REPORT

on a competition for occupation of the academic position "Associate Professor"
in professional field 4.5. Mathematics (Differential equations),
for the needs of Faculty of Mathematics and Informatics (FMI),
Sofia University "St. Kliment Ohridski" (SU),
announced in State Gazette No. 65/16 August, 2019 and on the web sites of FMI and SU

This review has been prepared by Prof. , PhD, Andrey Ivanov Zahariev, Plovdiv University "Paisii Hilendarski" , as a member of the scientific jury for the competition in professional field 4.5. Mathematics (Differential equations), according to Order No. РД 38-593/11.10.2019 of the Rector of the Sofia University "St. Kliment Ohridski".

For participation in the announced competition, documents were **submitted by only one candidate**, namely Chief Assistant Professor Tsvetan Dimitrov Hristov, PhD – Faculty of Mathematics and Informatics at Sofia University "St. Kliment Ohridski ".

I. General description of the procedure and submitted documents

1. General description of the materials presented

The documents of the applicant comply with the requirements of the Act of the Development of the Academic Personnel of the Republic of Bulgaria (ADAPRB), the Rules for the Implementation of the Act of the Development of the Academic Personnel of the Republic of Bulgaria (RIADAPRB) and the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at SU (RTCAADOAPSU).

The applicant has submitted for the competition:

- Autobiography;
- Copy of diploma for higher education and the annex thereto;
- Copy of diploma for educational and scientific degree "Doctor";
- Copy of certificate for occupation of the academic position "Chief Assistant Professor";
- Certificate of internship in the speciality;
- Documents proving the fulfillment of the requirements of Art. 115, para. 1, item 2 of RTCAADOAPSU: official note from an employer;
- Medical certificate certifying mental health;
- Medical certificate certifying physical health;
- Criminal record certifying that there is no penalty imposed on "deprivation of the right to pursue a particular profession or activity";
- Lists of publications (list of all publications and list of publications presented at the competition);
- List of publications, presentations, projects and supervision activities, generated by the information system of SU;
- Reference form for the fulfillment of the minimum national requirements for the relevant scientific field and professional field (4.5. Mathematics), and additional requirements of Sofia University "St. Kliment Ohridski " with the necessary evidence:

citation proofs and Web of Science and Scopus extracts for scientometric indicators of relevant scientific publications;

- List of citations with a complete bibliographic description of the cited and the citing publications;
- Reference for original scientific contributions with the necessary evidences;
- Habilitation extended reference;
- Reference for the degree of fulfillment of the indicators under Art. 122, para. 2 of RTCAADOAPSU with the necessary evidences (with inventory and applications): certificate for scientific supervision of a graduate student, certificate for successful pedagogical activity, certificate for participation in projects, reference for general study and auditing employment at FMI of Sofia University from 2014/15 to 2018/19 including;
- Scientific papers submitted for participation in the competition: 13 publications in scientific journals and materials for Handbook of Differential equations;
- Abstracts of the publications presented at the competition (in Bulgarian and in English);
- Copy of the competition announcement in the State Gazette;
- Recommendation from Prof. DSc. Nedyu Popivanov;
- Declarations by co-authors in collective publications - by Prof. DSc. Nedyu Popivanov and Assoc. Prof. Dr. Alexey Nikolov.

The documents are well prepared.

2. Details of the applicant

Tsvetan Hristov was born in 1974. In 1998 he obtained a master's degree from the Faculty of Mathematics and Informatics of Sofia University “St. Kliment Ohridski ", specialty "Mathematics", specialization "Differential equations". In 2006, the Higher Attestation Commission adjudicated him the educational and scientific degree “Doctor” after he defended his dissertation on "Singularities of solutions to hyperbolic equations in domains with characteristic boundaries." From 1998 to 2001 he worked as a mathematician in the section "Differential Equations" at IMI-BAS. Since 2005, he has worked in the Department of Differential Equations at FMI-SU consecutively as an Assistant Prof. (until 2007) and Chief Assist. Prof. (since 2007).

3. General characteristics of the applicant's scientific work and achievements

The candidate Dr. Tsvetan Hristov participates in the competition with 13 scientific papers, which do not repeat these from the previous procedures for acquiring the educational and scientific degree “Doctor” and for the academic position of “ Chief Assistant Professor”. All scientific papers presented are in English. All scientific papers presented are in English. They are an in-depth and multilateral study of multidimensional boundary value problems for degenerating hyperbolic equations (such as Tricomi and Keldysh type) that are related to the name of M. Protter. They are a natural continuation of the topic, from the dissertation of the candidate.

