REVIEW

on the competition for the academic position

"Associate Professor"

in the professional field 4.5. Mathematics (Differential Equations),

for the needs of Sofia university „St. Kliment Ohridski" (SU),

Faculty of Mathematics and Informatics (FMI),

announced in SG No. 65 from 16 august 2019 and on the Internet sites of FMI and SU

The review was prepared by Assoc. Prof. Dr. Mariya Georgieva Karatopraklieva-Koleva, Sofia university „St. Kliment Ohridski", Faculty of Mathematics and Informatics, professional field 4.5. Mathematics (Differential Equations), as a member of the Scientific Jury for the competition under Order No РД 38-593 /11.10.2019 of the Rector of the Sofia University

Only one applicant has submitted documents for participation in the competition: Chief Assistant Prof. Dr. Tsvetan Dimitrov Hristov, Faculty of Mathematics and Informatics of the Sofia University "St. Kliment Ohridski".

I. General description of presented materials

1. Information about the presented materials

The documents of the applicant comply with the requirements of the Act of the Development of the Academic Personnel of the Republic of Bulgaria (ADAPRB), the Rules for the Implementation of the Act of the Development of the Academic Personnel of the Republic of Bulgaria (RIADAPRB) and the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at SU (RTCAADOAPSU).

There are submitted on CD originals or copies of all documents of Dr. Tsvetan Hristov required by Art. 107 of RTCAADOAPSU:

 professional autobiography,
 higher education diploma with the attachment thereto,
 diploma for educational and scientific degree "Doctor",
 certificate for occupation of the academic position " Chief Assistant Professor ",
 certificate of seniority in the specialty,
 official notice on the beginning and length of service in the Sofia University "St. Kliment Ohridski" with a description of the positions held and the dates of their appointment to them;
 medical certificate attesting to his mental and physical health;
 criminal record certifying that no punishment was imposed for "deprivation of the right to pursue a particular profession or activity"
 a/ list of all publications
 b/ list of publications submitted for the competition,
 list of publications, conferences, projects and scientific supervision activities, generated by the information system "Authors" of SU;

 model reference for meeting the National minimum requirements and the requirements of SU for the scientific field 4. Natural sciences, mathematics and informatics, professional field 4.5 Mathematics, for the academic position "associate professor" and the additional requirements of SU; attachments evidence of Groups of indicators А, В, Г, Д and the citations of publications, and with Web of Science and Scopus excerpts on science metrics for editions

 List of citations with full bibliographic description of the cited and citing publications
 Reference for original scientific contributions,
 Reference for the degree of fulfillment of the indicators under Art. 122, para. 2 of RTCAADOAPSU, with appropriate evidence with description and the following attachments: *Certificate of scientific guidance of a graduate student; Certificate of successful pedagogical activity, Certificate for participation in projects; Information on general study and classroom employment in FMI of SU from 2014/15 to 2018/19 including; Study materials on Differential Equations.*

 publications presented at the competition,
 abstracts of the publications presented at the competition (in Bulgarian and in Eng-lish),
 the competition announcement in the State Gazette.

The following materials are also included on this CD:

- Habilitation extended reference for scientific contributions according to RIADAPRB, from Annex to Art. 1a, Para. 1, Note 12;
- Recommendation from a professor in the field;
- Co-authors declarations - 2 items.

All submitted documents and materials are well prepared, numbered, named and arranged, which makes it easy to work with them.

2. Information about the candidate

Tsvetan Dimitrov Hristov was born on April 16, 1974 in Cherven Bryag. Since 1993 he had been a student at the Faculty of Music at Sofia University “St. Kliment Ohridski ” and in 1998 he graduated with qualification "Master" Mathematics with specialization Differential equations. He acquired the Doctor's Degree in Differential Equations in 2006 with a dissertation on "Singularities of solutions of hyperbolic equations in regions with characteristic boundary".

