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A simple method for determination of the density of granular materials

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Abstract
A simple experiment using low cost equipment for the determination of the density of granular materials, without immersing them in a liquid, is presented. It is based only on the ideal gas state equation, so it is a good experimental task for undergraduate and high school students.

Introduction

Determination of the density of irregular shaped objects has been a challenge to researchers since the time of Archimedes [1]. The most direct approach is to immerse the object in a liquid, usually water, and to compare its weight to that in air. An alternative to this method is measurement of the momentum of forces [2–4] instead of mass and volume. The same procedure can be applied also to granular and porous materials which represents the more frequent case. Unfortunately measurement in liquid has several disadvantages: it is applicable only to materials that are insoluble in the corresponding liquid; the material should not contain voids and there should be a good wetting of the material. Some of these problems can be solved by replacing the liquid with gas. Several authors have proposed this technique where a test body with a known volume is needed [5, 6] for calibrating the chamber volume.

This task can be reduced to the determination of only the volume of the material, if it is possible to measure the mass independently. There are commercially available apparatuses for the determination of the volume of solid particles [7, 8].

Here we propose a simple piece of experimental equipment which can be easily assembled and used by students to introduce them to this method. The proposed experimental task is based only on the ideal state equation, so it is a good experimental task for undergraduate and high school students. The experiment trains the students in how to collect experimental data and also how to work with frequently used laboratory equipment. It was given as an experimental problem to undergraduate students at the National Physics Olympiad of Bulgaria in 2011.

Equipment

The equipment needed for the experiment consists of a 50 ml syringe, a pressure sensor, a multimeter, and a granular material (analytical grade NaCl from Valerus-BG, but table salt can be used as well) with known mass, which in the present case is 20 g. A photograph of all of the equipment is shown in figure 1.

The differential pressure sensor SPD030G (Smartec Pressure Sensor http://home.comet.g/datasheets/Sensors/DSSPD01v3.pdf) used in the

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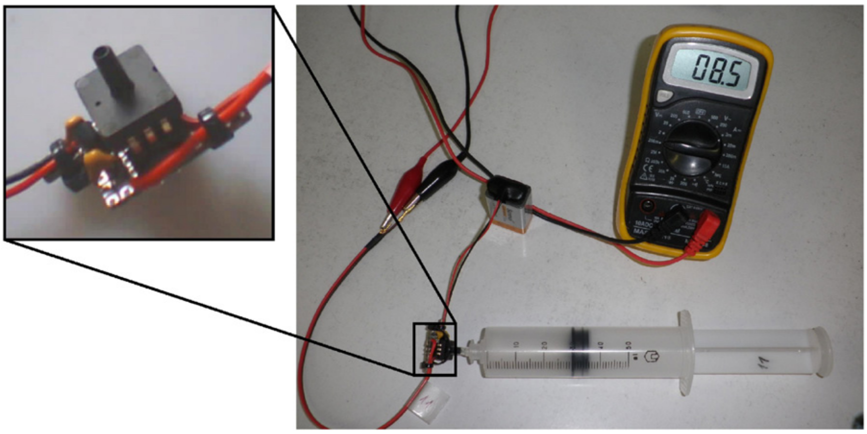

Figure 1. Photograph of the entire set-up used for the experiments. A large image of the pressure sensor is shown on the left.

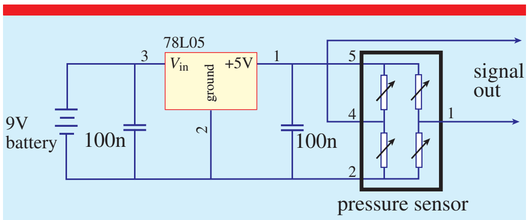

Figure 2. Schematic of the pressure sensor, powered by a 9V battery through a 5V voltage stabilizer.

current experiments measures the difference between the pressure (p) on its input and atmospheric pressure (p_0). The sensor is based on a resistive bridge and its output signal is a voltage proportional to the pressure difference. The typical full-scale span is 140 mV. The pressure range of the sensor is 0–2 bar. The offset (at a fixed temperature) is between -50 and $+50$ mV. The sensor is powered by a 9 V battery through a 5 V 78L05 voltage stabilizer for better stability—figure 2. The relatively large parameter variations and offset require individual calibration of each sensor, which is one of the experimental tasks given to the students.

The syringe tip is connected via a small plastic tube to the sensor input. There is a small piece of cotton in the tip, in order to prevent the sensor from being filled with the granular material. The syringe piston is greased, so there is no net loss or gain of gas.

Experimental tasks

Students are given the following experimental tasks.

(1) Sensor calibration.

- (1.1) Suggest and theoretically describe a procedure for sensor calibration. It should allow the relative pressure $p/p_0 = f(U)$ to be determined from the measured voltage. Think about the necessary precautions you will have to take in order for your assumptions to be valid.
- (1.2) Record the measured values in a table and express your data graphically.
- (1.3) From the graph, obtained in subtask (1.2), determine the function $p/p_0 = f(U)$ in explicit form.

