

Opinion on the doctoral thesis
(за придобиване на титлата доктор на науките)

entitled “*Fragmentability and functional-analytic approach to necessary optimality conditions*”

by Nadezhda Kostadinova Ribarska

In the presentation of my opinion, I follow the guidelines of the Faculty of Mathematics and Informatics of the Sofia University “St. Kliment Ohridski”.

The thesis consists of two chapters presenting two different research directions of Ribarska: the first one is devoted to issues in the general theory of Banach spaces, the second one addresses abstract, but more directly application-motivated problems in optimization theory. Ideas and technical knowhow developed in the first chapter are of substantial importance for the results in the second chapter.

Being a non-specialist in the area of the first chapter, I still find as impressive the result of Ribarska published in “Mathematika” (1987), which intrinsically characterizes the fragmentability of a topological space by a metric. Numerous implications of this result are demonstrated in the thesis. The abovementioned characterization allowed to prove that the space of Borel measures on a fragmentable compact Hausdorff topological space is fragmentable. A remarkable result by Ribarska published in “Proceedings of AMS” (1992) proves that the weak-star topology in the dual of a smooth Banach space is fragmentable by a metric.

In the second chapter of her thesis, Ribarska investigates necessary optimality conditions for extremal problems in infinite dimensional spaces, using functional-analytic approaches. This investigations target problems of calculus of variations and optimal control of distributed (partial differential equations) systems, which have important applications in economics and engendering. A starting point in the investigations of Ribarska in this direction are infinite-dimensional optimal control problems with terminal constraints, which lead to the introduction of the notion of “quasisolidity” as a replacement of the usual (rather strong) condition of finite co-dimensionality. This allowed for obtaining new optimality conditions for classes of PDE problems. The results are published in SIAM J. Control and Optim. (2011). They are based on the study of the classes of needle and diffuse variations and the corresponding approximations of the reachable set around a reference point. The main ideology (following Sussmann and Ioffe) is to base the derivation of optimality conditions on a non-separation property of the approximating cone to the reachable set

and the target. A further important step is the introduction of the “uniformly tangential” sets and cones and the proof of a non-separation theorem formulated in these terms. This is done in a paper published in *SIAM J. Control and Optim.* (2017), where the result is used to prove an abstract Lagrange multiplier rule. The latter is applied for obtaining optimality conditions for a rather general infinite dimensional optimal control problem. It seems the obtained results have a high potential for analysis of PDE-constrained optimization problems, which, I hope, will be proved in the future work Ribarska and/or her coauthors.

Most of the results in both chapters of the thesis of N. Ribarska have been published in highly reputable journals, among which *Marthematika*, *Proceedings of the AMS*, *J. Math. Anal. and Appl.*, *SIAM J. Control and Optim.*, *Set-valued and Variational Analysis*, *Studia Mathematica*, etc. The overall list of publications of Ribarska (also regarding works not included in the thesis) and the list of citations are impressive, and I am sure that any reasonable formal bibliometric requirements should be satisfied (those of the FMI are largely oversatisfied). Knowing personally most of the coauthors of Ribarska and their opinion about the joint work with her, I am completely convinced that her contributions are substantial, even decisive in many of the papers. I also mention that Ribarska has published in high level journals a number of papers as a single author, which proves her high quality as a mathematician.

I have attended numerous presentations by Ribarska at international conferences, and can claim that the results she reported attracted attention and induced positive comments by top specialists in the respective fields.

The “Autoreferat” correctly represents the content of the thesis, is very well written, and provides complementary and rather enlightening explanations.

Based on the doctoral thesis of Ribarska, and consistently with my overall impressions on her scientific contributions and activities, I am deeply convinced that she completely deserves the title “Doctor of Sciences” (доктор на науките).

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