grossberg & Schmajuk, 1987) was used alongside ART neural networks in a theory of consumer motivation (Leven & Levine, 1996). CogEM was further developed and became the precursor of the more general MOTIVATOR model (Draniias, Grossberg, & Bullock, 2008), which explained how cognitive–emotional resonances occur between brain areas that code subjective value and form the basis of behavioural choices. Levine (2006) proposed a neural model for the interaction of selfishness and empathy in economic actions.

This line of research owes a lot to Grossberg’s theoretical method, which consists of a number of iterative steps (Grossberg, 2006) when studying a particular cognitive phenomenon. These involve: top-down analysis of behavioural data, discovery of underlying principles, using them to develop mathematical models, refining the latter by computer simulations, and eventually identifying the most adequate theoretical model. The building blocks of all models are three nonlinear ordinary differential equations, characterizing the fundamental neural interactions. Complex brain processes are modelled by recombining the three equations as well as by embedding simpler models into more sophisticated ones in an evolutionary way. This method was adopted by other neuroscientists with interest in economic behaviour (notably, Levine), who carried on using it successfully even in the era of fMRI data.

Indeed, in the 2000s, fMRI studies helped to clarify how elements from the classical psychological theories of heuristic judgement and decision making, including prospect theory, could be mapped on brain processes (Tom, Fox, Trepel, & Poldrack, 2007; Trepel, Fox, & Poldrack, 2005). Complementary theoretical and experimental advancements led to the creation of new and more

sophisticated neural models. For example, Levine (2012) proposed an elaborate nonlinear neural network accounting for the biasing effects of emotion on probabilistic choice. It was composed of ART modules and gated dipoles, and was based also on fuzzy-trace theory, findings from fMRI studies, and traditional psychological experiments. Levine (2009) developed a similarly complex neural network that explained a variety of instances of emotionally influenced decision making. It was named DECIDER and combined a neural representation of Maslow’s hierarchy of needs with a system of ART modules, conceptually grounded in facts about brain processes.

All of these modelling efforts aimed at producing theoretical knowledge and were generally not intended for direct applications. Trying to use one of Grossberg’s or his disciples’ theories to guide a novel laboratory experiment or R&D work usually leads to unforeseen obstacles that call for introducing simplifications and ad hoc adaptations, trading off theoretical rigour and beauty for practical viability. This is necessary for at least two reasons: first, the target domain may be quite different from the theory’s territory of origin, both conceptually and methodologically, which makes the communication between the two problematic. In that respect, a good example is the attempt to employ instruments from mathematical neuroscience to examine economic decision making. Secondly, in applied science, an essential goal of the knowledge transfer is to create models for prediction, often in real time. However, this is a return to the imperfect empirical world, full of artefacts and contaminating factors, and almost always involving work with noisy data both as calibration and validation sets.

In the present study, we investigate experimentally the abilities of the Grossberg–Schmajuk CogEM model to explain and predict individual economic choice under laboratory conditions close to real markets. The main element in CogEM is a neural network called READ (REcurrent Associative gated Dipole). We suggest that READ may be remotely related to the “intuitive” system, or “Type I” process, as understood by the majority of the dual-process theories, and indeed may be seen as its hypothetical neural substrate. It must be stressed though, that the Grossberg–Schmajuk model is conceptually independent from these theories and can in no way be affected by any controversies around their empirical validation. In fact, when future research clarifies the difference between primitive intuition and gist-based expert intuition, READ could turn out to be just as useful for modelling the former, or may be even both.

Further, because the intuitive and the logical systems (or processes) are locked together in a loop of intensive mutual communication, if READ is to model that, it must be augmented with additional elements, as in Levine’s (2009, 2012) approach. However, even as it stands now, this neural network can account for the more primitive aspects of intuitive decision making. Precisely that is what it is used for in the present study.

Previously, READ successfully predicted 87% of people’s binary preferences in a simple experiment, thus surpassing some state-of-the-art econometric tools (Mengov, Egbert, Pulov, & Georgiev, 2008). Here we develop a more complex economic experiment, which involves profit maximization by choosing one among four competing suppliers of a good. Our goal is to use READ as a vehicle to connect real people’s market behaviour with some of the established theoretical concepts of decision science. Along the way, practical problems call for combining the gated dipole with econometric variables, thus obtaining a hybrid neural model of economic choice. A separate model is calibrated for each individual and is then tested with validation data to establish to what degree personal choices are predictable. In summary, what sets this study apart from many others is that it utilizes a sophisticated neural model to predict real people’s economic choices *person-by-person* in a relatively complex economic game.

## 2. Equations of the READ neural circuit

READ is a representative of Grossberg's set of neural models and combines three basic elements: short-term memory (STM) identical with neuron activity; medium-term memory (MTM) comprising synapses with neurotransmitter gates; and long-term memory (LTM) storing information patterns contained in a population of neurons, active during certain time intervals. The former two are sufficient to build a gated dipole—a model of opposite emotions, arising due to external stimuli and rebalancing each other. LTM is necessary to memorize the link between emotions and stimuli that caused them. All three elements are described by ordinary nonlinear differential equations. Sometimes, an issue about terminology has been raised on the grounds that it is not clear what the memory correlates for medium-term memory would be. The answer is given by the MTM's defining mathematical equation below, which characterizes it as another type of long-term memory.

READ, as implemented in the present study with four market players, consists of a system of 18 differential equations that cannot be solved analytically. However, a number of simplifying modifications are possible. Because short-term memory is typically two or three orders of magnitude faster than the other memories, in computational models it can be assumed to reach equilibrium instantaneously. In practical terms this means that when the entire system of equations is solved, all STM derivatives can be set to zero. At the same time, recurrent solutions of the two slower types of memories, if exist, can be used alongside the STM algebraic equations. Grossberg and Gutowski (1987) and Grossberg and Schmajuk (1987) have shown that if the neural activations and transmitters obey MTM and LTM equations such as (1) and (2) below, the mathematical product of instantaneous input signal and slower transmitter release can adequately approximate the conveyed signal.

Here the following notation is used. Neurons and their activities, i.e., the STM are denoted by  $x_1, \dots, x_8, x_A, \dots, x_D$ ; Quantities  $y_1$  and  $y_2$  designate MTM transmitter gates; Long-term memories are  $z_{7k}, \dots, z_{8k}$ , where  $k = A, B, C, D$  denote four mutually exclusive categories, here associated with four economic alternatives.

The three types of equations are introduced as follows, and a more elaborate discussion on them can be found in Grossberg (2013).

*STM: neuron activation:*

$$\frac{dx_i}{dt} = -a_1x_i + (a_2 - x_i)J_i^+ - (x_i + a_3)J_i^- \quad (1)$$

Eq. (1) is a variant of the classical Hodgkin–Huxley equation and describes the activity  $x_i$  of the  $i$ th neuron in the network. Term  $J_i^+$  is the sum of all incoming signals from other neurons activating  $x_i$ . Similarly,  $J_i^-$  sums all inhibitory signals from other neurons. Constants  $a_1, a_2, a_3$  are real and positive. All neuron equations used in this article's model can be regarded as special cases of Eq. (1).

*MTM: neurotransmitter gate:*

$$\frac{dy_i}{dt} = b_i(1 - y_i) - c_ix_iy_i \quad (2)$$

Eq. (2) describes neurotransmitter depletion and recovery in a neuron  $x_i$ , sending out a signal in response to received signals as per Eq. (1). Transmitter quantity  $y_i$  is a function of the strength of the emitted signal. In the absence of external activation, quantity  $y_i$  accumulates at fixed rate  $b_i$  until reaching its maximum of 1. Constants  $b_i, c_i$  are real and positive, and due to biological plausibility, in computer simulations may sometimes be considered slightly different for each gate  $i = 1, 2$ .

*LTM: gated learning:*

$$\frac{dz_{ik}}{dt} = x_k(-h_1z_{ik} + h_2[x_j]^+) \quad (3)$$

Eq. (3) is a variant of the gated learning law, which differs from that in Grossberg (2013), but is identical with the one used in Grossberg and Schmajuk (1987). It shows how a single LTM component  $z_{ik}$  between neurons  $x_i$  and  $x_k$  changes under the influence of a third neuron  $x_j$ . Operator  $[\cdot]^+$  denotes rectification:  $[\xi]^+ = \max\{\xi, 0\}$ , and  $h_1, h_2$  are real and positive constants. (In simulations, sometimes  $h_2 = 1$ .)

Assuming  $t$  is discrete time with sufficiently high sampling rate (which is achieved in the experiment below, see Section 4), the existing analytical solution of Eq. (2) can be rewritten straightforwardly in this recurrent form:

$$y_i(t) = y_i(t-1) \exp(-c_ix_i(t) - b_i) + \frac{b_i}{b_i + c_ix_i(t)} (1 - \exp(-c_ix_i(t) - b_i)), \quad (4)$$

where  $i = 1, 2$ .

Similarly, the recurrent solution to Eq. (3) is

$$z_{ik}(t) = z_{ik}(t-1) \exp(-h_1x_k(t)) + \frac{h_2}{h_1} o_l(t) (1 - \exp(-h_1x_k(t))). \quad (5)$$

Here, quantity  $o_l(t)$  with  $l = 1, 2$  is the READ-predicted consumer emotion of satisfaction  $o_1 = [x_5]^+$  or disappointment  $o_2 = [x_6]^+$ . Note that in Eqs. (4) and (5), each of  $x_i, o_l$ , and  $x_k$  is taken at moment  $t$  rather than  $t - 1$  because it is assumed to react instantaneously.