From October 5, 1998 to January 31, 2001, Dr. Hristov worked as a mathematician at the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences. From 01.08.2005 he is an assistant, and since 16.04.2007 he is a chief assistant professor at SU at the Department of Differential Equations at FMI. From 1998 until his appointment as an assistant, he worked part-time at FMI.

He has been a member of the Faculty Board of FMI since 2018 and of the General Assembly of Sofia University since 2015. He was the secretary of the Department of Differential Equations from 2008 to 2018. He participates in the creation and development of the 3D Visualization Lab at FMI.

Dr. Tsvetan Hristov has been in the teams of 4 projects at the Research Fund of the Ministry of Education and Science, and since 2017 he is a member of a bilateral cooperation agreement between the Bulgarian NSF and the Russian NSF. Since 2003, he has been involved in the projects at has so far participated every year in the projects at the Research Fund of Sofia University “St. Kliment Ohridski"; he was Head of Contract No 184/26.04.2010.

3. General characteristics of the applicant's scientific work and achievements

Dr. Tsvetan Hristov participates in the competition with 13 publications in Bulgarian or foreign scientific editions - 12 scientific articles and an article related to the university training in Differential Equations. The list of all 25 publications annexed to his competition papers contains another 11

scientific publications - numbered 14 to 24, and numbered 25 - a training article on Equations of Mathematical Physics.

All the scientific papers of Dr. Tsvetan Hristov are in the field of partial differential equations. The boundary problems considered can be grouped into three main groups according to the type of equations to which they relate, namely: boundary problems for strictly hyperbolic equations, boundary problems for degenerate hyperbolic equations of Tricomi type, boundary problems for degenerate hyperbolic equations of Keldish type. In all those papers the investigations are in three- or four-dimensional domains whose boundaries contain characteristic surfaces for the equations. In the papers [1] - [12] of the list of Dr. Tsvetan Hristov for participation in the competition, some of the boundary problems formulated by Protter (also called Protter-Morawetz problems), as well as some of their generalizations, mainly for degenerating hyperbolic equations of Tricomi-type or of Keldysh-type have been investigated. For the investigated boundary problems, results were obtained for the uniqueness or existence of different types of nonclassical solutions, for the growth of generalized solutions or of their first partial derivatives around parts of the characteristic boundaries of the areas under consideration, for the incorrectness of some of the adjoint boundary-value problems in the search for classical solutions. The results greatly contribute to the development of some aspects of the theory of degenerate hyperbolic equations in the three-dimensional and four-dimensional cases and generalize the results of Dr. Tsvetan Hristov's doctoral dissertation on strictly hyperbolic equations and generalizations or additions to a number of results by other scientists on degenerating hyperbolic Tricomi- or of Keldish- type equations in the two-dimensional case or in spaces of dimensions 3 and 4.

a) The scientific papers meet the minimum national requirements (under Article 2b, paragraphs 2 and 3 of ADAPRB) (by some indicators exceeding them significantly) and the additional requirements of SUfor occupation of the academic position “Associate Professor” in the scientific field and professional direction of the competition:

From **Appendix 2: Group of Indicators B** and the evidence to it is clear, that Tsvetan Hristov has 111 points from scientific publications with numbers [1] and [2] from the List for participation in the competition, representing the Habilitation work for Indicator 4. These two publications are in the journals of those included in Q1 (Mathematics) and Q4 (Mathematical Physics) at Web of Science and have IF 1,156 - 2017, IF 0,71 - 2017 respectively.

From Appendix 3: Group of Indicators Г, Indicator 7. Scientific publications in editions that are referenced and indexed in world-renowned scientific information databases (Web of Science and Scopus) follows that of the List for the contest other two publications are in journals included in the Web of Science in Q4, and 8 more are in journals included in Scopus and having SJR. In Indicator 7, the scores are 312 and significantly exceed the minimum score of 200 for the Indicator Group Г.

