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Entanglement Entropy in Quantum Many-Body Systems

Research Group: Quantum Many-Body Dynamics

Research field: Quantum Information, Many-body physics, Quantum Metrology, Quantum Sensing

Head of the research group

Assoc. Prof. Peter A. Ivanov

Members of the group

Yoana R. Chorbadzhiyska

Venelin P. Pavlov

Bogomila S. Nikolova

Nadejda Y. Markova

Project guidelines

The study of the dynamics of many-body quantum systems is of key importance in various fields of physics, such as condensed matter physics, quantum physics, quantum informatics, and statistical physics. A key feature of an interacting many-body quantum system is the creation of entangled quantum states. Although the initial state of the quantum system can be written as a product of the states of the two subsystems, the unitary evolution under the action of the many-body Hamiltonian can generate entanglement. Entangled states are those that cannot be represented as a product of single-particle states. Applications of quantum entangled states range from practical to purely fundamental in nature. For example, quantum entangled states play an important role in quantum algorithms and quantum computing. On the other hand, quantum entanglement is the basis of the understanding of the non-locality of quantum mechanics and of the thermalization process of an isolated quantum system. The main idea of the current project is to investigate the entropy of entanglement and thermalization in multi-particle quantum systems, which can be experimentally realized in a system of laser-cooled ions in a Pauli trap.

Introduction

An isolated non-integrable quantum system prepared in an out-equilibrium state undergoes a process known as quantum thermalization. One of the most successful approaches for describing this intriguing quantum phenomenon is the Eigenstate Thermalization Hypothesis (ETH), which assumes that the expectation values of an observable calculated in the basis of eigenstates of the non-integrable Hamiltonian are equal to the average calculated with the microcanonical ensemble. The validity of the ETH has been studied in various quantum many-body systems by means of exact diagonalization. Moreover, there have been key experiments demonstrating quantum thermalization in isolated systems using ultracold atoms, superconducting qubits, and trapped ions. Usually, the quantum thermalization in these systems is associated with increasing the complexity, because it requires a large Hilbert space which grows exponentially with the system size.

In this work we study the emergence of quantum thermalization in a small system consisting only of a single spin and two bosonic degrees of freedom which is described by Jahn-Teller (JT) model. The JT model describes electronic orbitals coupled to vibrational modes either in molecules or solids. The JT effect is related to a structural instability in electronically degenerate states of molecules, where electron-phonon interaction shifts the potential minima of the nuclei, which causes distortion of the molecular configuration. Here, we show that the quantum-optical analog of JT model undergoes a finite size quantum phase transition. Then we proceed to study the ETH in the different quantum phases of JT model.

Methodology

We consider a model consisting of a single spin-1/2 system with energy splitting Δ and a two-dimensional quantum oscillator with mass m and frequencies ω_a and ω_b which interact via Jahn-Teller coupling. The introduction of a pair of creation and annihilation operators $\hat{a}^\dagger$, $\hat{a}$ and $\hat{b}^\dagger$, $\hat{b}$ for each oscillator allows us to write the following Hamiltonian:

$$\hat{H}_{\rm JT} = \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \frac{\Delta}{2}\sigma_z + g_a \sigma_x(\hat{a}^\dagger + \hat{a}) + g_b \sigma_y(\hat{b}^\dagger + \hat{b}).$$

The total Hilbert space is spanned in the basis $\{|s\rangle|n_a, n_b\rangle\}$, where $|s\rangle$ ($s = \uparrow, \downarrow$) is the eigenstate of σ_z and $|n_i\rangle$ is the Fock state of the bosonic mode with occupation number n_i. In this work we consider the limit when the ratio of level-splitting Δ to bosonic frequencies ω_i grows to infinity $\eta_i = \Delta/\omega_i \to \infty$, which essentially plays the role of an effective thermodynamic limit in our model. We define dimensionless spin-boson couplings $\lambda_i = 2g_i/\sqrt{\omega_i \Delta}$ and investigate both analytically and numerically the different quantum phases of JT model.

Then we connect the quantum phase transition to the emergence of quantum thermalization in the system by verifying that the conditions of ETH are met in the superradiant phase. We study the effective time-averaged dimension of the JT model, which is a measure for ergodicity.