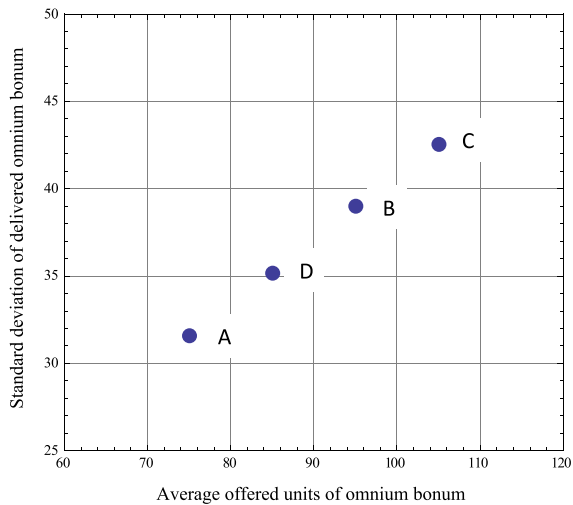
## 3. Economic experiment

The main objective of the experiment was to establish to what extent people act in the affective–intuitive way, accounted for by the READ neural model, when they are put in a moderately complex economic situation. Briefly, the content of this lab study is as follows. In a number of rounds, subjects had to choose repeatedly one among four suppliers of a fictitious commodity called *omnium bonum* (a good for everyone, in Latin) seeking to obtain as much as possible of it. At the end of the game the accumulated units of the commodity were exchanged for real money. Typically, one could earn € 4–8 in about 20 min, the duration of the entire procedure. The participants were rewarded for their ability to orient themselves in an environment with insufficient information by subjectively developing in their minds adequate profiles of the suppliers and using them as choice factor. The instruction was written so as to avoid any role-assigning in any concrete economic or business circumstances, thus escaping potential confound effects (Zizzo, 2013).

The four suppliers provided *omnium bonum* of equal quality, but were not always reliable in delivering it—they could offer certain amount, but often deliver less, or even more. Their profiles are summarized in Fig. 1, which shows how the supplier offering the most on average was also the least dependable. In essence, this was an implementation of the economic idea that higher profit goes hand in hand with more risk-taking. The subjects were unaware of this feature and could only discover it by trial and error.

The exact figures of the design were chosen such as to meet the following requirements:

1. There had to be real competition among the suppliers so that the participants could be facing real choices. Only that could make prediction by any method meaningful.
2. Each supplier had to remain competitive throughout the game lest the number of options diminished. Conversely, no supplier ought to come close to monopolizing the market.
3. Each supplier had to be in a position to form a distinct image in the eyes of the participant. One way of achieving this was by introducing the two-dimensional design shown in Fig. 1.



**Fig. 1.** Four suppliers offered and delivered different units of omnium bonum, whereby the riskiest (the one with the largest standard deviation) was most rewarding.

4. The game had to be long enough for the suppliers to become recognizable, yet it could not be too long lest the participants got used to it to the point of routine or boredom, which would be two additional confounding effects.

There might be many ways to implement the above demands and here is what we did. The number of rounds was fixed at twenty. They were divided in subsequences of five rounds each, with every supplier acting in each subsequence as follows:

- On one occasion they delivered omnium bonum exactly as offered;
- On two occasions more was delivered than offered;
- On two occasions less was delivered than offered.

The order of these events was unpredictable for the participant, who did not even know that such an arrangement existed. No transaction costs were involved in abandoning one supplier for another. The four suppliers were put on the computer screen as shown in Fig. 2.

In each five-round batch, the total amount offered equalled the amount delivered, should a supplier be chosen repeatedly. Of course no subject was obliged to do so, and was never told about this design feature. Further, in any given round at most two suppliers behaved similarly, e.g. by delivering exactly as having offered. In half of the experimental treatments a continuous economic growth was simulated, implemented by raising the offered and delivered quantities by 10 units after each five rounds (i.e. after rounds ##5, 10, and 15). Thus the subjects' motivation and involvement were maintained until the end of the game. The ratio between smallest and biggest offer in the first five rounds was 1.5 and diminished to 1.33 in the last five. This was needed to keep the absolute difference between them constant at 30 units of omnium bonum in all rounds. Such an arrangement ensured that the supplier with the most modest offer was able sometimes to deliver more than what was offered by the frontrunner. For example, in round #9, Supplier A offered 70 units and would deliver 102 if chosen, which would be more than the 100 units offered by Supplier C.

All these delicate balances created genuine competition as well as enough attractiveness for each supplier with no one becoming a monopolist. In a pre-test condition with 34 subjects (producing 680 decisions) Supplier A was chosen in about 14% of the cases and Supplier C in approximately 40%.

Half of the treatments simulated economic growth followed by economic crisis. This was achieved by reducing the quantities of offered and delivered omnium bonum in the last five rounds by all suppliers. Again, all figures were chosen so as to maintain the above balances.

An essential element of the experiment was measuring the consumer satisfaction, self-assessed and reported immediately after each omnium bonum delivery. On a separate screen not shown

<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px; text-align: center;">           Supplier A Offer: 70 units of omnium bonum         </div> <div style="border: 1px solid black; height: 40px; margin-top: 5px;"></div>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px; text-align: center;">           Supplier B Offer: 90 units of omnium bonum         </div> <div style="border: 1px solid black; height: 40px; margin-top: 5px;"></div>
<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px; text-align: center;">           Supplier C Offer: 100 units of omnium bonum         </div> <div style="border: 1px solid black; padding: 5px; margin-top: 5px; text-align: center;">           The omnium bonum actually delivered will be 145 units         </div>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px; text-align: center;">           Supplier D Offer: 80 units of omnium bonum         </div> <div style="border: 1px solid black; height: 40px; margin-top: 5px;"></div>
<div style="display: flex; justify-content: space-around; margin-bottom: 10px;"> <div style="border: 1px solid black; padding: 5px; text-align: center; width: 40%;">I Choose A</div> <div style="border: 1px solid black; padding: 5px; text-align: center; width: 40%;">I Choose B</div> </div>	<div style="display: flex; justify-content: space-around; margin-bottom: 10px;"> <div style="border: 1px solid black; padding: 5px; text-align: center; width: 40%;">I Choose C</div> <div style="border: 1px solid black; padding: 5px; text-align: center; width: 40%;">I Choose D</div> </div>
<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="text-align: left;">           Omnium bonum accumulated So far: 530 units         </div> <div style="text-align: right;"> <div style="border: 1px solid black; padding: 5px; width: 50px; height: 20px; display: flex; align-items: center; justify-content: center;"> </div> </div> </div>	

**Fig. 2.** Each round began with an experimental screen showing four offers. In the example, sometime through the game the participant examined them and chose Supplier C with a mouse click. Immediately, the actual quantity of omnium bonum was 'delivered' and added to the total.

here the subjects had to express their feelings about how they were treated in the current round in this Likert-type scale: “Extremely disappointed (−4)—Very disappointed (−3)—Disappointed (−2)—More disappointed than satisfied (−1)—As much satisfied as disappointed (0)—More satisfied than disappointed (+1)—...—Extremely satisfied (+4)”. The adverb–adjective compounds were chosen such as to make the measurement scale interval by ensuring that the segments between neighbouring points were equidistant in line with Cliff’s multiplicative rule (Cliff, 1959). Finally, the timing of each mouse-click was recorded alongside the action associated with it, and was later used in calibrating the system of differential equations.

Immediately after the game, the software application demanded answers to a sequence of open and closed questions about the subject’s strategy, tactical reasoning, and other issues, potentially relevant for the individual’s decision-making style. Obviously these components were unrelated to the CogEM model, but were included to investigate what factors could be linked with a potential predictive success by a READ-based neural model. A large part of them are of qualitative nature.

In particular, the debriefing session started with the open question, “Please describe how you chose the suppliers of omnium bonum”. The next question was, “Did you have a strategy? Yes/No”. This was followed by: “If you did have a strategy, please explain when and how it was formed”. The next question was, “Did you use different strategies throughout the game? Yes/No”. That line of questioning was continued by, “If you did change your strategy during the game, what made you do so?” The last question of the series was, “If we gave you the opportunity to play again, would you adopt a strategy different from the one you just used?” Apparently, there was a lot of redundancy in these questions, but it was introduced to ensure that all potentially relevant information was captured. In addition, our pre-test experience taught us that some subjects needed more time and questioning to give the best explanations they could formulate.

Another question asked was if the subject was loyal to a supplier. Finally, the participant was asked to assess the degree to which any of a number of factors influenced their decisions. Some of those factors were: The immediately preceding choice of a supplier; the decisions taken in the last couple of rounds; the difference between initially offered and eventually supplied omnium bonum.

#### 4. A hybrid neural model of economic decision making

Here we explain how READ was connected with the experimental data. Our study involved psychological but no neurobiological data, yet they had to be made tractable by the neural model. This challenge called for some assumptions that might seem a bit too speculative, however, results discussed in Section 5 show that the introduced simplifications were justified—they did not hamper the adequate prediction of subjects’ behaviour. In addition, some of our postulates touch upon the issue of whether the brain computes ‘economic’ value or utility, and if yes—is it absolute or relative, or is the brain choosing by performing just a mere comparison among alternatives without computation. The question has not received a definitive answer yet (Vlaev, Chater, Stewart, & Brown, 2011), but our technical solutions do not seem to contradict any of the plausible hypotheses.