In the Indicator Group Д, Indicator 11. Citations in two tables are noted the cited works of the candidate:

1. Citation in Impact Factor Scientific Papers (JCR - WoS) - 10 citations (= 80 points) of papers with numbers [1], [2], [7], [8], [6] and [4] from the List of publications for participation in the competition. The citations are in articles published in the period 2016 - 2019 in journals, two of which are in Q1 (Mathematics), IF 1,188 – 2018 and Q2(Mathematics), IF 0,828 – 2017 of Web of Science.

2. Citations in scientific publications with Impact rank (SJR - Scopus) - 50 citations (= 400 points) of works with numbers [1], [2], [9], [7], [8], [4] and [3] from the List of publications for participation in the competition and with numbers [23], [24] from the List with all publications of the applicant.

Points of citations from the first table clearly exceed the minimum number of 50 points for the Group of indicators Д.

b) From the reference form for the fulfillment of the minimum national requirements under Art. 26 of ADAPRB Appendix 1: Group of indicators A and The Profile of Ch. Ass. Prof. Dr. Tsvetan Dimitrov

Hristov in NACID is visible that the articles from the List for participation in the competition were not used for the acquisition of the educational and scientific degree “Doctor”.

c) There is no proven lawfully plagiarism in submitted in the competition of scientific works.

4. **General characteristics of the applicant's teaching activities**

Dr. Tsvetan Hristov's teaching activities are extremely diverse. He has taught and continues to deliver lectures, seminars, or computer-based exercises not only in compulsory or optional subjects in the fields of Differential and Partial Differential Equations for Bachelor Degrees: Software Engineering, Informatics, Applied Mathematics , Statistics , Mathematics and Informatics and Mathematics at FMI, but also in the compulsory subjects in the field of Mathematical Analysis for the specialties of Bachelor Degree of the Faculty of Mathematics and Faculty of Physics. From the *Information on general study and classroom employment in FMI of SU from 2014/15 to 2018/19 including* it is clear that each of these school years he has significantly exceeded the annual standards for general teaching and classroom employment.

Tsvetan Hristov has led seminars in eight and lectures on four compulsory subjects in the Bachelor degree in FMI or Faculty of Physics. I will note his careful preparation of computer exercises and project topics, and his detailed structured teaching of students in "Differential Equations and Applications" for the specialty "Software Engineering" from 2012 to 2017, when I gave lectures in this discipline. Taking those lectures after me, he manages the computer exercises team responsibly and applies a number of innovative methods and ideas to increase the effectiveness of student learning.

In the work [13] of the applicant to participate in the competition reported on 44th International Conference AMEE (2018) and published in AIP Conf. Proc. 2048, the results of the application of a model of electronic assessment of students' knowledge and skills are analyzed, within the framework of the TeSLA project of the European Commission. The work is included in the list of publications located at https://tesla-project.eu/papers/

In 2007/2008 he delivered in English the exercises of discipline "Variational methods in mathematical physics", which is mandatory in master's programs "Equations of mathematical physics" and "Equations of mathematical physics and applications".

In more detail, from the information in Item 1 of the Indicator Report under Art. 112, Para. 2 for the disciplines in which he taught exercises or gave lectures, it is evident that since 2007 Dr. Hristov has had had classes at Bachelor's Degree in FMI or Faculty of Physics in the following disciplines:

compulsory - seminars / computer exercises in: "Equations of Mathematical Physics" (specialty "Applied Mathematics") - 2007/2008, 2008/2009, 2011/2012 and 2012/2013, 2018/2019; "Partial differential equations" (specialty Mathematics) - 2007/2008 and 2008/2009; "Mathematical Physics Equations Using a Scientific Calculation System" (specialty "Software Engineering") - 2009/2010 and 2010/2011; “Differential Equations and Applications” (Specialty “Software Engineering”) - every academic year from 2011/2012 up to now; Differential Equations (Specialty “Statistics”) - 2015/2016, 2016/2017 and 2019/2020; "Differential equations and applications" (specialty "Informatics") - every school year; "Differential and Integral Calculus 1" (specialty "Mathematics and Informatics") - 2012/2013; "Mathematical Analysis of Functions of Many Variables" (Specialty "Engineering Physics", "Nuclear Engineering and Nuclear Energy", "Astrophysics, Meteorology and Geophysics") - 2008/2009 and 2011/2012;