Results

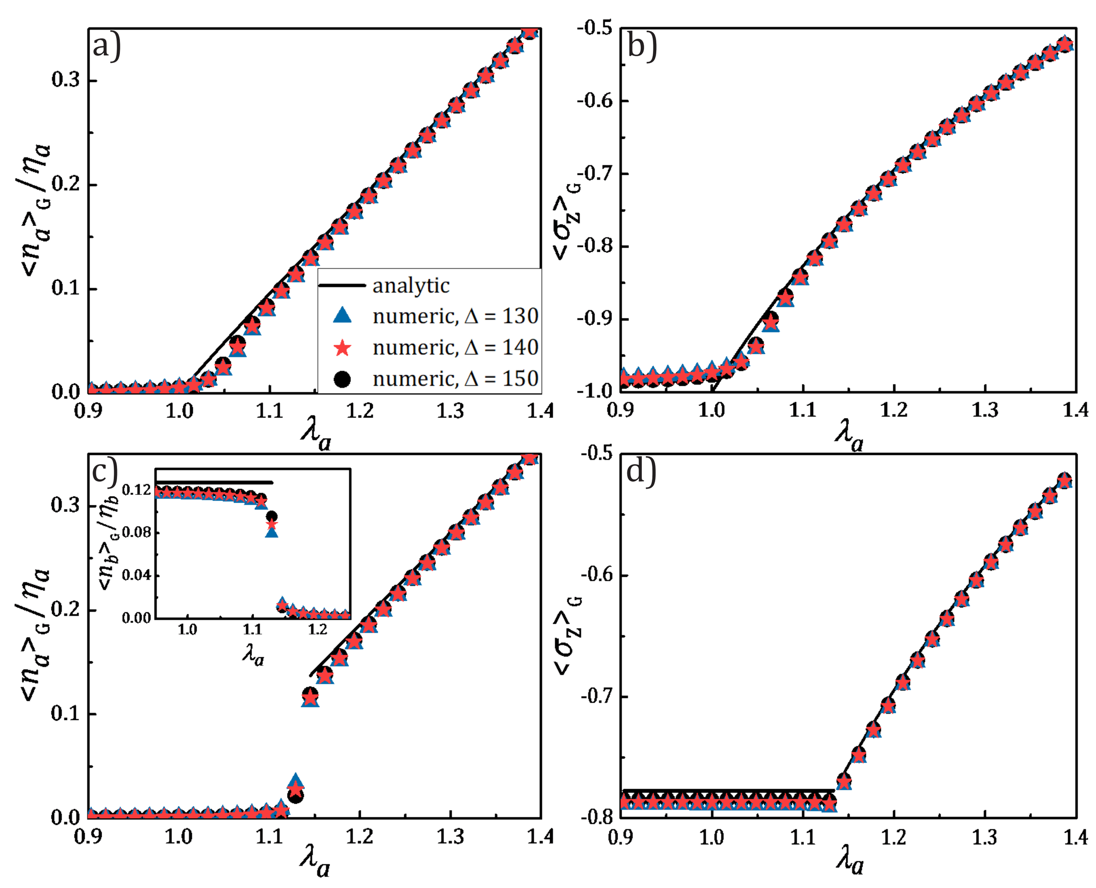

Figure 1: Mean bosonic excitation $\langle \hat{n}_a \rangle_G$ and mean-field value of $\langle \sigma_z \rangle_G$ as a function of the dimensionless spin-boson coupling λ_a for various Δ. a) and b) Second order quantum phase transition between normal phase to a-superradiand phase, where we set $\lambda_b = 0.96$. c) and d) First order quantum phase transition between b-supperadiant phase and a-supperadiant phase. We set $\lambda_b = 1.13$ and vary λ_a across the transition point $\lambda_a = \lambda_b$. The mean bosonic excitations $\langle \hat{n}_a \rangle_G/\eta_a$ and $\langle \hat{n}_b \rangle_G/\eta_b$ (inset) show jump at $\lambda_a = \lambda_b$. The bosonic Hilbert space is truncated at $n_{\rm max} = 80$ for a- and b-modes.