The data were fed into READ via two channels: information about omnium bonum got in through neurons  $x_1$  and  $x_2$ , while the identity of the currently active supplier was submitted through neurons  $x_A, \dots, x_D$  as shown in Fig. 3.

Neurons  $x_1$  and  $x_2$  are described by the following equations, which show how the supplier offers are incorporated in the model.

$$\frac{dx_1}{dt} = -x_1 + J_{base} + \delta J_{\Delta+} + \delta_1 J_{eye}^{(k)} + Mx_7 \quad (6)$$

$$\frac{dx_2}{dt} = -x_2 + J_{base} + \delta J_{\Delta-} + Mx_8. \quad (7)$$

Term  $J_{base}$  is the baseline signal, also known as tonic signal in the gated dipole. Term  $J_{eye}^{(k)}$  with index  $k = A, B, C, D$  accounts for the omnium bonum commodity  $q_k$ , offered by the four suppliers. By submitting  $J_{eye}^{(k)}$  to the on-channel (for positive events and emotions) we implemented the idea that under the game circumstances, looking at offers of omnium bonum was inherently pleasant.

Signal  $J_{eye}^{(k)}$  is greater than zero only while the subject eyeballs the offers on the screen and deliberates which one to choose. All quantities of the commodity are scaled to become  $J_{eye}^{(k)} = q_k/10 \max(q_k)$  and not exceed 10% of the maximum STM value of 1. The two signals are related:

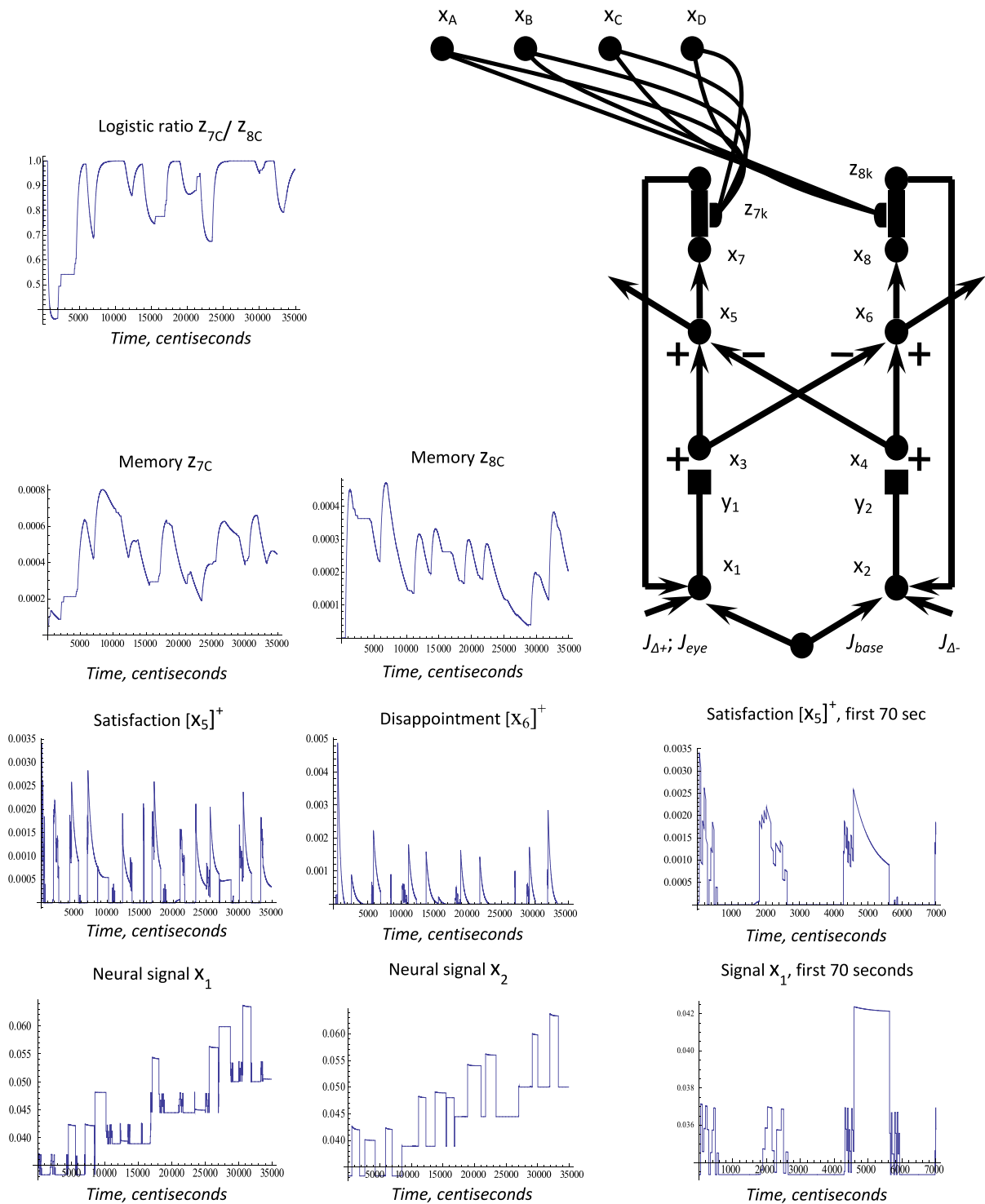
$$J_{base} = \min \{ J_{eye}^{(k)} \}_k, \quad (8)$$

with the logic that  $J_{base}$  is neural activation caused by the weakest offer. Similarly, terms  $J_{\Delta+}$  and  $J_{\Delta-}$  reflect a positive difference (more omnium bonum delivered than promised) and a negative difference (less delivered than promised), and are also scaled down to  $\Delta q_k/10 \max(\Delta q_k)$ . Eqs. (6)–(7) contain positive constants  $\delta, \delta_1$  which due to previous experience are fixed in advance at 0.4 and 0.2 respectively, and  $M$ , which is determined together with some other constants in a simulated annealing procedure.

Eqs. (6)–(8) embody a number of assumptions that had to be introduced in order to link neural and economic variables. First, Eq. (8) may not be the only way to establish the baseline signal but it rests upon sufficient common sense: Eyeballing the four offers, a participant is likely to view the weakest one as a reference point for assessing the potential benefit of the remaining three. An alternative approach would have used some form of averaging of the four, but that could add little and would be an unnecessary complication.

Next, the model implementation of eyeballing signal  $J_{eye}^{(k)}$  involved perhaps another amount of scientific speculation. In the absence of brain scanner or eye-tracking facility, and having only records of economic decisions, self-assessed satisfactions, and their timing, that was but necessary. One assumption was that in each eyeballing-and-deliberation period all four options were sufficiently well understood. In reality, it may have taken several glances at each of them, and maybe more attention was paid to one than to another, especially after some initial experience. We postulated, however, that in each round the subject divided her/his attention (i.e. time) among the four suppliers *equally*, and cast exactly *three* glances at each of them *at random*. Thus, the eyeballing period was split into 12 equal parts, during which  $J_{eye}^{(k)}$  propelled the offers inwards as per Eq. (6). This happened in synchrony with signals  $x_A, \dots, x_D$ , switched on and off accordingly. Of course it cannot be claimed that such was the real sequence of events, but only that this was a reasonable way to model how the options were adequately grasped.

The entire cognitive mechanism is described by Eqs. (9)–(24) below and can be summarized with the help of Fig. 3 as follows. A signal  $x_k$  (with  $k = A, B, C, D$ ) indicates which of the four suppliers is currently in the focus of attention. If, for example, this is Supplier A, then  $x_A = 1$ , and  $x_k = 0$  for all the other  $k$ s. The participant reacts to the supplier’s performance with various degrees of satisfaction  $[x_5]^+$  or disappointment  $[x_6]^+$ . When the relevant



**Fig. 3.** Dynamics of the neural circuit. **Bottom left and middle plots:** Signals  $x_1$  and  $x_2$  are the responses to the incoming stream of offers, deliveries, and to the participant's eyeballing before choosing. In the example, the person kept choosing Supplier C in the first three rounds and received extra omnium bonum in the third, which is reflected in the two "first 70 seconds" plots (**third column, below the neural circuit**). There,  $x_1$  produced "ripples" at the onset of each round and then jumped around the 4500th centisecond (45th s) due to the surplus delivered. Around the 2000th centisecond, the corresponding  $[x_5]^+$  signal shows that eyeballing four positive options can cause satisfaction, almost as intense as that of the actual lavish treatment. **Upper-left plots:** The memory for positive emotions  $z_{7C}$  initially rose negligibly due to eyeballing, and then increased around the 50th second after the generous delivery in the third round. In contrast, the memory for negative emotion  $z_{8C}$  rose steeply in the first round and remained high in the next due to disappointingly unfulfilled promises. The supplier's dynamic reputation was defined by the logistic ratio  $z_{7C}/z_{8C}$  reflecting the two memories' joint action.

gating signal  $x_k$  is switched on and for instance the customer gets disappointed (positive  $[x_6]^+$ , zero  $[x_5]^+$ ), neuron  $x_8$  is active and the corresponding memory component  $z_{8k}$ , described by its concrete equation among Eqs. (21)–(24), undergoes learning due to

signals  $x_A$  and  $x_6$ . If the emotion were positive,  $z_{7k}$  would have been updated. Coming back to the example of Supplier A disappointing the subject, Eq. (21) below shows how  $z_{8A}$  changes to account for the situation.

