On invariant polynomials in free associative algebras over a field of arbitrary characteristic

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Let $K\langle X_d\rangle$ be the free associative algebra of rank $d \geq 2$ over a field K. In 1936 Wolf proved that the algebra of symmetric polynomials $K\langle X_d\rangle^{\mathrm{Sym}(d)}$ is infinitely generated. Koryukin (1984) equipped the homogeneous component of degree n of $K\langle X_d\rangle$ with the additional action of $\mathrm{Sym}(n)$ by permuting the positions of the variables. He proved finite generation with respect to this additional action for the algebra of invariants $K\langle X_d\rangle^G$ of every reductive group G. We established that over a field of characteristic 0 or of characteristic $p > d$ the algebra $K\langle X_d\rangle^{\mathrm{Sym}(d)}$ with the action of Koryukin is generated by (noncommutative version of) the elementary symmetric polynomials. We proved that if the field K is of positive characteristic at most d then the algebra $K\langle X_d\rangle^{\mathrm{Sym}(d)}$ taking into account the Koryukin's action is infinitely generated and describe a minimal generating set.

The problem we are solving is: whether the algebra of invariants $K\langle X_d\rangle^G$ where $G = A_n$ or D_n is finitely generated, with the Koryukin action and considering different field's characteristic.