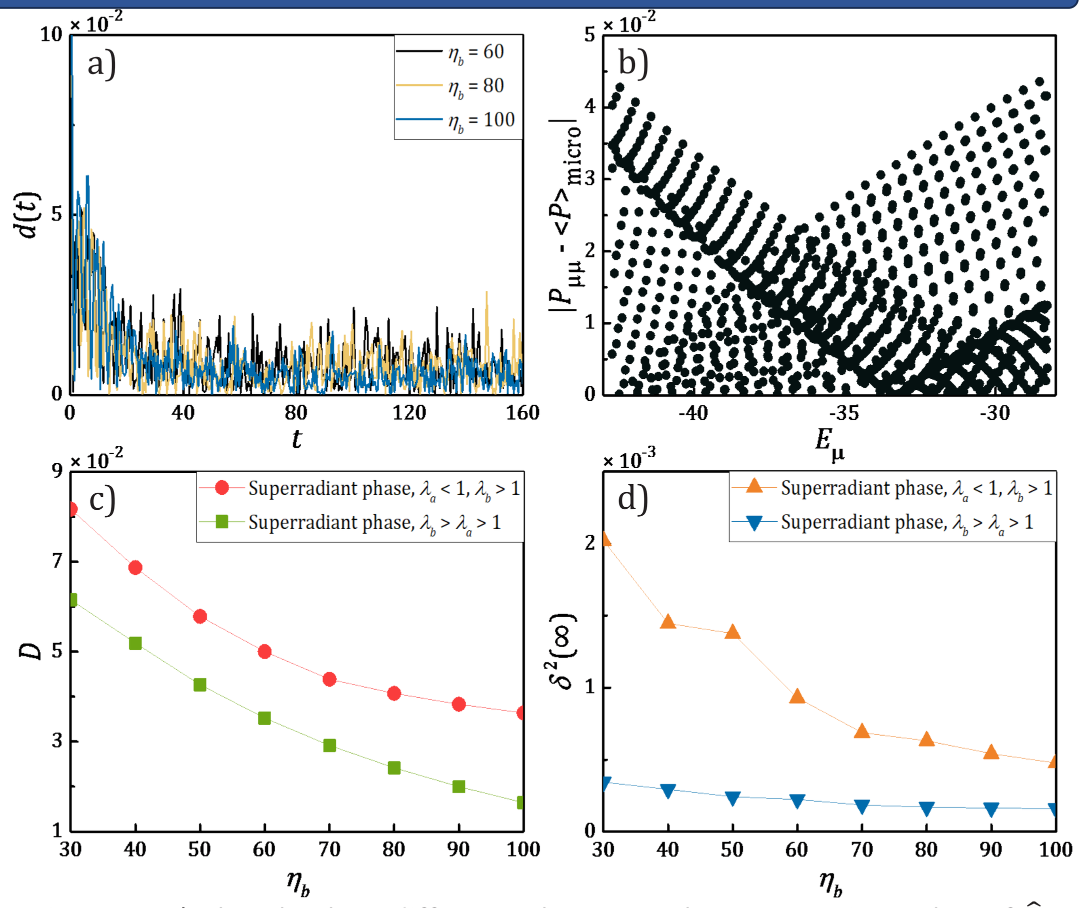

Figure 3: a) The absolute difference between the expectation value of $\hat{P}$ and the average predicted by the DE for various η_b. We set $\lambda_a = 1.096$ and $\lambda_b = 1.10$ such that the system is in a b-superradiant phase. b) The difference between the eigenstate expectation values and the microcanonical average for $\eta_a = 100$ in a b-superradiant phase. c) Comparison between the DE and ME averages of $\hat{P}$ as a function of the effective system size. d) The long-time fluctuations of the spin population. The bosonic Hilbert space is truncated at $n_{\rm max} = 150$ for a- and b-modes.

Finite size quantum phase transition

The JT system is in a *normal phase* when $\lambda_i \leq 1$, which is characterized by zero mean-field bosonic excitations of the two bosonic modes and polarized spin along the z axis, namely

$$\lim_{\eta_a \to \infty} \frac{\langle \hat{a}^\dagger \hat{a} \rangle_G}{\eta_a} = 0,\ \lim_{\eta_b \to \infty} \frac{\langle \hat{b}^\dagger \hat{b} \rangle_G}{\eta_b} = 0,\ \langle \sigma_z \rangle_G = -1.$$

The excitations are $\epsilon_{\rm np} = \omega_a\sqrt{1-\lambda_a^2} + \omega_b\sqrt{1-\lambda_b^2}$ which are real for $\lambda_i \leq 1$.

For $\lambda_a > 1$ and $\lambda_b < 1$ the system undergoes a second-order quantum *phase transition to a-superradiant phase* (Fig. 1 (a) and (b)) where the a-bosonic mode is macroscopically excited and the spin state is rotated with respect to its orientation in the normal phase:

$$\lim_{\eta_a \to \infty} \frac{\langle \hat{a}^\dagger \hat{a} \rangle_G}{\eta_a} = \frac{\lambda_a^4 - 1}{4\lambda_a^2},\ \lim_{\eta_b \to \infty} \frac{\langle \hat{b}^\dagger \hat{b} \rangle_G}{\eta_b} = 0\ ,\langle \sigma_z \rangle_G = -\frac{1}{\lambda_a^2}.$$

The excitations now are $\epsilon_{\rm sp} = \omega_a\sqrt{1-\left(\frac{\lambda_a}{\lambda_b}\right)^2} + \omega_b\sqrt{1-\lambda_b^{-4}}$ which are positively defined for $\lambda_b > 1$ and $\lambda_a \leq \lambda_b$.