**Table 1**  
Response times recorded as intervals between mouse clicks. Data are over 131 participants.

		Round in the economic game					
		#1	#2	#3	...	#19	#20
$T_1$ , seconds (Time for eyeballing and choice)	Max	157.62	104.25	83.05	...	31.96	50.62
	<b>Median</b>	<b>29.92</b>	<b>13.60</b>	<b>10.03</b>	...	<b>4.29</b>	<b>4.59</b>
	Min	7.43	2.12	1.87	...	0.76	0.76
$T_2$ , seconds (Time for comprehending the delivery)	Max	72.06	31.42	16.01	...	32.14	9.19
	<b>Median</b>	<b>7.38</b>	<b>4.29</b>	<b>3.65</b>	...	<b>1.34</b>	<b>1.28</b>
	Min	1.34	0.69	0.62	...	0.59	0.56
$T_3$ , seconds (Time for assessing own satisfaction)	Max	104.69	33.15	22.98	...	21.18	11.28
	<b>Median</b>	<b>12.42</b>	<b>4.74</b>	<b>3.71</b>	...	<b>1.93</b>	<b>1.65</b>
	Min	3.42	1.25	1.01	...	0.70	0.75
$T_4$ , seconds (Time for starting next round)	Max	12.36	10.90	7.16	...	3.51	3.18
	<b>Median</b>	<b>1.70</b>	<b>1.25</b>	<b>1.23</b>	...	<b>0.80</b>	<b>0.76</b>
	Min	0.70	0.47	0.42	...	0.30	0.30

Beyond  $x_1$  and  $x_2$ , the other equations of the READ model, adapted for the present experiment, are as follows:

$$\frac{dy_1}{dt} = b_1(1 - y_1) - c_1x_1y_1 \quad (9)$$

$$\frac{dy_2}{dt} = b_2(1 - y_2) - c_2x_2y_2 \quad (10)$$

$$\frac{dx_3}{dt} = -x_3 + x_1y_1 \quad (11)$$

$$\frac{dx_4}{dt} = -x_4 + x_2y_2 \quad (12)$$

$$\frac{dx_5}{dt} = -x_5 + (1 - x_5)x_3 - (x_5 + 1)x_4 \quad (13)$$

$$\frac{dx_6}{dt} = -x_6 + (1 - x_6)x_4 - (x_6 + 1)x_3 \quad (14)$$

$$\frac{dx_7}{dt} = -x_7 + G[x_5]^+ + L(x_{A,Z7A} + x_{B,Z7B} + x_{C,Z7C} + x_{D,Z7D}) \quad (15)$$

$$\frac{dx_8}{dt} = -x_8 + G[x_6]^+ + L(x_{A,Z8A} + x_{B,Z8B} + x_{C,Z8C} + x_{D,Z8D}) \quad (16)$$

$$\frac{dz_{7A}}{dt} = x_A(-h_1z_{7A} + [x_5]^+) \quad (17)$$

$$\frac{dz_{7B}}{dt} = x_B(-h_1z_{7B} + [x_5]^+) \quad (18)$$

$$\frac{dz_{7C}}{dt} = x_C(-h_1z_{7C} + [x_5]^+) \quad (19)$$

$$\frac{dz_{7D}}{dt} = x_D(-h_1z_{7D} + [x_5]^+) \quad (20)$$

$$\frac{dz_{8A}}{dt} = x_A(-h_1z_{8A} + [x_6]^+) \quad (21)$$

$$\frac{dz_{8B}}{dt} = x_B(-h_1z_{8B} + [x_6]^+) \quad (22)$$

$$\frac{dz_{8C}}{dt} = x_C(-h_1z_{8C} + [x_6]^+) \quad (23)$$

$$\frac{dz_{8D}}{dt} = x_D(-h_1z_{8D} + [x_6]^+) \quad (24)$$

Eqs. (9)–(10) represent neurotransmitters and are identical with Eq. (2), while Eqs. (11)–(16) are neural activations that can be viewed as special cases of Eq. (1).

There exist a number of approaches to integrate the system of equations (6)–(24). Typically (Grossberg & Raizada, 2000; Grossberg & Seitz, 2003; Grossberg & Williamson, 2001) the fastest cell reactions are computed at steady state, other activity equations are solved with the Runge–Kutta–Fehlberg 4–5 method, and MTMs

and LTM are solved at a reduced time scale with Euler's method. In a model similar to the present one, Mengov et al. (2008) reported implementing a version of the Runge–Kutta–Fehlberg 4–5 method with improved precision (Gammel, 2004) for computing the entire system of differential equations.

Here we adopted a different approach by introducing discrete time and then computing all neuron activations at steady state while solving all MTM and LTM with Eqs. (4) and (5). To do so adequately, one must use sufficiently high “sampling” rate, in the spirit if not the letter of the sampling theorem. In practical terms this meant considering the following points. On one hand, no neural signals but only moments of mouse clicks were recorded, which were four in number for each round of the game. This means that each participant had to eyeball each offer, choose one by a mouse click, then comprehend the delivery, do another mouse click, and then assess their own satisfaction or disappointment, which is done by a mouse click on the Likert-type scale measuring the emotional valence and intensity. A final mouse click is needed to start the next round. Table 1 gives an overview on the response times in a sample of 131 subjects. Apparently, the interval medians diminished systematically as the subjects got used to the game settings.

The adopted twelve-look scheme implied that  $T_1$ , the time for examining offers and taking a decision, had to be divided by 12. This was a plausible assumption with regard to the overwhelming majority of response times, but needed a closer look in the boundary case of the quickest responses. In particular, the shortest recorded  $T_1$  was 530 ms, which happened twice and was achieved by one and the same subject towards the end of the game. This number divided by 12 gives approximately 44 ms and puts on display the following limitation of the scheme validity. It is hard to imagine that in half a second the subject glanced over each of four options three times—such a speed is reminiscent of Saul Sternberg's discovery (Sternberg, 1966) of an internal automatic scanning process for image recognition operating at a rate of one sample per 33–40 ms. However, this speed could not leave enough time for the slower and more complex cognitive process involving economic reasoning. Most probably, on that occasion the subject glanced only once at each offer before choosing. A more detailed data examination revealed that early in the game she/he had tried only two suppliers and had remained loyal to Supplier C ever since. Therefore, a predisposition towards one option had a powerful effect on the speed of the two fastest decisions.

Although the plausibility of a twelve-look scheme may be questioned in that specific case, it still remains unproblematic from a modelling point of view. Opening the gate for a fixed time interval in any of Eqs. (17)–(24), and in particular Eqs. (19) and (23) for Supplier C, would have resulted in the same LTM value regardless of whether that interval was divided into three parts (corresponding to three separate looks at the supplier), or was uninterrupted.

A time step of 10 ms (1 centisecond) was used for computing the entire system of equations. It apparently provided high enough sampling rate even for 1/12th of the shortest recorded intervals.

An illustration of the numerical solution for a subject can be seen in Fig. 3, where the bottom right plot shows  $x_1$  in a typical situation at the opening rounds of the game. During these first 70 s the participant played four rounds, beginning with screen eyeballing and responding accordingly to the four levels of arousal, three times to each in random order. Initially, Supplier C was chosen and apparently misbehaved because  $x_1$  returned to  $J_{base}$  instead of jumping to  $J_{base} + \delta J_{\Delta+}$ . A positive  $J_{\Delta-}$  appeared in Eq. (7) for  $x_2$  (not shown in a 70 s plot). Much the same happened in the second round, approximately between the 20th and 30th second, but this time eyeballing took a bit longer. Only in the third round  $x_1$  jumped up due to a surplus delivery of omnium bonum, reflected by  $J_{\Delta+}$  in Eq. (6).

#### 4.1. Three-factor decision rule

Here we show how the READ neural model was employed to predict choices of suppliers by the participant. Clearly, the current offers and the past experiences in the game were the two major drivers of each decision. They had to be suitably operationalized and combined in a unified decision rule. We developed a three-factor rule consisting of the following elements:  $F_1$ : momentary reaction to the four omnium bonum offers;  $F_2$ : past experiences with the suppliers accumulated in the customer's long-term memory;  $F_3$ : emotion, remembered after the last deal with a particular supplier regardless of how far in the past it happened. Factors  $F_1$  and  $F_2$  were directly based on READ components and interactions, while  $F_3$  was the result of an independent econometric study. Thus, the decision rule took this form:

$$\mathbf{D} = d_1 \mathbf{F}_1 + d_2 \mathbf{F}_2 + d_3 \mathbf{F}_3 \quad (25)$$

$$K = \max(\mathbf{D}) . \quad (26)$$

In Eq. (25),  $\mathbf{F}_i = [F_i^{(A)}, F_i^{(B)}, F_i^{(C)}, F_i^{(D)}]^T$  is a vector of real numbers between 0 and 1 computed as discussed below, and  $i = 1, 2, 3$ . The three factors were weighted by positive constants  $d_1, d_2, d_3$ .

