

# REVIEW

Of Materials Submitted for Participation in Contest  
For Occupying Academic Position Full Professor  
in Professional Field 4.5. Mathematics  
(Operations research),  
At Sofia University “St. Kliment Ohridski”,  
Faculty of Mathematics and Informatics  
Candidate: Ass. Prof. Nadia Pejcheva Zlateva, DSci  
Reviewer: Prof. Nadezhda Kostadinova Ribarska, DSci  
September 4th, 2020

Associate Professor Nadia Pejcheva Zlateva, DSci is the only one candidate in the contest for occupying the academic position “full professor” in: scientific area 4. Natural Sciences, Mathematics and Informatics, professional field 4.5. Mathematics (Operations Research) for the needs of Sofia University “St. Kliment Ohridski”, Faculty of Mathematics and Informatics, announced in State Gazette no. 21 of 13 March 2020 and on the Internet sites of FMI and Sofia University “St. Kliment Ohridski”. She is born on August 31, 1969. She graduated with honors in FMI of Sofia University “St. Kliment Ohridski” in 1993. In the period March 1993 - December 1996 she was a doctoral student at the Faculty of Mathematics and Informatics at Sofia University with a break during the period February – July 1994 for a six months specialization under the guidance of Prof. Robert Deville at the University of Bordeaux, France. She defended her PhD thesis “Subdifferential calculus and variational methods in non-smooth analysis” in 1999 and acquired a PhD educational and scientific degree. Her scientific advisor was Prof. Pando Georgiev. In the period February 2002 - July 2003 she obtained an eighteen months post-doctoral position at the University of Montpellier II, France, as Marie Curie Fellow in collaboration with Prof. Lionel Thibault. In the period October 2004 - March 2005 she held a six-month post-doctoral position in University of Brittany, France, as part of a research network “Evolution Equations” under the VI Framework Program of the EU. From February 1997 to February 2006 she worked at the Institute of Mathematics and Informatics of Bulgarian Academy of Sciences. Since 1994 she has been working at the Faculty of Mathematics and Informatics of Sofia University “St. Kliment Ohridski”. In the beginning (from 1994 till 1999) she was a Lecturer (from 1994 till 1999), next she was a visiting associate professor (in 2000) and after that she held a permanent position as Assistant Professor (from 2000 till 2005) and Associate Professor (from 2005 till now) in the department of Probability, Operations Research and Statistics. Associate Professor Zlateva defended a dissertation on “Variational analysis: Methods

and applications” and acquired the degree “Doctor of Mathematical Sciences” in 2018.

Assoc. Prof. Nadia Zlateva has submitted for participation in the contest a full set of documents and materials, which complies with the Regulations on the Academic Staff Development in the Republic of Bulgaria, the Regulations for their application and with the respective regulations for acquiring scientific degrees and holding academic positions of Sofia University “St. Kliment Ohridski”.

The research interests of Assoc. Prof. Zlateva are in the fields of Optimization and Mathematical Analysis, more precisely in Variational Analysis. This modern mathematical discipline emerges as a successor to calculus of variations and covers relatively new areas such as non-smooth analysis, analysis of multi-valued mappings, variational principles and others. The works of the applicant are highly appreciated by the mathematical community.

Assoc. Prof. Nadia Zlateva has been a university lecturer for more than twenty years. Her teaching career began in 1994 as a part-time assistant at the Faculty of Mathematics and Informatics of Sofia University “St. Kliment Ohridski” and continues till now. Since her acquiring of a full-time position in 2000, she lectured on Linear Optimization and Nonlinear Optimization. These courses are compulsory for the students majoring in Applied Mathematics and Statistics. In addition, Assoc. Prof. Zlateva lectures on Operations Research, a compulsory course for students majoring in Informatics and Computer science and an optional course for students majoring in Software Engineering and Information Systems. She has prepared and regularly updates lecture notes for these courses. Her lecture notes are extremely helpful for students. Assoc. Prof. Zlateva is an active participant in the seminar of the master’s program “Optimization”. She is an important member of the teaching staff, responsible for this program. Matei Konstantinov, who graduated from the master’s program “Optimization” in 2019, since February 2020 is a full-time doctoral student under Assoc. Prof. Zlateva’s guidance in the doctoral program “Operations Research” at the department of Probability, Operations Research and Statistics of FMI at Sofia University.

Associate Professor Zlateva devotes a lot of energy and time to the service to the academic community. She was Head of the department of Probability, Operations Research and Statistics from March 2008 to March 2012. In the period June 2011 - January 2017 she was Vice-Dean for Research, project activities and doctoral training at the Faculty of Mathematics and Informatics. Associate Professor Zlateva has participated in the creation and implementation of numerous research projects, detailed information on which can be found in her CV. Personally, I have always been able to rely on her advice and assistance concerning any administrative or project issues.

Associate Professor Zlateva submitted for the participation in the present contest for Full Professorship 8 papers in peer-reviewed scientific journals, of which 6 have already been published and the remaining 2 are accepted for publication. All articles are in journals with impact factor and impact rank (Journal of Optimization Theory and Applications - 2, Journal of Convex Analysis - 4, Proceedings of the American Mathematical Society - 1, Comptes rendus de l'Académie bulgare des Sciences - 1). The scientific papers presented by the applicant have not been used in previous applications. These articles have been published during the period 2008 - 2020 in co-authorship with Assoc. Prof. Milen Ivanov. I have no doubts that Assoc. Prof. Zlateva has contributed an equal share to all joint publications.

The scientific papers submitted for the competition satisfy the minimal requirements of the law (under Art. 2b, para. 2 and 3 of ZRASRB) and the additional requirements of Sofia University "St. Kliment Ohridski" for occupying the academic position "Full Professor" in the scientific area and the professional field of the competition.