The quantum *phase transition between b- and a-superradiant phase* when $\lambda_a > 1$ and $\lambda_a > \lambda_b > 1$ is of first order. Discontinuity of the order parameters at the critical point $\lambda_a = \lambda_b$ is observed (Fig. 1 (c) and (d)).

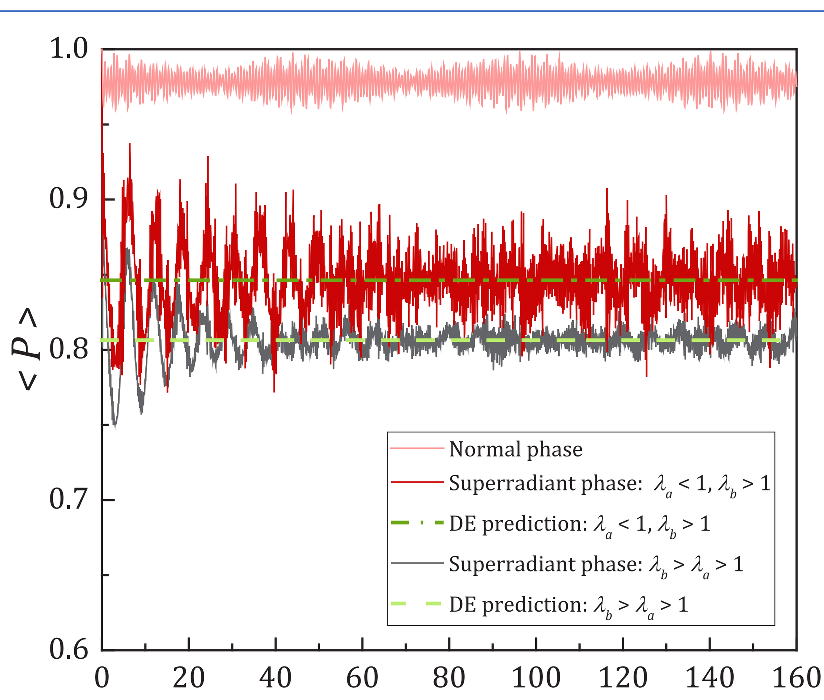

Figure 2: Time-evolution of the spin population $\langle P(t) \rangle$ compared to the DE prediction in various quantum phases. The initial state is $|\Psi_0\rangle = |\downarrow\rangle|5_a, 10_b\rangle$ and the parameters are set to: $\omega_a\Delta = 90$, $\omega_b\Delta = 100$ and $g_a = 1.5$, $g_b = 2.0$ (normal phase); $g_a = 1.5$, $g_b = 5.5$ ($\lambda_a < 1$, $\lambda_b > 1$); $g_a = 5.2$, $g_b = 5.5$ ($\lambda_b > \lambda_a > 1$).

Quantum thermalization in Jahn-Teller system

We assume that the system is initially prepared in an out-of-equilibrium state $|\Psi_0\rangle$ with mean energy $E_0 = \langle \Psi_0 | \hat{H}_{\rm JT} | \Psi_0 \rangle$ which evolves under the action of the unitary propagator. The long-time average of an observable $\hat{O}$ is given by

$$\langle \bar{O} \rangle = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau \langle \Psi(t) | \hat{O} | \Psi(t) \rangle\, dt = {\rm Tr}(\hat{O}\hat{\rho}_{\rm DE})$$

where $\hat{\rho}_{\rm DE}$ is the density matrix of the diagonal ensemble (DE). We investigate the time evolution of the spin observable $\hat{P} = |\downarrow\rangle\langle\downarrow|$ in the different quantum phases (Fig. 2). In the normal phase the spin population is nearly frozen around the initial value. But in superradiant phase $\langle \hat{P} \rangle$ shows oscillations which subsequently decrease and tend to the long-time average value. The increase of the effective thermodynamic parameter ameliorates this result (Fig. 3 (a)).