We postulated that  $K$ , the predicted choice of supplier, would be the one with the maximum value among the four scalars in vector  $\mathbf{D}$ . The three constants were constrained as follows:  $d_1 \in (0, 1)$ ;  $d_2 \in (0, 1 - d_1)$ ; and  $d_3 = 1 - d_1 - d_2$ . These limitations ensured that each weight could vary within the open interval (0, 1) at the expense of the other two. Now let us discuss the three factors in more detail.

##### Factor #1: STM and dynamic neural balances in response to economic options

Following Mengov et al. (2008), we model the first factor – participant reaction to the current offers – with a function of neuron activations  $x_7$  and  $x_8$ , as these integrate the two most important influences: (1) emotional responses  $[x_5]^+$  and  $[x_6]^+$  to the omnium bonum quantities, and (2) the input from emotional memories  $z_{7k}$  and  $z_{8k}$ , reflecting past experiences. Eqs. (15)–(16) illustrate this position clearly. However, Mengov et al. (2008) dealt with only two suppliers and could model the selection process as a choice between the status quo and a change. With four options in the present study this is not possible. In addition, the decision procedure now had to accommodate the eyeballing phase with three glances at each offer. This was achieved by defining the following quantities:

$$\mathbf{x}_7^l = [x_7(t_A^l), x_7(t_B^l), x_7(t_C^l), x_7(t_D^l)]^T \quad (27)$$

$$\mathbf{x}_8^l = [x_8(t_A^l), x_8(t_B^l), x_8(t_C^l), x_8(t_D^l)]^T, \quad (28)$$

$l = 1, \dots, n$ , where  $l$  is index for the values of  $x_7$  and  $x_8$  during the last of three consecutive exposures of the dipole to each offer in each round. Here  $n = 20$  is the number of rounds. In other words,  $\mathbf{x}_7^l, \mathbf{x}_8^l$  are vectors containing  $x_7$  and  $x_8$  activations in the last four of the 12 periods of eyeballing in the  $l$ th round. It is assumed that around the last 1/3rd of interval  $T_1$  the subject already has an established opinion about the options.

There was, however, an additional complication to be taken into account. In simulations, it happened that due to exhausted neurotransmitter in the on-channel, the feedback loops  $x_7 \rightarrow x_1$  and  $x_8 \rightarrow x_2$  sometimes propelled the offers to the off-channel, where it was  $x_6$  (and not  $x_5$ ) that got activated; the off-channel reacted to the best option (the largest amount of omnium bonum on offer) with the relatively smallest signal among the four, in effect treating it as the least unacceptable; similarly, the worst option provoked the biggest response, making it the most unacceptable. Apparently, this phenomenon is not only psychologically plausible but is also easy to understand technically, when one considers that  $x_6$  depends on  $(-x_3)$  as per Eq. (14), and in equilibrium  $x_3$  is proportional to  $x_1$  as seen from Eq. (11).

Therefore, it was necessary to construct a criterion that would be channel-invariant. We solved the problem by implementing the following simple algebraic operations. First,  $\mathbf{x}_8^l$  was inverted so that in the position of its largest element, in the new vector  $\hat{\mathbf{x}}_8^l$  was placed the new smallest element, and vice versa:

$$\hat{\mathbf{x}}_8^l = \boldsymbol{\mu}_8^l - \mathbf{x}_8^l \quad (29)$$

In Eq. (29),  $\boldsymbol{\mu}_8^l$  is a vector of four real numbers, all equal to the largest element,  $\max(\mathbf{x}_8^l)$ , of the activity vector  $\mathbf{x}_8^l$ . The obtained  $\hat{\mathbf{x}}_8^l$  represents the 'off-channel point of view' on the options and is compatible with  $\mathbf{x}_7^l$  in the sense that both have their largest, second-largest, etc. elements in the same positions. Eq. (30) shows how both were summed and divided by two to obtain their average,  $\bar{\mathbf{x}}_{7,8}^l$ , which is one form of the needed channel-invariant representation:

$$\bar{\mathbf{x}}_{7,8}^l = (\mathbf{x}_7^l + \hat{\mathbf{x}}_8^l) / 2. \quad (30)$$

Finally, to place all  $\mathbf{F}_1$  elements between 0 and 1, vector  $\bar{\mathbf{x}}_{7,8}^l$  was normed by dividing its elements by the largest among them:

$$\mathbf{F}_1 = \bar{\mathbf{x}}_{7,8}^l / \max(\bar{\mathbf{x}}_{7,8}^l). \quad (31)$$

Operations (29)–(31) were certainly not the only way to neutralize the channel-alternating effect. Perhaps a more 'neuronal' and sophisticated approach could have been implemented, in contrast to the present algebraic one. Yet, what was done was simple and robust—two qualities needed for the computationally intensive stochastic optimization, to be described in Section 4.2.

##### Factor #2: Emotional long-term memories and economic reputations

The decision function (25) required that all past experiences with a supplier – positive and negative – be united in a single quantity to represent its market reputation. One way of characterizing a supplier  $k$ 's reputation was this:

$$\mathbf{F}_2 = 1 / (1 + \exp(-z_{7k}^l / z_{8k}^l)), \quad (32)$$

with  $k = A, B, C, D$ . Eq. (32) ensures that the widely varying ratio  $z_{7k}^l / z_{8k}^l$  of positive and negative aspects of the reputation stays constrained between 0 and 1 as can be seen in the simulation with real data in Fig. 3, top left plot. Alternative approaches involved other functions of the LTM ratios, their logarithmic transformations, their scaled logistic transformations, LTM differences as arguments of logistic or other nonlinear functions, etc. Eventually, Eq. (32) was selected as the best compromise between simplicity and adequacy for the task.



### Factor #3: Remembered particular consumer satisfaction

The two factors discussed already dealt with (i) current emotion and (ii) reputation, or emotional memory, the latter being a kind of time-average over all interactions with a supplier. However, neither (i) nor (ii) covered a particular type of memory regarding customer satisfaction, which was about what the subject remembered from their last interaction with a supplier, no matter how far in the previous rounds this had happened. An econometric study revealed the importance of this variable and here we had it as a decision factor in its own right. It is, therefore:

$$\mathbf{F}_3 = \Psi_{last}^{t-1} \quad (33)$$

Here,  $\Psi$  is a vector of a participant's self-assessed emotions of *disappointment* or *satisfaction*. The superscript in the *rhs* of Eq. (33) indicates that  $\mathbf{F}_3$  keeps track of every emotion from the first to the penultimate round,  $t - 1$ , which was experienced after a supplier delivery. For example, if the game is now in its round #13, Supplier A may have been chosen for the last time in the second round when it made the participant *Very disappointed* ( $-3$ ), Supplier B in the 12th whereby the participant felt *More satisfied than disappointed* ( $+1$ ), Supplier C in the 7th when the participant was *Disappointed* ( $-1$ ), and Supplier D in the 11th when the participant felt *Extremely disappointed* ( $-4$ ). It is the values of the disappointment–satisfaction scale in these rounds that comprise the current content of  $\Psi_{last}^{t-1}$ . Again, all values were rescaled from  $[-4, 4]$  to  $[0, 1]$ .

Apparently the three factors are related, but each carries an aspect of information that is distinct from the other two. Using them together as independent variables in regression equations would of course be problematic; however, the hybrid neural model poses no such restriction. In fact, it is well known that simple heuristic rules perform no worse than regression techniques precisely because they do not demand lack of correlation among the input factors (Gigerenzer, Todd, & The ABC Research Group, 1999).

#### 4.2. Calibration of the model

Because this study sought to identify the degree in which people acted intuitively as accounted for by the hybrid neural model, it was person-by-person behaviour that had to be analysed. Be the model successful in predicting a participant's decisions, then that person must have chosen suppliers in the simplest affective–intuitive way without any strategizing—otherwise the model would have failed. Testing this proposition practically meant having the equations calibrated with approximately half of a participant's data, and then assessing model performance with the other half. Naturally, the records used for calibration had to be chronologically first, while those for testing had to cover the later rounds, a fact that brought in some additional complications.

As the game unfolded, participants could not only form opinions about suppliers and generally adjust themselves to the circumstances, but could also begin to act routinely and even become bored. To counter that, as already explained, the experimental design offered constantly increasing quantities of *omnium bonum* in half of the treatments, and a scheme with growth followed by downturn in the other half. However, both these schemes implied an evolving course of events introducing differences between the calibration and test samples. Thus the READ-based model was challenged to the limit of its capabilities and naturally, its predictions of unknown data could not be as good as with a stationary process.

For each participant a model was calibrated with simulated annealing—a method for stochastic optimization. The objective was to make the model emulate the real decisions taken and customer satisfactions declared. Optimal values were sought for constants  $M, b_1, b_2, c_1, c_2, G, L, h_1$  in the READ Eqs. (6)–(24) and constants  $d_1, d_2, d_3$  in the decision rule defined by Eq. (25). Because

our setting contained twenty rounds, we took the first twelve for calibration and left the remaining eight for testing.