(2) Determination of the unknown true volume of a granular material of NaCl.

- (2.1) Suggest a procedure for the determination of the unknown true volume of the NaCl granular material.
- (2.2) Record the measured values in a table and express your data graphically.
- (2.3) Using the measured data and those obtained in task (1), determine the exact volume of the NaCl crystals.

(3) Using the results obtained in task (2) determine the density of the NaCl.

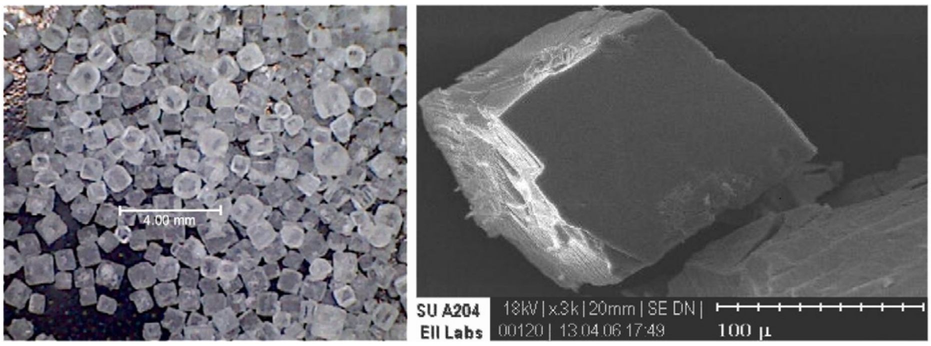

Figure 3. Optical and scanning electron microscopy images of NaCl microcrystals.

At every stage of the experiment the students are required to find the absolute and the relative uncertainty of the determined parameters.

To give an impression of the microstructure of the investigated material and stimulate their interest, the students were provided with optical and scanning electron microscopy images of NaCl microcrystals, the true density of which was to be determined—figure 3.

Before starting the experiments, it was discussed with the students that they should assume the gas in the syringe as ideal. To deal with the present experimental task the students need to know only the equation of state for an ideal gas.

Exemplary results and discussion

The calibration procedure consists of the determination of the explicit expression of the function $p/p_0 = f(U)$. As was stated above, the pressure sensor measures the difference between the pressure in the syringe and the atmospheric pressure, which is proportional to the output voltage. In the general case this relationship has the form

$$\Delta p = p - p_0 = a + bU, \tag{1}$$

where a is the offset and b is the coefficient of linearity. The introduction of a dimensionless pressure simplifies the task and also excludes the atmospheric pressure from the calculations, which is considered as unknown, but constant during the experiment. The dimensionless pressure can be expressed as

$$\frac{p}{p_0} = \frac{p_0 + a}{p_0} + \frac{b}{p_0}U \tag{2}$$

or

$$\frac{p}{p_0} = A + BU, \tag{3}$$

where $A = (p_0 + a)/p_0$ and $B = b/p_0$.

Each pressure sensor reading must be taken at one and the same temperature—the room temperature. Precautions must be taken to keep it constant during the experiment; for example, direct sunlight should be avoided. The students must take special care not to hold the body of the syringe in their hand during the measurements and wait long enough for temperature equilibration after changing the volume. For an isothermal process it follows that

$$p/p_0 = V_0/V. \tag{4}$$

Combining equations (3) and (4) it follows that

$$\frac{V_0}{V} = \frac{p}{p_0} = A + BU, \tag{5}$$

which gives the explicit form for the function $p/p_0 = f(U)$. The initial volume V_0 is measured at atmospheric pressure.

By varying the volume of the syringe, the students obtain different values for the pressure in terms of voltage. The experiment can be carried out in the compression or the expansion regime, as in the working range of pressures the gas in the syringe behaves as ideal. The students usually choose to carry out the experiment by compressing the volume and starting from the maximum available volume (50 ml in our case). In that case the piston has the largest moving range.

Initially the students are not familiar with the voltage range of the pressure sensor, so they have to carry out an 'empty' experiment. It is important for the students to choose an appropriate voltage

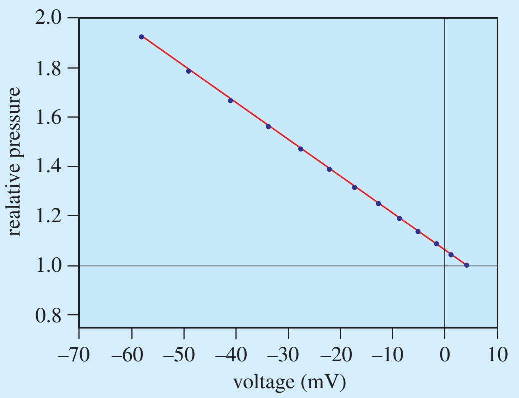

Figure 4. Experimental values of the relative pressure as a function of the output voltage. The solid line is the linear fit.

