**PEER REVIEW**

**on the Thesis Submitted for Awarding**
**the Scientific Degree "Doctor of Sciences"**
**in the Professional Field 4.5. Mathematics**

**Author of the Thesis: Prof. Ph.D. Alexandra Andreeva Soskova** from the Department of Mathematical Logic and its Applications of the Faculty of Mathematics and Informatics at the Sofia University "St. Kliment Ohridski ".

**Title of the Thesis:** "Computable Structure Theory: Jump of Structure, Coding and Decoding".

The peer review is written by **Prof. D.Sci. Vesselin Stoyanov Drensky, Full member of the Bulgarian Academy of Sciences**, working in the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences, Professional Field 4.5 Mathematics, as a member of the Scientific Jury for the procedure by Order No. РД 38-613/21.12.2020 of the Rector of the Sofia University.

## 1. General characteristics of the D.Sci. Thesis and the presented materials

The presented D.Sci. Thesis is written in English. It contains 270 pages and consists of an introductory chapter, one chapter with preliminary information, 5 chapters with the main results of the D.Sci. Thesis and 16 pages of bibliography with 162 titles. In addition to the D.Sci. Thesis and the author's abstract in Bulgarian and English, the documentation includes a list of all publications and of the publications included the D.Sci. Thesis, a list of citations of the works of the applicant, copies of the publications included in the D.Sci. Thesis, CV, diploma for higher education, diploma for candidate of mathematical sciences (which according to the Act on Development of the Academic Staff in the Republic of Bulgaria is equivalent to the educational and scientific degree "Doctor"), declaration for contributions and use of materials in other procedures, information for fulfillment of the minimum national requirements under the Act on Development of the Academic Staff in the Republic of Bulgaria, as well as two recommendation letters by Acad. Sergey Goncharov, Director of the Sobolev Institute of Mathematics of the Siberian Branch of the Russian Academy of Sciences in Novosibirsk and by Assoc. Prof. Ekaterina Fokina from the Technical University of Vienna.

## 2. Information and personal impressions for the applicant

The applicant Alexandra Soskova graduated the FMI of SU and received a Master of Science diploma in Mathematical Logic. Later, she completed the Ph.D. program at the FMI of SU and defended her Ph.D. Thesis on "Effective Algebraic Systems". With M.Sci. Diploma she worked for national computer firms. After the defence of her Ph.D. Thesis Alexandra Soskova visited for two years UCLA. Later and up till now she works at the Department of Mathematical Logic and its Applications of the FMI of SU, consecutively as an Assistant Professor, Associate Professor and Professor.

She was for two terms Chairperson of the Department and two years Deputy Dean of FMI. She has a rich administrative and organizational experience as a coordinator and principal investigator of national and international projects, chairperson and member of program committees and others.

I know Prof. Alexandra Soskova for almost 30 years. In that time the Bulgarian groups in Algebra and Mathematical Logic had many contacts and several times we had joint conferences. These contacts became stronger when the Sections of Algebra and Mathematical Logic of the Institute of Mathematics and Informatics were integrated in the Section of Algebra and Logic. I have excellent personal impressions for Prof. Soskova. In particular, she was the main driving force in the organization of the joint project between the University of Ghent and the Institute of Mathematics and Informatics in the frames of the bilateral cooperation between the Bulgarian Academy of Sciences and the Research Foundation ¬ Flanders. As a result the project which continued two times for three years each, was very successful both for the Belgian colleagues and the Bulgarian mathematicians working in Mathematical Logic, Algebra, and Algebraic Geometry, not only in the Bulgarian Academy of Sciences but also in the Sofia University and other Bulgarian universities. At the beginning of this year Prof. Soskova was one of the initiators of the extremely successful joint celebration of the Third World Logic Day by FMI–SU, IMI–BAS and the Institute of Philosophy and Sociology–BAS. Since I was a referee for the professorship of Prof. Soskova, I know very well her scientific achievements. She is the author or coauthor of 31 papers (10 included in the D.Sci. Thesis, and the remaining 9 are in journals, 10 in conference proceedings and 2 are biographical articles), 2 books in Bulgarian and has been the editor of the proceedings of 5 international conferences.