I will follow the grouping proposed by the candidate in my analysis of her achievements. The numbering of the articles is as in the list of publications submitted for participation in the contest.

1. *Use of perturbation spaces for minimization of integral functionals* ([26] and [36]).

In the paper [26] the following problem is considered

$$(V_{\|\cdot\|}) \begin{cases} \int_0^\infty (\|v(t)\|^2 + f(u(t))) dt \rightarrow \min \\ u(t) = a + \int_0^t v(s) ds, v \in L^2([0, \infty], X), \end{cases} \quad (1)$$

where  $X$  is a Banach space,  $f$  is a convex function with nonnegative values and closed epigraph,  $f(0) = 0$  and  $f \geq k\|\cdot\|$  for some constant  $k > 0$ .

In the paper [36] is considered the problem

$$(P_f) \begin{cases} \int_0^1 (f(u(t), t)) dt \rightarrow \min \\ u : [0, 1] \rightarrow X \text{ is 1-Lipschitz function with } u(0) = 0, \end{cases} \quad (2)$$

where  $X$  is a Banach space,  $f : X \times [0, 1] \rightarrow \mathbb{R}$  is a bounded below function with closed epigraph,  $f(0) = 0$ , such that  $\int_0^1 f(0, t) dt < \infty$ .

The above minimization problems of an integral functional on an infinite-dimensional space may have not a solution, as simple examples show. The presented articles prove that "arbitrarily close" to a given problem there is a problem which has a solution. Of course, it could be done easily if one only wants to show "closeness" of the perturbed problem to the original one

in some sense, but it could change the type of the functional. In the articles presented by the applicant, the results proved are much stronger: the perturbed problem is obtained from the original one by an appropriate perturbation of the integrand. To achieve this goal, a new variational principle has been developed which is of interest by itself. The axioms for the perturbation space used in the second article are less general but easier to check.

*2. Surjectivity of mappings in Fréchet spaces ([56] and [66]).*

The classical Nash - Moser theorem is an inverse function theorem in Fréchet spaces and it is often used for proving solvability of some nonlinear problems. Since the classical theorem works only for  $C^2$  functions and under some additional assumptions, a suitable generalization of this theorem under weaker smoothness assumptions has long been of interest (at least its surjectivity part). Breakthrough in this direction was implemented by Ekeland in 2011. Articles from this group presented by the applicant significantly develop the techniques proposed by Ekeland.

In the article [56] a result of Nash - Moser type concerning surjectivity of multi-valued maps with closed graph is proved. The new feature is the evaluation of all semi-norms. This result is formulated in terms of a suitably defined graphic derivative of a multi-valued mapping whose domain is a linear metric space. In its proof an abstract iterative scheme is used, which is developed by Assoc. Prof. Zlateva and Assoc. Prof. Ivanov. The relation between the mappings satisfying the standard assumptions of Nash - Moser theorem and metrically regular mappings is clarified. Article [66] provides a direct proof of an important particular case of Nash - Moser theorem for a single-valued mapping. Again evaluation of all semi-norms is provided, which enhances Ekeland's results.

*3. New proofs of some known results in Variational Analysis ([16], [46], [76] and [86]).*

A new proof of the classical theorem of Moreau and Rockafellar (any proper convex lower semi-continuous function on a Banach space is determined up to a constant by its subdifferential) is presented in [16]. The new proof makes use of some ideas from the proof of the classical calculus theorem about Riemann integrability of monotone functions.

In [46] a new proof of the maximal monotonicity of the subdifferential of a convex function is given. This is a classical theorem and many mathematicians have proposed different approaches to its proof. The proof of Assoc. Prof. Zlateva and Assoc. Prof. Ivanov demonstrates a "return to the roots" from the 70s of the last century.

In the article [66] a characterization of the metric regularity property of a multi-valued map, defined on a metric space and taking values in a Banach space, is obtained. This characterization is in terms of the so-called contingent variation of the multi-valued map. This result extends a result of Frankowska from 1990 in the following sense: Frankowska's article contains a characterization of the local modulus of regularity, while in [66] the characterization is

global.

The article [86] proposes a new approach to the proof of the following theorem of Correa, Jofré and Thibault: Let  $X$  be a Banach space,  $\partial$  is admissible subdifferential and  $f : X \rightarrow \mathbb{R} \cup \{+\infty\}$  is a proper lower semi-continuous function. Then  $\partial$  being a monotone operator implies that  $f$  is a convex function. This approach is based on the use of barrier functions and on the Ekeland's variational principle.

In total, Associate Professor Zlateva published 29 articles in peer-reviewed scientific journals, which altogether have over 270 citations and h-index 7 (in each of the WoS and Scopus systems). The quantity of the citations is impressive and implies that the works of the candidate are highly appreciated by the mathematical community.

I have no doubt that Associate Professor Zlateva has an equal contribution to all joint articles.

No plagiarism was found in the publications submitted for assessment.

#### **CONCLUSION:**

The scientific achievements of Associate Professor Nadia Zlateva, both in terms of quality and quantity, do not leave any doubt in the positive assessment of her research and pedagogical activity.

**I strongly recommend the scientific jury to prepare a report-proposal to the esteemed Faculty Council of the Faculty of Mathematics and Informatics of Sofia University to elect Associate Professor Nadia Zlateva for the academic position "Full Professor" of Sofia University "St. Kliment Ohridski" in Scientific Area 4. Natural sciences, Mathematics and Informatics, Professional field 4.5. Mathematics (Operations Research).**

04.09.2020

(Prof. N. Ribarska, DSci)