REPORT

by Prof. D.Sc. Mancho Hristov Manev
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on a dissertation for awarding
the scientific degree "Doctor of Sciences"

Area of higher education: 4. Natural Sciences, Mathematics and Informatics
Professional field: 4.5 Mathematics
Scientific speciality: Geometry and Topology

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Title: **The Geometry of Quaternionic-Contact Manifolds and the Yamabe Problem**

By Order No. RD 38-113/19.02.2020 of the Rector of Sofia University "St. Kliment Ohridski" (abbr., SU) I was appointed a member of the scientific jury in connection with the above procedure.

I have received the administrative and the scientific documents submitted by the candidate and I think that they are in accordance with the requirements of the legal framework, namely the Development of the Academic Staff in the Republic of Bulgaria Act (abbr., DASRB Act), the Regulations for the Application of the DASRB Act, and the Rules on the Terms and Conditions for Acquisition of Scientific Degrees and Holding Academic Positions at Sofia University "St. Kliment Ohridski" (abbr., Rules on TCASDHAP at SU).

1. General description of the dissertation

The submitted dissertation of Dr. Ivan Minchev is written in English and presented on about 200 pages. It consists of an introduction, 18 sections in 5 chapters and a bibliography of 91 titles. The subject matter and content of the dissertation are completely in accordance with the declared scientific speciality.

The introduction briefly informs the reader about the topic of the thesis and the content of its individual chapters and sections. Here the author points out that the results presented in the dissertation were published in 4 papers denoted by

[IMV10], [IMV12], [IMV14] and [IMV16], all in co-authorship with Stefan Ivanov and Dimitar Vassilev, published in the period 2010–2016 in prestigious mathematical journals: Memoirs of the American Mathematical Society (2014) , Mathematical Research Letters (2016), Journal of the European Mathematical Society (2010), Annali Della Scuola Normale Superiore Di Pisa - Classe Di Scienze (2012).

Chapter 1 of the dissertation (entitled *Preliminaries*) introduces the topic by summarizing the known main results on these $(4n+3)$-dimensional manifolds and it shows the reader the author's motivation to study quaternionic-contact (abbr., QC) geometry.

In Chapter 2 (*Geometry of quaternionic-contact manifolds*), main for the thesis, the author develops the basic concepts in the differential geometry of QC manifolds, which are fundamental in getting the results further: the torsion and the curvature of the Biquard connection, QC Einstein QC structures, conformal deformations of a QC structure, special functions and pseudo-Einstein QC structures, infinitesimal automorphisms and especially the QC Yamabe problem, i.e. to find all QC structures that are QC-conformal to a given QC structure and have constant QC scalar curvature. The main results here are Theorems A, B and C. In the former two theorems, the author finds all the conformal deformations that transform the standard QC Einstein structure on the Heisenberg group into a QC Einstein structure and give a partial solution to the QC Yamabe problem on the quaternionic Heisenberg group assuming that the obtained quaternionic structure has an integrable vertical space. In the latter theorem of the series above, QC Einstein manifolds are characterized locally as 3-Sasakian or by vanishing the torsion of their Biquard connection.

In Chapter 3 (*Quaternionic-contact Einstein manifolds*), the author extends some results about the QC scalar curvature of a QC Einstein manifold M, namely by Theorem D it is proved that this curvature is always a constant for the special case of dimension 7, i.e. $n = 1$. The latter assertion together with Theorem 5.9 in the case of $n > 1$ prove the constancy of the QC scalar curvature on M. Then, it is established that a 7-dimensional QC Einstein manifold of nowhere vanishing QC scalar curvature is locally QC-homothetic to a 3-Sasakian manifold. Moreover, using a metric connection $\widetilde{\nabla}$ on the vertical bundle V (spanned by the triple of Reeb vector fields), it is given a characterization of QC Einstein manifolds by the flatness of $\widetilde{\nabla}$ (Theorem 11.3). Next, the author describes locally structure equations of the QC Einstein structure by conditions for the contact form $\eta = (\eta_1, \eta_2, \eta_3)$ (Theorem 12.1). At the end of the section, the related Riemannian geometry of QC Einstein spaces is commented on.

In Chapter 4 (*Solving the QC Yamabe equation on S^7*), Theorem E strengthens the result of Theorem B on the unit sphere S^{4n+3} in the case of $n = 1$ by neglecting the integrability condition of the vertical space of η. Then, Theorem F determines the best (optimal) constant and extremals in L^2 Folland-Stein embedding theorem for the 7-dimensional quaternionic Heisenberg group. A complete solution of the QC Yamabe problem on the 7-dimensional quaternionic Heisenberg group is obtained by the results of Theorem A, conformal transformations of η and the divergence formula in Theorem 15.4.

In Chapter 5 (*The optimal constant in the L^2 Folland-Stein inequality on the quaternionic Heisenberg group*), the main results are given in Theorem G that generalizes Theorems E and F in the case of any dimension. Theorem G provides the best constant in the Folland-Stein embedding theorem on the quaternionic Heisenberg group of arbitrary dimension and the non-negative extremal functions.

2. Compliance with legal requirements

According to the documents presented by Dr. Ivan Minchev, the minimum national requirements in relation with this procedure are fulfilled with his theses for "Doctor" and "Doctor of Sciences" (Indicators 1 and 2, respectively), 2 publications ([IMV14] and [IMV16]) in scientific journals refereed and indexed in Web of Science and Scopus (Indicator 7) for 135 points at 100 points required and finally 13 citations in scientific journals refereed and indexed in Web of Science and Scopus (Indicator 11) for 104 points at 100 points required.

Although the paper [IMV14] in Memoirs of AMS was used in the procedure for "Associated Professor" and it is now used in the procedure for "Doctor of Sciences", it does not violate the letter of the legal requirements. The usage of the latter paper is according to the Register of Academic Positions and Dissertations of the National Centre for Information and Documentation.

Despite the fact that the articles on which the dissertation is based are co-authored, I believe that the dissertation thesis is prepared independently. Moreover, in accordance with the legal requirements, the thesis submitted does not repeat the topic and content of the thesis presented for awarding of his Doctor's degree entitled "Differential geometry of metric connections with torsion". The latter opinion is based on the annotation of his Doctoral dissertation given in the Register mentioned above.

I have not noticed any form of plagiarism in the dissertation thesis and the publications submitted for evaluation by Dr. Ivan Minchev.

3. Conclusion

I my opinion, the dissertation of Dr. Ivan Minchev contains theoretical generalizations and solutions to major scientific problems that are relevant to modern achievements and make a significant and original contribution to differential geometry.

The submitted documents meet all the requirements of the DASRB Act, the Regulations for Applications of DASRB Act and the Rules on TCASDHAP at SU.

Because of the above, I am assured of my **positive assessment** of the research presented by the dissertation, authored abstract, results achieved and contributions presented above, and I propose to the Honorable Scientific Jury **to award Dr. Ivan Minchev Minchev** the scientific degree of "Doctor of Sciences" in Area of higher education: 4. Natural Sciences, Mathematics and Informatics; Professional field: 4.5. Mathematics; Scientific speciality: Geometry and Topology.

7th May 2020 Signature:

Plovdiv, Bulgaria

(Prof. D.Sc. Mancho Manev)