## 3. Analysis of the scientific and scientific-applied achievements of the applicant in the presented D.Sci. Thesis and the publications included in it

According to the introduction of the presented D.Sci. Thesis, the D.Sci. Thesis is in the field of computable structure theory and considers the relationship between definability and

computability in mathematical structures. The D.Sci. Thesis is devoted to the following natural questions: How difficult is it to represent a structure and can it be presented computably? How difficult is to compute relations reflecting different properties of the structure? These questions have been asked and have been answered in the study of the algebraic and combinatorial properties of the structure. From a formal point of view, the D.Sci. Thesis is in the computability theory, which is considered as a part of mathematical logic. But since the studied structures are algebraic and combinatorial objects and in particular groups, rings, fields, graphs or linear orderings, in my opinion the D.Sci. Thesis is in a boundary area of mathematical logic with algebra and combinatorics.

**The first chapter** of the D.Sci. Thesis is introductory and contains the motivation for the research, a description of the obtained results and in which publications they were obtained. I shall note that in the beginning of every chapter the applicant synthesizes in few lines the motivation for the investigations and then she describes in detail the obtained results and their relationship with the research of other authors.

**The second chapter** of the D.Sci. Thesis introduces the basic concepts, methods and facts necessary for the further exposition. It is written in a language understandable to a reader with minimal knowledge of mathematical logic, which makes the D.Sci. Thesis accessible not only to limited number of experts in the field, but also to a larger mathematical audience.

**The third chapter "Jump of a Structure"** is the first chapter containing new scientific contributions. Classical results of Turing allow to introduce a partial ordering between sets of natural numbers: $\boldsymbol{A} \leq_{\boldsymbol{T}} \boldsymbol{B}$ if there is a Turing machine which calculates the characteristic function of the set $\boldsymbol{A}$ using an oracle that can answer the question whether an integer $n$ belongs to the set $\boldsymbol{B}$. This defines an equivalence relation in the sets of natural numbers. The Turing jump $\boldsymbol{A}'$ of the set $\boldsymbol{A}$ consists of all integers $e$ for which the $e$-th Turing machine (with respect to some ordering of all Turing machines), equipped with an oracle associated with the set $\boldsymbol{A}$, stops working after a finite number of steps, starting its work from the integer $e$. The jump operation is also transferred to the equivalence classes. The natural question arises whether it is possible to transfer the jump operation in the case of enumerable structures preserving their algebraic and combinatorial properties. Several independently obtained answers to this question are known. The D.Sci. Thesis defines the jump of structures, using the ideas of Moschovakis and defines a new predicate, which is analogous to the stop set of Kleene. A jump inversion theorem is established based on Marker extensions and forcing, which is analogous to the classical jump inversion theorem without structures. Relativized versions of the results are given, as well as applications of the obtained results. In fact, the results are formulated and proved in terms of

spectra. (The spectrum of a structure is the set of all Turing degrees generated by the representations of the structure.)

In **the fourth chapter "Strong Jump Inversion"** a more precise notion than jump inversion is studied – the strong jump inversion. It is known that this property is shared by a number of structures, including Boolean algebras, some structures with equivalence relation, some abelian *p*-groups, countable models of differentially closed fields of zero characteristic, etc. In the D.Sci. Thesis theoretical-model conditions for the structure are given which imply the strong jump inversion. A general result is established, which guarantees the fulfillment of this property in terms of saturation, enumeration properties of the sets of types with formulas of low arithmetic complexity and effective type completion. When the structure is low relative to an oracle, information is obtained about the complexity of the isomorphisms between the structure and its computable copies with the help of the oracle. This result is applied to a number of classes of structures such as some classes of linear orderings, Boolean algebras, trees, differentially closed fields. In addition to the new results, refinements of results of other authors have been obtained.

**The fifth chapter "Effective investments and interpretations"** is devoted to the following question. For the known effective codings of one class of structures into another class, is there effective or more difficult decoding for special classes? Special attention is paid to the class of linear orderings and to nilpotent groups of nilpotency class 2. (These two classes and the class of undirected graphs are at the top of the Turing computable embeddings because every other class of structures is embedded in them.) Examples are given of graphs that cannot be reduced to a linear ordering and a jump of linear ordering. On the other hand, it is proved that every graph can be coded in the second jump of a linear ordering.

In 1960, Maltsev gave a computable definition of rings in the class of nilpotent groups of class 2 using parameters. In the D.Sci. Thesis it is proved that the class of fields can be interpreted in the class of nilpotent groups but without parameters, giving two proofs of embedding in the Heisenberg group (which consists of the 3 x 3 upper unitrigular matrices). In this way the result of Maltsev in the case of the fields is specified and improved.