Each time a solution to the hybrid model equations was computed, the READ constants were generated as uniformly distributed random numbers from within certain intervals. Therefore all the solutions were independent realizations of the simulated annealing optimization process. The boundaries of most of these intervals were initially determined as roughly  $\pm 50\%$  around the values, used in the numerical simulations in (Grossberg & Schmajuk, 1987). With experience, more promising intervals were identified. After each 20 random solutions, the intervals were shrunk by  $\eta = 0.0001$  which ensured that, e.g., after 10,000 iterations the intervals became about 36.79% of their initial size. (Indeed,  $(1 - 0.0001)^{10,000} = 0.3679$ .) Whenever a new best solution in terms of the objective function (defined below) was found, it replaced the previous solution as determinant of the constants' intervals. For example, if  $\hat{c}_1$  was the new value of  $c_1$  for which the new best solution was achieved, the new lower and upper bounds of the interval for that constant would become  $c_1^{L(new)} = \eta(\hat{c}_1 - c_1^{L(old)})$ , and  $c_1^{U(new)} = \eta(c_1^{U(old)} - \hat{c}_1)$  respectively. This rule has the effect of reducing the interval in proportion of the proximity of the new best value to the border—the greater the proximity, the smaller the reduction. As discussed in Section 4.1, constants  $d_1, d_2, d_3$  were chosen by a slightly different scheme, whereby the new intervals for  $d_1$  were obtained in the way described on the example of  $c_1$ , but the intervals for  $d_2$  had lower and upper bounds zero and  $(1 - d_1)$  respectively, while  $d_3$  was always  $d_3 = 1 - d_1 - d_2$ .

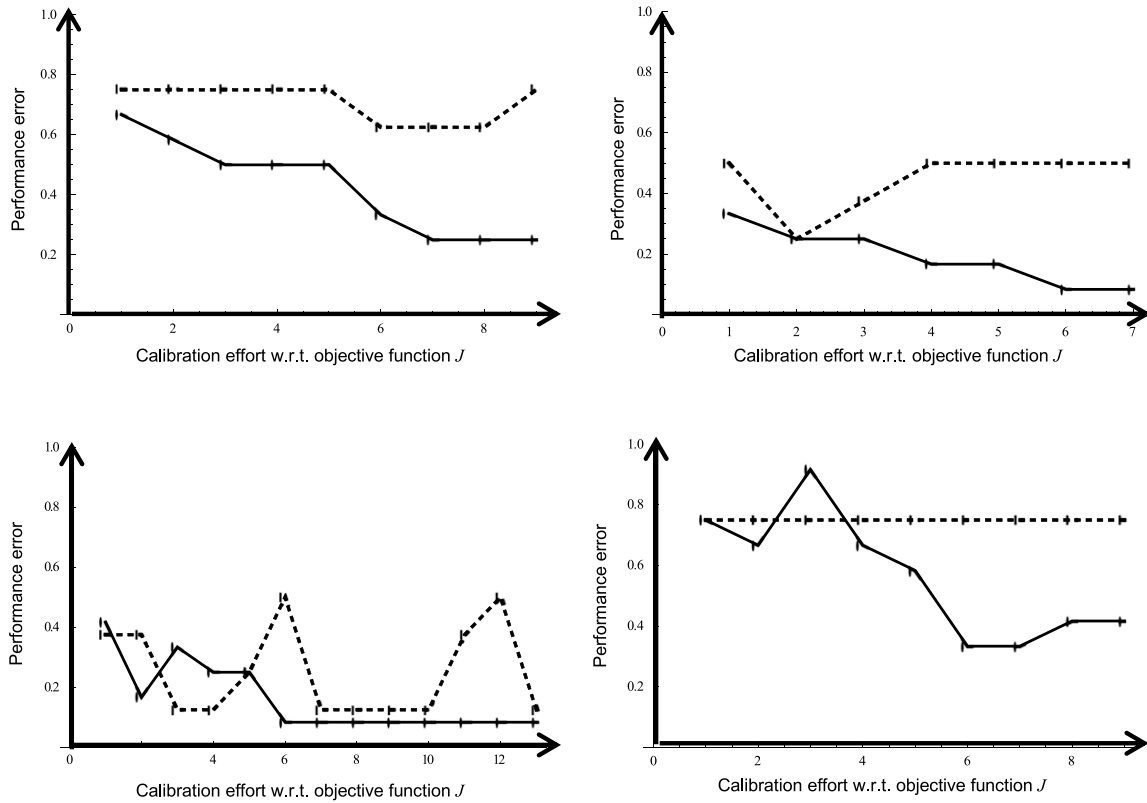
In a previous study, Mengov et al. (2008) implemented an objective function, optimized with respect to both emotional self-assessments and supplier choices, each carrying the same weight. Here we adopted a different approach because the present experiment involved more nonstationarity, which had to be addressed explicitly. This was achieved by defining an objective function that favoured the calibration rounds immediately preceding the test sample records. In particular, simulated annealing maximized this objective function:

$$J = \sum_{i=1}^{m_1} I_i + \gamma \sum_{i=1}^{m_2} I_{m_1+i} + R(\Psi(\mathbf{t}_{DS}), \mathbf{o}(\mathbf{t}_{DS})). \quad (34)$$

In Eq. (34),  $I_i$  is an indicator equal to 1 if in round  $i$  the model chose a supplier in the sense of Eq. (26) exactly as the participant, and 0 otherwise. The first summation is over the initial  $m_1$  rounds, while the second is over the next  $m_2$  rounds in the calibration sample. After some experimenting it was established that a good division comprised  $m_1 = 8$  and  $m_2 = 4$ , which left for test sample the last eight from the entire sequence of twenty.

The nonstationarity of the process was mitigated by enhancing the impact on  $J$  of the final  $m_2$  calibration rounds, whose correct predictions were weighted more ( $\gamma > 1$ ) than the initial  $m_1$ . Coefficient  $\gamma$  was chosen heuristically but not arbitrarily—it had to ensure a good balance between the two calibration subsets. Because of the crucial position of the second subset, it was decided that a combination of its all four correct choices contributing to  $J$ , together with totally incorrect choices in the first eight rounds would be valued more than another combination of all correct first eight plus only half correct among the next four. This amounted to  $0 + 4\gamma > 8 + 2\gamma$ , which meant that the smallest prime number to satisfy the inequality was  $\gamma = 5$ . This design implied that combinations of the same number of correct choices were treated differently depending on their configuration in the two calibration subsets and, say, four successes might occasionally be preferred to ten, as in the above example.

Which of two solutions with identical number of guesses in the two calibration subsets should be preferred? That answer is given by the third term in Eq. (34): the correlation



**Fig. 4.** Empirical error curves for the calibration sample (solid line) and the test sample (dotted line). On the y-axis is shown the prediction error in a zero-to-one scale with 'zero' meaning correctly guessed 12 calibration choices, and 'one'—not a single success. The x-axis gives the number of times objective function  $J$  achieved a new maximum during calibration with simulated annealing. At all such points, the obtained model parameter values were used to compute also the test sample error, whereby 'zero' stands for eight guesses. Although the panels show the new  $J$  maxima at equal distances, in practice every two neighbouring points were separated by several to several thousand computations of  $J$ . The two top plots show how excessive calibration may degrade model performance with test data. The bottom left plot shows a participant predicted with error of about 10%. More precisely, the correct guesses from 6th to 13th improvement of  $J$  were 91.67% for the calibration sample, and were 87.50% in the 7th, 8th, 9th, 10th, and 13th improvement for the test sample. The bottom right plot presents another person whose test sample choices were estimated correctly only 25% (error of 0.75), which is equivalent to a random guess.

$R(\Psi(\mathbf{t}_{DS}), \mathbf{o}(\mathbf{t}_{DS}))$  between all self-assessed customer emotions  $\Psi(\mathbf{t}_{DS}) = [\Psi(t_{DS}^{(1)}), \dots, \Psi(t_{DS}^{(m)})]^T$  and their model-predicted counterparts  $\mathbf{o}(\mathbf{t}_{DS}) = [o_1(t_{DS}^{(1)}), \dots, o_l(t_{DS}^{(m)})]^T$  at the actual self-reported moments  $\mathbf{t}_{DS}$ . Subscript  $DS$  indicates that  $\mathbf{t}_{DS}$  are the moments when the subject reported her/his disappointment ( $D$ ) or satisfaction ( $S$ ) with a mouse click. Here  $m = m_1 + m_2 = 12$ . As we explained in Section 2, quantities  $o_l(t)$  are the READ-predicted satisfaction  $o_1 = [x_5]^+$  and disappointment  $o_2 = [x_6]^+$ . In  $\mathbf{o}(\mathbf{t}_{DS})$  all  $o_2$  values are taken with negative signs. Now, if two different solutions during the simulated annealing procedure yield two identical choice predictions in the 12 calibration rounds, correlation  $R$  tips the balance in favour of the one that better accounts for the participant's emotions. Thus, the objective function of Eq. (34) selects solutions fitting the calibration data with increasing precision both with regard to choices and emotions. Using the latter for tie-breaking also prevents falling in traps of local error minima during the simulated annealing stochastic optimization.

## 5. Results and discussion

### 5.1. Solutions of the model

In this section we discuss the numerical solutions to the hybrid neural model. Each participant's calibration data were used to fit the model in a simulated annealing process of 5000–10,000 solutions. Characteristically, all runs of the algorithm virtually always yielded the same levels of prediction for a given person.

For example, if somebody's choices were predicted 50% correctly in the calibration sample and 38% in the test sample these figures were arrived at repeatedly, with very few exceptions, at the end of every new run. For most subjects, the task was computed at least twice, while for those producing either very poor or very precise forecasting the number of runs was extended to about ten. This was done to check the stability of the results, but as stated, turned out to be unnecessary.