compulsory - lectures on "Differential equations" (specialty "Statistics") - 2015/2016, 2016/2017, 2019/2020; "Differential and Integral Calculus 1" (specialty "Mathematics and Informatics") - 2013/2014; "Differential and Integral Calculus 2" (specialty "Mathematics and Informatics") - 2012/2013; "Differential Equations and Applications" (specialty "Software Engineering") - 2017/2018 and 2018/2019;

selectable by list - lectures and exercises on: "Partial differential equations" (specialty "Mathematics and Informatics") - 2008/2009 and 2011/2012; for the specialty "Mathematics and Informatics" - part-time

training: "Partial differential equations" - 2011/2012 and 2014/2015; “Differential Equations” - 2012/2013 academic year; "Selected Chapters in Mathematical Analysis" - 2009/2010;

electives - lectures and exercises in: "Partial differential equations and applications"(for all specialties without "Mathematics" and "Applied mathematics") - 2015/2016 - 2018/2019.

In connection with his teaching activities, Dr. Hristov **has developed the following lectures**: in 2017/2018 - "Differential equations and applications" for the specialty "Software Engineering", in 2008/2009 - "Partial differential equations" for the specialty "Mathematics" and computer science "; in 2015/2016 - "Partial differential equations and applications"; in 2011/2012 - "Partial Differential Equations" and in 2009/2010 - "Selected Chapters in Mathematical Analysis", both of which are elective on the list for the specialty "Mathematics and Informatics" - part-time training, and computer exercises in "Differential Equations" and applications "for" Software Engineering "with 2 hours of computer exercises per week, and" Differential equations and applications "for" Informatics" with 1 hour of computer exercises per week.

In item 4 of the Indicators Report under Art. 112, para. 2 there are given data on the activities of Tsvetan Hristov to the teams for the development of teaching aids and applications for the needs of training in FMI, the results of which are described and analyzed in detail in points**:**

4.1 Training materials on differential equations, from which he prepared 1.3 "Mechanical oscillations" of Chapter 1 "Linear differential equations and systems. Mechanical oscillations "and Chapter 2" Wave processes and wave equations. Correct and incorrect problems of mathematical physics' without 2.1.

4.2 Interactive Learning Application with SimCoDi Consumer Documentation, created in 2013 under the project "Computer Applications for Simulation of Real Processes" within the program to improve the quality and effectiveness of training at FMI - EOS, with participation of students from the Master’s program " UMF and Applications" under the guidance of Tsvetan Hristov, which application is used in training in "Equations of Mathematical Physics" and "Differential Equations and Applications with Mathematica, Matlab or Maple ".

4.3 A set of demonstration programs with user documentation for the course "Differential Equations and Applications", specialty Informatics, developed under the project "Applications and Differential Equations" within the EOS - FMI program in 2015 and containing a set of training and interactive applications, suitable for use in computer-based exercises in this discipline and for students' self-study.

4.4 A set of interactive training applications for "Mathematical Physics Equations" for the specialty Applied Mathematics and "Differential Equations and Applications" for the Specialty Software Engineering, prepared in 2018 by a team led by Tsvetan Hristov on the project "Applications and Equations of Mathematical Physics” within the FMI's EOS program for computer-based exercises in these disciplines, using Matlab to numerically solve the problems of the seminars and visualize various objects related to them. At the beginning of the semester, providing the materials from the set of study applications to the students in both courses enables the computer-based exercises to focus on analyzing and interpreting the behavior of the models under consideration, and to demonstrate more and more mathematical models of real processes, instead of writing code.