The third result of this chapter gives an interpretation of an algebraically closed field of characteristic 0 in the group of matrices of the second order with determinant 1. Finitary existential formulas with two parameters are used. As a consequence, an interpretation is obtained, which is defined by elementary formulas of the first order.

**The sixth chapter "Cohesive Powers"** discusses effective variants of the model-theoretical constructions of ultraproducts and ultrapowers. An infinite set of natural numbers *C* is cohesive

if for every computably enumerable set $W$ almost all elements of $C$ are either in $W$ or in the complement of $W$. Cohesive powers of computable structures play the role of ultrapowers, where cohesive sets are analogs of ultrafilters. There are computable structures that have in-computable copies. In this connection, the natural question arises whether the cohesive powers of two isomorphic structures are elementary equivalent. The D.Sci. Thesis considers the case of two copies of a computable linear ordering. Conditions are given under which a computable structure is isomorphic to its cohesive power and the type of isomorphism of the cohesive powers of some computable linear orderings is investigated.

In the last **seventh chapter "On Cototality and the Skip Operator"** the substructures of the structure of the enumeration degrees $\mathcal{D}_e$ are studied, which strictly contains the structure of the Turing degrees $\mathcal{D}_T$. As noted in the D.Sci. Thesis, in many cases $\mathcal{D}_e$ is more convenient than $\mathcal{D}_T$ for analyzing the complexity of effective mathematics objects. The main focus is on the substructure of the cototal degrees. It is known that the topic has an interesting connection with the theory of simple groups. The set of non-identity words in a finitely generated simple group is cototal. Cototal degrees are strongly related to the skip operator, which is used as a tool in the investigations. In some sense, it is an analogue of the jump operator in the Turing degrees, and the D.Sci. Thesis proves an analogue of the jump theorem. Many examples are given of classes of enumeration degrees that guarantee or, conversely, prohibirt cototality. The chapter closes with a list of open problems. The importance of the obtained results is indirectly confirmed by the fact that the paper on which the results are based in the chapter was published two years ago, and there are already 10 citations.

The proofs use a rich arsenal of techniques from mathematical logic and related fields of mathematics. The obtained results are original, interesting and occupy an important place in the field. The nature of the scientific contributions of the applicant is in the development of new methods and enrichment of existing knowledge. They have already been highly estimated by experts, used in research by other authors and I have every reason to believe that their use will continue in the future. Since the results are also relevant to theoretical informatics, they would have an indirect impact on the application of the scientific results in practice.

Although not an easy reading, the D.Sci. Thesis is written with great care for the reader, which undoubtedly shows excellent pedagogical skills of the applicant. I believe that the text may be used successfully for introduction in the field of masters and doctoral students. I think that the applicant should consider writing a monograph, which is based on the D.Sci. Thesis, but also including similar results of other authors. Finally, I would recommend to post the D.Sci. Thesis

in arXiv to make it available to a larger audience of mathematicians both in Bulgaria and abroad.

**4. Analysis of the publications on the D.Sci. Thesis**

Prof. Soskova has included in her D.Sci. Thesis results from 10 of papers published in the period 2007–2020. One paper is without coauthors, 3 are with one coauthor, 1 with 2 coauthors, 2 with 5 and 6 coauthors and 1 with 8 coauthors. She has declared that the joint papers are written with the equal participation of all coauthors. The coauthors are Vatev and Harizanov in 4 of the papers, Soskov and Morozov in 3 of the papers, Calvert, Knight, Dimitrov and Shafer in 2 of the papers, Frolov, McCoy, Alvir, Goodman, Russell Miller, Weisshaar, Andrews, Ganchev, Kuyper, Lempp, Joseph Miller and Maria Soskova in 1 of the papers. In my peer review on the professorship of Prof. Soskova, I mentioned that until 2017 all her joint papers were with Bulgarian coauthors, and then things changed and she began to work actively with foreign mathematicians. This tendency is also demonstrated in the works included in the D.Sci. Thesis. All papers published since 2018 are with essential participation of foreign coauthors. The papers included in the D.Sci. Thesis are published in J. Logic and Computation – 2, J. Symbolic Logic –2 (paper No. 7, which is said to be submittted for publication, is accepted in this journal), Trans. AMS – 1, Lect. Notes Comp. Sci. – 2, in proceedings of conferences in Greece – 2 and 1 paper is posted in arXiv and submitted for publication. Five of the articles have an impact factor (1 in quartile Q1 and 4 in quartile Q3 with a total impact factor of 3,945). These 5 papers and 2 more have a total SJR of 5,791, and of the papers published in the last few years, 4 are in quartile Q1 and 1 is in quartile Q2 of Scopus. The applicant has presented a list of 78 citations, including citations of world-renowned scientists, of 17 of her papers, Ph.D. Thesis and Master Thesis. Of these citations 48 are of papers included in this D.Sci. Thesis, and one of the papers with Ivan Soskov is cited 25 times. The results have been presented many times in Bulgaria and abroad. I want to add the comment that I am impressed by the recommendation letter of the world-famous specialist in mathematical logic, Acad. Goncharov. It is 5 pages long and its content can easily be considered as an additional positive peer review. My inspection shows that:

a) The scientific works meet the minimum national requirements (under Art. 2b, para. 2 and 3 of the Act on Development of the Academic Staff in the Republic of Bulgaria) and respectively the additional requirements of Sofia University "St. Kliment Ohridski" for obtaining the scientific degree "Doctor of Sciences" in the scientific field and professional direction of the

procedure. Moreover, with minimum requirements of 100 points in two of the groups of indicators, the applicant has submitted evidence for 150 and 136 points, respectively.

b) Five of the scientific papers included by the applicant in the D.Sci. Thesis are used in the procedure for the academic position "Professor". These are the results that underlie the methods presented in the D.Sci. Thesis (Chapters 3 and 4). If they were not included in the D.Sci. Thesis, the text would be not self-contained and difficult to read. I believe that the inclusion of these five articles in the D.Sci. Thesis only helps to improve the exposition and is not a procedural violation.

c) There is no legally proven plagiarism in the submitted D.Sci. Thesis and scientific papers in this procedure.

**5. Qualities of the abstract**

In my opinion, the abstract was prepared in a very good but unusual way. The Bulgarian version has 141 pages and the English version has 139 pages. In practice, this is a text that contains an extremely detailed description of the results, and many of the proofs are sketched, which allows the reader to trace in detail the ideas used to obtain the results. On the other hand, if the reader only wants to get briefly information for the achievements in the D.Sci. Thesis, it is enough to read the introduction, which is 11 pages in Bulgarian and 10 pages in English. I am sure that the abstract meets all the requirements for its preparation and correctly presents the results and the content of the D.Sci. Thesis.

**6. Critical remarks and recommendations**

I do not have essential critical remarks, but I noticed some small inaccuracies in the documentation. For example, paper No. 2 from the list of publications included in the D.Sci. Thesis is on pages 113–117, not 114–117 as written. For paper No. 6 the numbers of the volume, the issue and the pages of the paper are missing (volume 85, issue 2, pages 673–690), and the paper itself is presented in a preproof version. It would be nice if the applicant were presented the published version of the paper or at least were sent it additionally to the members of the scientific jury. The information for paper No. 7 is that it has been accepted for publication after a minor revision and the revised version should be presented instead of the one posted in arXiv. It would also be nice to number the citing articles in the list of citations to facilitate the work of the members of the scientific jury. But all these remarks do not change my positive opinion for the D.Sci. Thesis and the results obtained in it.

## 7. Conclusion

After reading the D.Sci. Thesis presented in the procedure and the accompanying scientific papers and based on the analysis of their significance and the scientific and scientific-applied contributions contained in them, I confirm that the presented D.Sci. Thesis and scientific publications to it, as well as the quality and originality of the results and achievements presented in them, meet the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its application and the respective Regulations of Sofia University "St. Kliment Ohridski" for awarding of the applicant with the scientific degree "Doctor of Sciences" in the Scientific Field 4. Natural Sciences, Mathematics and Informatics and Professional Field 4.5 Mathematics. In particular, the applicant satisfies the minimum national requirements in the professional field and no plagiarism has been established in the scientific papers submitted at the procedure.

Based on the above, I **recommend** the scientific jury to award Prof. Ph.D. Alexandra Andreeva Soskova with the scientific degree "Doctor of Science" in Scientific Field 4. Natural Sciences, Mathematics and Informatics, Professional Field 4.5 Mathematics (Mathematical Logic).

March 21, 2021 Referee:

(Prof. D. Sci. Vesselin Drensky, Full member of the BAS)