While the outcomes were stable, the paths to them were not. Fig. 4 shows typical realizations of the procedure on the example of four subjects and indicates the degree of variability in the learning process.

It was expected that more calibration effort, or number of times the model is computed, would lead to higher prediction accuracy in the calibration sample, and would be accompanied by a bow-like performance in the test sample. Exactly that happened, as can be seen in the two top panels of Fig. 4. The four plots also suggest that people's predictability depended on the person being predicted. The top left plot shows a gradually diminishing forecasting error in calibration due to more computational effort, alongside a test sample not giving in—its prediction was successful initially only 25% and later rose to 37.5% (error falling from 0.750 to 0.625). The plot on the right, in contrast, tells the story of a quickly achieved test sample error of only 25%, soon lost due to excessive fine-tuning with the calibration sample. The remarkable thing here was that somebody's choices of one among four options could be guessed with 75% accuracy.

Even more impressive was the success with the person represented in the bottom left plot of Fig. 4. There, with the 6th

**Table 2**  
Prediction rates of supplier choices obtained with different methods in pre-test.

Prediction method	Prediction rate	
	Calibration sample: First 12 rounds ( $n = 340$ )	Test sample: Last 8 rounds ( $n = 272$ )
No method (pure guessing)	0.25 <sup>a</sup>	0.25 <sup>a</sup>
Multinomial logit model 1	0.5618	0.3713
Multinomial logit model 2	0.4794	0.3787
Fuzzy ARTMAP neural network	0.9971	0.4228
READ neural network,	0.5619 <sup>b</sup>	0.4262 <sup>b</sup>
Person-by-person	(0.1945)	(0.1837)

Here  $n$  is number of observations. The logit model equations used independent variables such as customer satisfaction/disappointment from the current and two immediately preceding rounds. Therefore, only data from rounds ## 3–12, or 340 from all 408 records were available to assess logit model performance in the calibration sample. All methods' prediction of calibration data are reported for those 340 observations only.

<sup>a</sup> Theoretical estimate.

<sup>b</sup> Averages over 34 participants. Standard deviations are given in parentheses.

improvement of the objective function the model guessed eleven out of twelve calibration choices (91.67%), and with the 7th improvement went on to achieve 87.50% success (seven out of eight) with the test sample. That result was repeated in the next three improvements of  $J$ , and again in the 13th and last improvement.

That was not the highest achievement with test data: one person out of 131 was predicted 100% correctly in their test sample, with prediction success of 91.67% in the calibration sample. Two people were predicted 87.50% correctly in their test sample with respectively 83.33% and 100% correct predictions in their calibration sample. A close inspection of these three people's data revealed no apparent anomaly in their behaviour, like staying absolutely loyal to one supplier, or adopting some other deterministic pattern of choices.

There were some cases, in which the test sample was better predicted than the calibration one, but these were exceptional and as a rule it was the other way round. In fact, the bottom-right plot of Fig. 4 gives the extreme opposite example, with no improvement in test sample prediction beyond 25%, regardless of the calibration effort and achievement.

The success shown in the bottom left plot of Fig. 4 needs some further discussion. Even if the prediction instability of the first six improvements of  $J$  can be regarded as natural, the issue arises why the test sample error was erratic again in the 11th, 12th, and 13th improvements. A combination of two factors explains the effect. First, the number of parameters to be optimized simultaneously – eleven – was too large, and second, the nature of the computational procedure itself contributed to the problem, as follows.

Some studies (Marder & Taylor, 2011; Prinz, Bucher, & Marder, 2004) have shown that both neural network models and single neuron models with even moderate number of parameters exhibit a particular kind of instability: indistinguishable model behaviours can arise from multiple sets of parameter values. A stochastic optimization method such as simulated annealing, in which every new solution is based on random generation of uniformly distributed parameter values within certain boundaries, is instability generator par excellence.

Because of all that it was impossible to find one 'true' set of parameter values. Coming back to the bottom left plot in Fig. 4, all five best solutions are equally good candidates for prediction of that person's choices, should there be any game rounds beyond twenty. Moreover, all of them would most likely produce very good further forecasts at least until the process nonstationarity makes them less relevant. Within this computational framework, we speak about predictability of human decisions in the next section.

Another consequence of the existence of a multitude of parameter sets is that it is usually not possible to conduct a meaningful statistical analysis on only a subset of parameters—they change in sync with the others, all of them mutually compensating each other. For example, our study could not establish which of the

three decision factors in Eq. (25) was most influential, because their weights  $d_1$ ,  $d_2$ ,  $d_3$  were optimized simultaneously alongside eight other constants.

All the same, Fig. 4 clearly shows that prediction of economic choices is possible at the level of the individual and, moreover, it can be remarkably precise. How often this may happen is the topic of the next section.

## 5.2. Predictability of human economic choice

We compared a number of methods for choice prediction. Table 2 shows their achievements with data from a pre-test with 34 subjects from a pilot study with economic parameters (quantities of offered and delivered omnium bonum) exactly as in the main study with 131 subjects.

What sets the READ model apart from the others is that it uses only 12 records to calibrate, and then is tested on eight subsequent records, all 20 from one and the same person. The results for these 34 people are then averaged and reported in the last row of Table 2. The standard deviations for this person-by-person prediction are quite large, apparently due to people's different degree of predictability—a fact demonstrated already in Fig. 4.

In particular, one person's test sample choices were forecasted with 0.0% success, while with two other people the model achieved only 12.5%. It went below 25% (pure guessing) because what it had learned during calibration later turned out to be counterproductive in testing due to the nonstationarity of human behaviour. Simply put, these subjects changed their choice strategies dramatically throughout the game's 20 rounds.

In contrast to READ, all other models in Table 2 use the entire pool of 340 records to calibrate, and are tested on another 272 records. Now person-by-person prediction cannot take place—it makes no sense to forecast somebody's eight consecutive choices using data from other people.

The state-of-the-art method for predicting economic choice among more than two options is multinomial logistic regression (Gujarati, 2011). It uses empirical data to compute the probability of each option, and the most probable is declared winner. Independent variables are selected exactly as in classical linear regression. Applying this method to our data achieved predictability of approximately 0.48 in the calibration sample and 0.38 in the test sample (Table 2, Multinomial Logit Model 2). A more complicated Logit Model 1 did better in calibration, but lost some ability to generalize as seen from its test sample result. Then, a more sophisticated tool such as Fuzzy ARTMAP went above 0.99 and 0.42 respectively. Again it must be stressed that Table 2 presents results, achieved in two quite different ways: one is classical statistical inference based on hundreds of records (Logit models and Fuzzy ARTMAP), while the other (READ-based model) is a theory-guided attempt at forecasting from only a handful of observations. In other words, in our

**Table 3**  
Prediction success of the hybrid neural model with the test sample.

	Pilot study without economic aggregates ( $n = 80$ )	Design without economic aggregates ( $n = 63$ )	Design with economic aggregates ( $n = 68$ )	$p$ -value, $F$ -test of Association, with/without economic aggregates
Share of participants with $\geq 75\%$ Correctly predicted choices	8.75% (7)	7.94% (5)	4.41% (3)	0.2036
Share of participants with $\geq 62.5\%$ Correctly predicted choices	25.00% (20)	15.87% (10)	14.71% (10)	0.1881
Share of participants with $\leq 25\%$ Correctly predicted choices	30.00% (24)	33.33% (21)	44.12% (30)	0.0049

The number of participants is denoted by  $n$ . In parentheses below the percentages is given the actual number of subjects. The pilot study consisted of five treatments with variations in their economic parameters. Nonparametric  $F$ -test compared differences between percentages with or without economic aggregates (omnium bonum total production, and production forecast), but not between those two and the pilot study.

particular task the Grossberg–Schmajuk model needed 12 records to achieve the same average forecasting accuracy as other methods reached with at least 340.

As was already established, people in the omnium bonum game were amenable to forecasting in various degrees and therefore could be arranged in a continuum with regard to their choice predictability. In this section, we discuss the most interesting facts about their distribution and suggest what could explain the available observations. Table 3 gives some essential figures about the test sample – last eight rounds – in a number of variants of the experimental design.

To better understand the capabilities of the model, several treatments were employed. Our pilot study was conducted with a total of 80 subjects and five design variations that helped establish the optimal economic parameters, later used in the main study. Table 3, column ‘Pilot study...’, gives average numbers over these pre-tests and shows their similarity to those obtained from the actual experiment.

Recalling the main purpose of the READ-based model—to account for rudimentary intuitive thinking, it made sense testing it also with a slightly more complicated economic problem. To this end we devised a new design by putting two more pieces of information on the screen right above the four suppliers. The first was “Total production of omnium bonum in the last round” and simply gave the sum of the four potential deliveries. The second was a forecast in percentage points about the omnium bonum production change in the current round. Because virtually all subjects were undergraduates or graduates who had taken at least a couple of economics courses, they could easily recognize the former variable as something a lot like a country’s gross domestic product (GDP) or the world production of commodities such as oil, ores, corn, etc., and the latter variable as a standard economic forecast about GDP growth. While these cues were totally irrelevant for the game outcome, they could trigger associations with economic concepts and theories, which in turn could provoke strategic thinking and lead the subject far away from primitive–intuitive thinking. In Section 3 we outlined an experimental procedure, which sought to avoid any allusions to real economic systems. In contrast, here we deliberately introduced such a factor in order to examine in a controlled way how task complication would affect choice predictability.