I emphasize that the described tutorials and applications are available at intranet.fmi.uni-sofia.bg. As a former Lecturer in” Differential Equations and Applications” for Software Engineering, I reviewed these materials with interest, and I find them very useful for training in a number of disciplines at FMI.

To the reference for the indicators under Art. 112, para. 2, an assurance was received from Professor D.Sci. Nedyu Popivanov, the former head of the Master’s program "Equations of mathematical physics and applications", that Dr. Hristov was the supervisor of a successfully defended graduate student in 2013.

3. Scientific contributions.

The presented scientific works in the competition are a continuation and generalization in several directions of the main results of Dr. Tsvetan Hristov from his Doctoral dissertation. In short, it deals with the wave

equation of two spatial variables $\Delta_x u - u_{tt} = f$ in a domain Ω_0, surrounded by an uncharacteristic surface Σ_0 - a circle in the plane $t = 0$ with radius 1 and center in (0, 0, 0), and of the two characteristic surfaces - cones Σ_1 and $\Sigma_{2,0}$, located in the half-space $\{(x_1, x_2, t): t > 0\}$. The cone Σ_1 has a vertex in (0, 0, 1) and passes through the circle in the plane $t = 0$ with radius 1 and center at the point (0, 0, 0), and the cone $\Sigma_{2.0}$ has a vertex in (0, 0, 0). Several boundary value problems are posed for this equation in the domain under consideration. The problem of finding a solution in Ω_0 with Dirichlet boundary conditions on Σ_1 and Σ_0 is denoted by P1 and by P2 - with Neumann type conditions on Σ_0 and Dirichlet type on Σ_1. Their adjoint problems with Dirichlet boundary condition on the other characteristic cone $\Sigma_{2,0}$, instead of on Σ_1, are denoted by P1* and P2* respectively. If the Dirichlet condition is set on Σ_1 and the condition $u_t + \alpha u = 0$ is set on Σ_0, where $\alpha \in C(\Sigma_0)$, then the problem of finding the wave equation in Ω_0 under these boundary conditions is denoted by P_α.

The presented scientific papers for the competition study analogues of the above boundary-value problems for degenerating hyperbolic equations of two basic types - Tricomi and Keldish. An example of a Tricomi type equation is

$$t^m \Delta_x u - u_{tt} = f, \ 0 < m, \ (*)$$

and for a Keldish type equation:

$$\Delta_x u - (t^m u_t)_t = f, \ 0 < m < 2, \ (**)$$

as $t > 0$. It is essential that these equations have a parabolic degeneracy on the plane $t = 0$, and for Keldysh type equations it is a characteristic. These boundary-value problems are posed in regions Ω_m, that are located in the part of space at $t > 0$. In this case, one of the boundary surfaces is part of $t = 0$, and the other two are parts of suitably selected characteristic surfaces for the corresponding equations. There is a fundamental difference in the behavior of the approach of these characteristic surfaces to the plane $t = 0$. For Keldysh type equations, the boundary characteristic surfaces in their common parts with the plane $t = 0$ are tangent to it, with one of the surfaces only at the beginning of the coordinate system. For Tricomi-type equations there is no tangent to the plane $t = 0$, even one characteristic surface has a peak at the beginning of the coordinate system. This leads to differences in the formulation of the boundary value problems for the two types of equations and the restriction on the parameter m in the second equation, as well as the emergence of different difficulties in studying the behavior of their solutions. In the presented scientific papers for the competition, infinitely many classical solutions to some of the adjoint homogeneous problems for the above two equations are found for $f = 0$. This requires that in the study of the boundary value problems one should work in classes of non-classical solutions. Therefore, the concepts of quasi-regular and generalized solutions are introduced.