Table 1. Experimental data for the sensor calibration. The initial volume is set to 50 ml.

Point number	Volume V (ml)	$V_0/V = p/p_0$	Voltage U (mV)
1	50	1.00	4.2
2	48	1.04	1.2
3	46	1.09	−1.6
4	44	1.14	−5.1
5	42	1.19	−8.7
6	40	1.25	−12.7
7	38	1.32	−17.3
8	36	1.39	−22.1
9	34	1.47	−27.6
10	32	1.56	−33.8
11	30	1.67	−41.1
12	28	1.78	−49.2
13	26	1.92	−58.2

range for the multimeter in order to measure the voltage with the highest possible accuracy.

Exemplary experimental data are presented in table 1. The initial volume is set to 50 ml and this value is used for computation of the third column (the relative pressure—p/p_0). Figure 4 shows a plot of the relative pressure as a function of the output voltage (fourth column from table 1). From the offset and the slope of the graph the coefficients A and B (equation (5)) are determined.

In the general case just two measurements are sufficient to determine the unknown coefficients A and B. Finding them from the graph trains the student in how to process experimental data using linear fitting and provides more correct determination of the unknown coefficients.

Table 2. Experimental data and values computed from equation (5) for the relative pressure and the expression $V\frac{p}{p_0}$ used for the determination of the NaCl volume.

Point number	Volume V (ml)	Voltage U (mV)	p/p_0	Vp/p_0 (ml)
1	50	4.1	0.989	49.445
2	48	0.6	1.041	49.971
3	46	−2.9	1.093	50.288
4	44	−7.3	1.159	50.986
5	42	−11.9	1.227	51.547
6	40	−17.5	1.311	52.430
7	38	−23.6	1.402	53.262
8	36	−30.6	1.506	54.214
9	34	−38.8	1.628	55.356
10	32	−47.5	1.758	56.248

The calculated values of the coefficients A and B are $A = 1.05$, $B = -0.0149$ mV^{-1}. The corresponding uncertainties are also determined from figure 4. Their absolute values are $\Delta A = 0.02$ and $\Delta B = 0.0004$ mV^{-1} respectively.

The next task for the students is to find the real volume of the NaCl crystals. To do this they have to put the material into the syringe and measure the pressure sensor voltage as a function of the syringe volume readings. For the isothermal process

$$pV' = \text{const}, \tag{6}$$

where $V' = V - V_x$ is the gas volume in the syringe, V is the syringe volume reading, and V_x is the unknown volume of NaCl. Using again a dimensionless pressure, after some transformations equation (6) takes the form

$$\frac{p}{p_0}V = c + V_x\frac{p}{p_0}, \tag{7}$$

where $c = \text{const}/p_0$.

The relation for the relative pressure p/p_0 is computed from equation (5) using the already determined calibration coefficients A and B. The unknown volume is the linear coefficient in equation (7).

The experimental data are given in the second and third columns of table 2. The last two columns are the computed values for the relative pressure p/p_0 and the left side of equation (7)—$V\frac{p}{p_0}$.

Figure 5 shows the data from table 2. As follows from equation (7), the unknown volume can be easily determined from the slope of the plotted straight line.

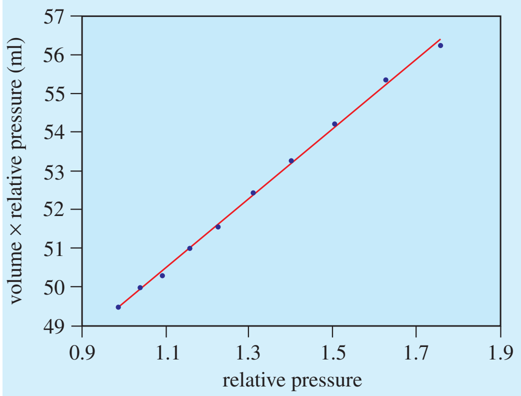

Figure 5. The computed values of $V\frac{p}{p_0}$ plotted as a function of the relative pressure p/p_0. The solid line is a linear fit to the experimental data.

Working carefully the students obtain the volume of NaCl with an uncertainty no greater than 4%. In our case the obtained value of the NaCl volume, using the calibration coefficients determined from figure 4 and the experimental data in table 2, is 9.1 ± 0.3 ml.

Having the mass of the material it is easy for the students to complete the final task (3). The calculated value for the density of the crystals of NaCl is 2.20 ± 0.07 g cm^{-3}, which agrees well with the literature data [6].

Conclusion

The experimental task presented here uses simple and low cost equipment to demonstrate a practical application of the ideal gas law. It is successfully used to determine the density of granular material.

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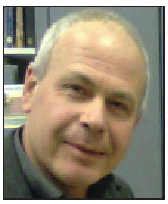

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