A less likely hypothesis was that on average, subjects might become overwhelmed by that information and would act more intuitively than those seeing only supplier offers and deliveries. However, the opposite seemed more plausible. The data in Table 3 are clearly in favour of the strategic thinking hypothesis: people who participated in a treatment with economic aggregates (omnium bonum total production in the current round, and a forecast about production change in the next round) turned out to be less predictable by the model. Fisher’s exact test of association (last column in Table 3), a conservative nonparametric test, approached significance for the differences between the shares of successfully predicted participants (above 75% or 62.5%), and was highly significant for those with whom the model failed. In particular, the

difference between approximately 8% and about 4% of very well predicted subjects (Table 3, first row) could not be statistically significant simply because those were only a handful of people.

In general, the hybrid neural model predicted extremely well the economic choices of 15 people out of 211, achieved solid results with many more, and was overall useful for about two thirds of them all. Those people for whom it worked apparently must have adopted a very simple approach with virtually no strategizing or expert economic thinking because these two modes were far beyond the model’s scope. Precisely that kind of decision making is what both popular culture and contemporary science call intuitive. Of course the scientific discourse has elaborated a lot on the notion of intuition and has identified a variety of phenomena that could be denoted by it (cf. Kahneman, 2011; Reyna & Brainerd, 2008). The present study deals with maybe the simplest and most primitive form of intuitive thinking about economic choice.

It is probably not surprising that a READ-based model had limited success with forecasting what agents did in a relatively complex economic game. From a different perspective, it is all the more remarkable that a computational model consisting almost entirely of equations for neurons and neurotransmitters, and utilizing behavioural data unrelated to neurobiology, could perform so well with so many people.

As we saw, the more economic information one received, the more one was inclined to think strategically; complicating the game invited participants to act less intuitively and be less predictable. The rate of successful model prediction correlated negatively ( $R = -0.26$ ) with the amount of accumulated omnium bonum in the treatments without economic aggregates (total production and growth forecast). It was even more so ( $R = -0.36$ ) in the treatments with aggregates, suggesting that the more intuition (of the simple type) one used in the game, the less successful one was economically.

As mentioned in Section 3, all 131 participants in the core study had to answer open questions about their strategies. Analysing formally these qualitative data is a challenge for a further study, yet some observations can still be outlined here. First, our expectation that subjects who acted intuitively might declare that explicitly was justified to some degree. Comments like, “I took my decisions mostly by intuition”, with variations, were not uncommon, though they were not widespread either. However, essentially the same phrases were sometimes used also by people with whom the model failed! In fact, the empirical material was not at all straightforward to interpret.

Reading through people’s answers led to the observations summarized in Table 4. It shows what were the main strategies identified in the debriefing and what was the distribution among them of the well- and poorly-predicted subjects by the neural model. The first finding is that both groups’ distributions share a lot of similarities. Plenty of those well predicted had developed their strategic thinking gradually, but so had done many among those poorly predicted. The situation is similar with the subjects who were quick to form a strategy, and also with those who were influenced by the

**Table 4**  
Most widely used strategies and model prediction success.

Type of strategy	Number of subjects with at least 62.5% correctly predicted test sample choices (Total of 20 subjects)		Number of subjects whose test sample choices were correctly predicted 25% or less (Total of 51 subjects)	
	Number of subjects	% of 20	Number of subjects	% of 51
Loyalty to one particular supplier	2	10.00	0	0.00
Developed strategy gradually and updated it throughout the game	6	30.00	20	39.00
Developed stable strategy very early in the game (before 3rd–4th round)	4	20.00	11	21.57
Decisions were influenced by the last one or two deliveries	3	15.00	5	9.80
Decisions were influenced by the gap between the last offer and actual delivery	3	15.00	5	9.80
Always chose the highest or second highest offer	5	25.00	3	5.88
Avoided the extreme (highest and lowest) offers	2	10.00	15	29.41
Choices were made randomly	1	5.00	2	3.92

From the 131 participants, only those 20 well-predicted and those 51 poorly-predicted are presented. Some subjects' reports were very elaborate and contributed to more than one category, while others were too brief and could not be included in any. The table does not include idiosyncratic strategies such as, "I deliberately exaggerated my disappointment to induce more favourable treatment".

actual omnium bonum deliveries. Both groups were not loyal to any particular supplier; both did not choose at random, etc. A hint at significant difference offered the category "Always chose the highest or second highest offer". Twenty five percent of the well-predicted subjects ascribed to it, while this was so far less than 6% of those for whom the model failed. A possible explanation may be that the two highest-offering suppliers were indeed those who on average delivered the largest quantities of omnium bonum (Fig. 1). Naturally, they left the most positive emotional memories in the participants, which is exactly the basis of READ success.

In general, Table 4 shows that the debriefing questions, elaborate and numerous as they were, could not uncover a clear discriminating factor, explaining why the READ neural model succeeded or failed. One hypothesis why that happened is that our model is relevant for a most primitive or elementary level of cognition, which is quite difficult to access with questions about self-assessed strategic thinking.

In addition, reading through the subjects' answers left the impression that the game had sometimes provoked thoughts unrelated to its content. Naturally, this was most often the case with participants in treatments with economic aggregates. Apart from that, two or three people had developed the idea that they could influence the behaviour of their suppliers, although the instruction did in no way suggest such an opportunity. One person confessed having deliberately and systematically exaggerated her customer disappointment by one or two degrees hoping to induce more advantageous omnium bonum deliveries.

## 6. Conclusions

Here we showed how a hybrid READ-based neural model was able to predict with great precision the choices of a significant minority of the participants in an experiment. Overall, the method proved its usefulness for about two thirds of all subjects. An important novelty of this study was that economic behaviour was analysed on a person-by-person basis.

It was remarkable that a model rooted in mathematical neuroscience was useful in predicting economic choices. This was possible because the model consisted of a neural circuitry underlying those cognitive–emotional interactions, which form the basis of what contemporary science considers a simple form of intuition.

The exact figures of the model performance are related to a concrete lab market with four suppliers of a good, and would have been different for a larger set. It was known (Mengov et al., 2008) that with only two economic options READ can achieve impressive forecasting heights. The current study seems to have reached the boundaries of that circuit's capabilities which probably makes it unnecessary to design more complicated experiments around it.

We showed that more difficult economic tasks call for application of more sophisticated neural networks.

At present, neuroscience has not yet reached the level of detail, allowing rigorous modelling of brain circuits and identifying a single most appropriate structure for cases such as our experiment. The model used here was only one of three possible variants outlined in Grossberg and Schmajuk (1987). Moreover, Gaudiano, Surmeli, and Wilson (1994) have spotted a striking resemblance of a neural architecture by Raymond, Baxter, Buonomano, and Byrne (1992) to READ which is both functional and structural. That network could potentially have served our purposes equally well. It might be expected that with more knowledge in the future, research efforts like this one will become more precise, and identifying the best neural network will be a more clear-cut exercise.

Our experiment shares many characteristics with real markets, in which companies with different business practices compete to attract and retain customers, while the private individual must take decisions under substantial uncertainty and has little or no opportunity to influence their behaviour. One important feature that real markets and this study have in common is that in both, agents are heterogeneous in the sense that they perceive differently what is at stake. When choices are considered important, much effort and strategic thinking is involved, while ordinary situations are tackled more intuitively.

The study presented here puts the economic agents in a new perspective as they may now be seen as reacting differently to the same options due to their different attitudes towards the importance of what is to be gained or lost. Alongside the existing dimensions of agent heterogeneity would be added a new one, incorporating the strategic vs. intuitive thinking continuum. This aspect is related to, but is at the same time different from the classical view of economics, positing that two agents in the same situation choose differently due to their different risk attitudes as expressed by the concavity/convexity of their utility functions. For many economists the question is, "How risk-averse, i.e., how afraid is the agent in the face of a risky prospect?" The present study outlined a different question, which is: "Is the choice important enough for strategic thinking to get involved, or would simple intuition be sufficient?"

Yet another dimension is added by the fact that psychology seems to have identified at least two different phenomena under the umbrella of "intuition". One is the most rudimentary intuition, produced by System I (Type I process) according to many dual-process theories, while the other is the gist-based intuition, characteristic of adults and experts, as formulated by fuzzy trace theory. All scientists, familiar with the Grossberg–Schmajuk theory would probably not be surprised if in the future, gated dipole models turned out to be useful in studies of both types of intuition.

The one envisaged by fuzzy trace theory would probably demand a more complex neural network around the dipole.

New questions with economic relevance may arise out of this theoretical framework: “When a choice is important enough, is the agent expert enough to address it with her quick gist-based intuition, or must she resort to slower deliberative thinking? And even if she is extremely competent and erudite, perhaps the situation is too unusual and complicated, and therefore demands more than just intuition?”

Finally, studies such as this one may help economic analysis in the effort to devise realistic models of large-scale agent-based systems. There, primitive–intuitive agents will coexist with gist-based intuitive agents; at the same time, more rational agents will be guided by a variety of strategies. Grossberg’s theoretical method of combining and recombining three basic differential equations to build complex models offers a research paradigm, which seems to be very useful for such endeavours. In addition, it fits comfortably in Von Neumann and Morgenstern (1944) view that science needs “...methods...which could be extended further and further”. Introducing neuropsychological knowledge in behavioural economics may become another powerful way to discover how socioeconomic systems function.

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