The papers [3], [4], [5] discuss several boundary-value problems, among which analogues of problems P1, P2, for Tricomi-type equations, the simplest of which is Equation (*). $\Sigma_{1,m}$ and $\Sigma_{2,m}$ denote the characteristic cones for this equation with vertices in (0, 0, 1) and (0, 0, 0) located in the half-space $\{(x_1, x_2, t): t > 0\}$, as $\Sigma_{1,m}$ is slammed down and passes through the circle in the plane $t = 0$ with radius 1 and center at the point (0, 0, 0). It turns out that the analogues of problems P1 and P2 for the equation (*) are not well posed, because their adjoint homogeneous problems have infinitely many solutions. Therefore, in [3], [4], [5], equations of the type of Tricomi with lower order terms are considered. An analog of the problem P_α is also formulated. Generalized solutions are defined in [3], published in BAS Reports, assuming that their first partial derivatives can have singularities over $\Sigma_{2,m}$. There are formulations and comments on the existence and uniqueness theorems of the introduced generalized solution and theorems about the behavior of this solution in the neighborhood of (0, 0, 0). An example of a solution with a singularity is given. The work [5] introduces quasi-regular solutions of the boundary value problems stated in [3] for Tricomi type equations with lower order terms. Results for the uniqueness of such solutions are presented. Appropriate examples are given in which the conditions of Theorem 3.1 (for $m = 0$) and Theorem 3.4 (for $m > 0$) for the uniqueness of quasi-regular solutions are fulfilled. It is interesting here that in the case of $m \geqslant 2$, after entering

the cylindrical coordinates in the equation, one of the lower coefficients there satisfies a condition of the type of Protter condition, whereas in the case of $m < 2$ no such restriction is imposed.

In [4], the uniqueness of a generalized solution to Tricomi-type equations with lower order terms for the analogue of problem P1 is proved. This paper also formulates the problem PK for the Keldysh equation $\Delta_x u - (t^m u_t)_t + r u = f$, $0 < m < 2$, and with the help of Hardy-Sobolev inequality theorems for the uniqueness of a quasi-regular solution are proved. There is required $t^m u_t \to 0$ as $t \to 0$, and no condition is set directly for u_t at $t = 0$. This is an important difference between the problem PK and the problem P2 for Tricomi-type equations. Lemma 3 lists infinitely many classical solutions to the adjoint problem PK* with a homogeneous equation and $r = 0$, and it is shown that hard constraints on the function f must be imposed when looking for classical solutions to the PK problem.

In [1], [2] and [6 -12], several boundary value problems for Keldysh type equations are investigated. In [7], generalized solutions of the PK problem for the Keldysh type equation (**) are defined for $0 < m < 4/3$. The significant difference between this type of solutions compared to the generalized solutions to the problems P1 and P2 for an equation of the Tricomi type is commented, in the second case for $m > 2$ their first partial derivatives are allowed to have singularities on the boundary of the domain and for the equations Keldysh-type only u_t could have a singularity near $t = 0$. In order to establish solvability in the class of generalized solutions, the problem PK is transformed by two appropriate changes of variables to the Goursat-Darboux problem for the Euler-Poisson -Darboux equation. Solving it by using the Riemann - Hadamard function the requirement arises $0 < m < 4/3$. Theorem 4.2 proves a generalized solution for an appropriately specified right-hand side of the equation, and Theorem 4.1 proves the uniqueness of the problem PK in the class of generalized solutions.

In [9], for $0 < m < 1$, a Keldysh type equation with lower order terms in the three-dimensional case is considered, and for it the problem PK_α is studied with the Dirichlet condition on the part of the boundary of the domain lying on the characteristic surface with a peak in (0, 0, 1), which touches the plane $t = 0$ at the points of the circle, with radius 1 and center at the point (0, 0, 0). The solution must have the property $t^m u_t + \alpha u \to 0$ as $t \to 0$. This is a fundamental difference from the problem P_α, where the condition is $u_t + \alpha u = 0$ at $t = 0$. Another important difference is that no condition is imposed as of the type of Protter condition. The main steps in the scheme for proving the existence of a generalized solution are given when the right side of the equation is a harmonic polynomial. Clearly, the proof is long and requires a high degree of mastery of a great deal of techniques from the apparatus not only of the partial differential equations, but also of the theory of special functions and of integral equations. Theorem 4 deals with the singularity of the generalized solution and her proof is sketched.