The equilibration of a closed quantum system into a thermal state implies that $\langle \bar{O} \rangle \approx \langle O \rangle_{\rm micro}$, where $\langle O \rangle_{\rm micro} = {\rm Tr}(\hat{O}\hat{\rho}_{\rm DE})$ is the microcanonical average of $\hat{O}$ taken over eigenstates with energies close to E_0. In this energy shell, in accordance with ETH, the diagonal elements of the observable take values close to the microcanonical average (Fig. 3 (b)). DE and ME averages agree (Fig. 3 (c) and (d)).

As a measure for ergodicity of our system we introduce an effective dimension $d_{\rm eff} = [{\rm Tr}(\langle \hat{\rho}(t) \rangle_t^2]^{-1}$. An increase with the spin-boson coupling is observed (Fig. 4 (a)). We study also its dependence on the initial state: $d_{\rm eff}$ becomes greater with the occupation number (Fig. 4 (b)). Higher values of $d_{\rm eff}$ lead to suppression of the temporal fluctuations and hence small dimension subsystems equilibrate.

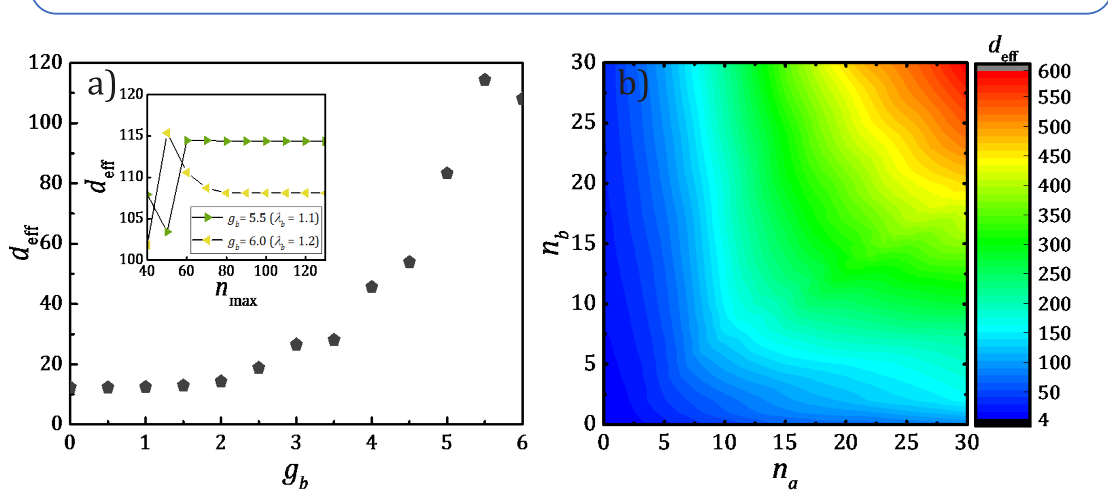

Figure 4: a) Effective dimension of the time-averaged density operator as a function of the spin-boson coupling g_b in the a-superradiant phase ($\lambda_a = 1.24$) where $\eta_b = 70$. The initial state is $|\Psi_0\rangle = |\downarrow\rangle|7_a, 8_b\rangle$. The bosonic Hilbert space is truncated at $n_{\rm max} = 90$ for each mode and the convergence of the results is verified (inset). b) Effective dimension of the time–averaged density operator as a function of the occupation numbers. The parameters are set to $\lambda_a = 1.096$, $\lambda_b = 1.100$, $\eta_b = 90$ and $n_{\rm max} = 150$.

Discussion

In the present work we have investigated the processes of thermalization in a small system consisting of single spin and two bosonic modes. We have found that the JT system undergoes finite size second order quantum phase transition between normal to superradiant phases and first order quantum phase transition between two superradiant phases. The latter phase transition is associated with a jump of order parameters at the critical point. We have studied the emergence of thermalization in the different quantum phases of JT system. In the normal phase the expectation value of the spin observable quickly oscillates around its initial value. In contrast, when the system is in a superradiant phase the observable quickly approaches its long time average value, which is given by the diagonal ensemble.

We have found very good agreement between the expectation value of the spin observable in the basis of eigenstates of JT Hamiltonian and the microcanonical average. This equality is the core of the ETH for the diagonal elements of an observable. We also have studied the effective dimension of our system, which is a measure for ergodicity. The large effective dimension implies that a large number of eigenstates contribute to the dynamics such that destructive interference causes suppression of the temporal fluctuations of the observable. We have found a large effective dimension in the superradiant phase. Finally, we have shown that the temporal fluctuation decays with the effective dimension.

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