In publications [10], [11], [1] and [2] generalizations of the problem PK are studied in three dimensions regarding spatial variables. The investigations are for $0 < m < 4/3$ and for an equation of Keldysh type without lower order terms. In [10], a number of non-trivial classical solutions to the adjoint homogeneous problem PK* were found. Therefore, in these works it is looked for generalized solutions to the problem PK of an appropriate class. Methods for establishing the existence of a generalized solution encounter the need to construct a Riemann-Hadamard function to solve the two-dimensional Cauchy-Goursat problem for the Euler-Poisson-Darboux equation obtained after several suitable changes in the variables in the initial Keldysh-type equation and applying the Fourier method to separate variables. The Euler-Poisson-Darboux equation turns out to be not considered before for such type right-hand side. The construction of the Riemann-Hadamard function is precisely done in the work [1], using substantially and with great skill the special function apparatus. For right-hand side of the initial equation of the special type of harmonic polynomial there is proved uniqueness theorems and the existence of a generalized solution and an estimate for its growth in the domain is derived, giving the maximum order of singularity around the point (0, 0, 0, 0). The restriction $0 < m < 4/3$ arises naturally in the proof of the existence of solutions to a number of auxiliary two-dimensional problems. Separately, the results obtained for the Cauchy-Goursat problem for the Euler-Poisson - Darboux equation generalize M. Smirnov's results (up to 1977) for this problem. Finally, I will point out the importance of the work [2], in which with exceptional skill, with the introduction of new ideas, with

detailed calculations, establishing appropriate identities and asymptotic developments, necessary and sufficient conditions are established for the growth of the generalized solution of the problem PK around (0, 0, 0, 0) at $t > 0$ over the boundary of the domain, depending on the type on the right-hand side of the equation $\Delta_x u - (t^m u_t)_t = f$, $0 < m < 4/3$.

The abstracts of the publications and the Reference for the original scientific contributions correctly reflect the contributions in the works of Dr. Tsvetan Hristov. The Extended habilitation report also shows an extremely good knowledge of the results of other scientists in the field of the studied problems for the boundary value problems under consideration, both in the plane case and in higher dimensional spaces, as well as for the basic problems for the Euler-Poisson-Darboux equation.

Citations of the works of Tsvetan Hristov are increasing, which is evident from the enclosed extensive reference to the citations. Particularly impressive are the citing articles in the first table - 1. Citations in scientific publications with Impact Factor (JCR - WoS) - with 10 citations of the works with numbers [1], [2], [7], [8], [6] and [4]. Important articles [1], [2] have according to both tables 8 and 5 citations respectively in the last 2 years! Much of the citation is in publications by scientists from Russia or the former Soviet republics. This is quite natural, because the two-dimensional cases has been versatile developed in these countries - for example E. Moiseev and T. Kalmenov have many scientific works in this direction. The number of citations to Tsvetan Hristov's publications is expected to increase as the topic of his research is growing more up-to-date. The problems and equations under consideration - especially those of the Keldysh type - have been studied at an increasing rate over the last 20 years as they find increasing applications in modeling real processes. Keldish-type equation problems models processes in cold plasma, as indicated by Th. Otway in a number of his works and in a 2012 monograph.

Twelve of Dr. Tsvetan Hristov's works for the competition have been published inscientific editions with Impact Factor (JCR - WoS) or Impact Rank (SJR - Scopus), namely1 article in Boundary Value Problems, 2017, IF (1.156 - 2017), Quartile: Q1 (51/310 Mathematics, 2017 JCR-WoS), 1 article in Advances in Mathematical Physics, 2017, IF (0.71 - 2017), Quartile: Q4 (48/55 Physics Mathematical, 2017 JCR-WoS), 2 articles in Comptes rendus de l'Académie bulgare des Sciences - IF (0.106 - 2007), Quartile: Q4 WoS; IF (0.27 2017), Quartile: Q4 WoS;7 articles are in AIP CP with SJR - Scopus; Article 1 is in Siberian Advances in Mathematics, 2011, SJR (0.169 - 2011) Scopus. An article is in a foreign refereed magazine.

Three works of the candidate submitted in the competition are his own, 2 works with 3 co-authors, 6 with 2 co-authors, 2 with one co-author. Declarations by Assoc. Prof. Dr. A. Nikolov and Professor D.Sci. N. Popivanov on the equality of participation of Tsvetan Hristov in their joint publications were presented. I have no doubt about his significant contribution to the results in all the publications presented. The results have been reported at a number of international conferences and other scientific forums in Bulgaria and abroad.

6. Critical notes and recommendations

I have one note about the structuring of the statement in the publication [9], namely, the constraints imposed on the coefficients of the lower-order terms in the equation and on the boundary condition function in relation to establishing the existence of generalized solutions to the original and the basic two-dimensional problem, to which the original is reduced, are described on page 5 of the article but are not present in the formulations of Theorems 2, 4, and 5 for existence.

I recommend Dr. Tsvetan Hristov to continue his research on the boundary problems he is considering for degenerate Keldish-type hyperbolic equations to clarify the question of the existence of generalized solutions for parameter values *m* between 4/3 and 2. I believe that the results will greatly contribute to the development of the theory of these type equations in space with dimensions of three upwards and to the possibilities for its application in solving real practical problems.

7. Personal impressions of the applicant

I have known Dr. Tsvetan Hristov since 1998. I have been with him many times in teaching teams and I have a direct impression of his presenting at conferences and scientific forums, defending his Doctoral dissertation, participating in Committees for candidate master's exams. He is hard working, very organized, well prepared in advance for his upcoming activities, meets the exact deadlines. He shows innovativeness, resourcefulness and persistence in the development of teaching materials. Dr. Hristov treats his students and colleagues at FMI in principle and correctly, following the rules of academic ethics.

I believe that Dr. Tsvetan Hristov is a prospective scientist with extensive knowledge in a number of fields of differential equations, mathematical and functional analysis, and has accumulated experience and brings useful ideas in solving non-standard scientific problems.

8. Conclusion on the application

Having become acquainted with the materials and scientific works presented in the competition and based on the analysis of their importance and the scientific and applied contributions contained therein**, I confirm** that scientific and other academic achievements meet the requirements of the Act of the Development of the Academic Personnel of the Republic of Bulgaria , the Rules for the Implementation of the Act of the Development of the Academic Personnel of the Republic of Bulgaria and the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at SU for appointment the candidate Tsvetan Hristov on the academic position “associate professor” in the scientific field and professional direction of the competition. In particular, the candidate satisfies the minimum national requirements in the professional field 4.5 Mathematics and no plagiarism has been found in the scientific papers presented at the competition.

I give a **positive assessment** of the application of Ch. Assistant Professor Tsvetan Hristov for Academic Position "Associate Professor" in Professional Degree 4.5 Mathematics (Differential Equations), for the needs of Sofia University "St. Kliment Ohridski ", Faculty of Mathematics and Informatics

II. OVERALL CONCLUSION

On the basis of the above, **I recommend that the scientific jury propose to the competent body of choice of the Faculty of Mathematics and Informatics at Sofia University “St. Kliment Ohridski "to to select Ch. Assistant Prof. Tsvetan Dimitrov Hristov to take the academic position of "Associate Professor" in the professional field 4.5 Mathematics (Differential Equations).**

11/12/2019 Prepared the review:

/Assoc. Prof. Dr. Maria Karatopraklieva/