The research activity of the candidate and the publications presented by him in the competition are in the Professional field 4.5 Mathematics, specialty "Differential equations". He is the single author in three of the submitted publications for participation [8, 12, 13], one

[3] is co-authored with N. Popivanov, one [11] is co-authored with A. Nikolov, five [4, 5, 6, 7, 9] co-authored with N. Popivanov and M. Schneider and three [1, 2, 10] co-authored with N. Popivanov, A. Nikolov and M. Schneider. The candidate presented declarations by Prof. N. Popivanov and Assoc. Prof. A. Nikolov, which attest to the equal personal contribution of the candidate in the collective publications. In fact, declaring an equivalent contribution can also be found in the text of publications [1, 2].

There is no proven plagiarism in the scientific works of Dr. Tsvetan Hristov, presented in the competition.

As a habilitation work, the candidate submitted one publication in the prestigious journal Boundary value problems (IF2017 = 1.156, Q1-WoS) and one publication in Advances in mathematical physics (IF2017 = 0.710, Q4-WoS). The other publications are as follows: two publications in the "Comptes rendus de l'Academie bulgare des sciences" (IF2007 = 0.106, Q4 - WoS and IF2017 = 0.270, Q4 - WoS), i.e. 4 publications in Impact Factor journals with overall impact factor **2.242.** In scientific journals, referenced and indexed by Web of Science and Scopus (with SJR) are published 8 articles, as one in Siberian Advances in Mathematics and seven in conference volumes published in the renowned AIP Conference proceedings series. One publication in the scientific peer review journal “Doklady AMAN”.

The candidate has provided evidence for 60 citations in journals, referenced and indexed in world-famous scientific information databases (Web of Science and Scopus). Ten of these are in Impact Factor journals (JCR - WoS) and 50 are in Impact Rank journals (SJR-Scopus).

The scientific papers of Dr. Tsvetan Hristov, submitted for participation in the competition, meet the minimum national requirements (under Art. 2b, para. 2 and 3 of the ADAPRB) and the additional requirements of Sofia University “St. Kliment Ohridski" for the academic position of Assistant Professor in the scientific field and professional field of the competition. According to the submitted documents, the candidate for the competition has the following point asset:

Group of indicators	**Content**	**Associate Professor**	**Points of Tsvetan Hristov**
„А“	Indicator 1	50	50
„В“	Indicators 3 or 4	100	111
“Г“	Sum of Indicators from 5 to 10	200	312
“Д”	Sum of the points in the Indicator 11	50	480

Dr. Tsvetan Hristov has participated in a number of scientific projects, has reported on numerous international and national conferences, and has been on several specializations abroad at reputable universities in Germany, France and Italy. At the 8th Young Scientist School in Nalchik, Russia, in 2011, he was honored with the award for scientific novelty and originality of his report. The data for these activities can be systematized as follows:

Participation in research projects:

- Member of the scientific teams of 6 projects at Bulgarian NSF, one of which under a program for bilateral cooperation with Russia, with partners - a scientific team of the Moscow State University “M. V. Lomonosov ";
- Head of a project at Science Fund of Sofia University;
- Member of the scientific teams of 16 projects at Science Fund of Sofia University.

Conference talks:

- International - 36, including conferences held in Norway, Germany, Russia, Portugal, Czech Republic, Greece, Slovakia, Macedonia;
- National – 18.

Specializations abroad:

- Karlsruhe Universitaet, Germany, September 2002 – August 2003
- Universita Degli Studi Della Tuscia, Viterbo, Italy, September 2007
- Universite Louis Pasteur, Strasbourg, France, September 2008.
- Universite Louis Pasteur, Strasbourg, France, September 2010.

4. General characteristics of the applicant's teaching activities

Dr. Tsvetan Hristov's teaching activity is large in scope and diverse in content. It started in 1998 and continues to this day. From the submitted information under Art. 112 (2) of RTCAADOAPSU and the appendices thereto, it is clear that he has teaching activity such as seminars and lectures at the FMI in various courses in the field of ordinary and partial differential equations and mathematical analysis:

- Lectures and seminars on the compulsory courses "Differential equations and applications", "Differential equations", "Differential and Integral Calculus 1", "Differential and Integral Calculus 2 ", as well as on the elective courses "Partial Differences Equations and applications", "Selected Chapters of Mathematical Analysis";
- Seminars on the compulsory courses "Partial differential equations", "Equations of mathematical physics", "Variational Methods in Mathematical Physics (in Bulgarian and English, for master degree), "Mathematical Analysis of the Functions of Many Variables", "Differential and Integral Calculus 1 and 2" (Faculty of Physics of SU).

The candidate has teached the seminars on “Advanced Mathematics 1” in English for Mechanical Engineering at Karlsruhe Universitat, Germany, in 2002/2003.

The candidate has developed part of computer exercises for the courses "Differential Equations and Applications" for the bachelor programs "Informatics" and "Software Engineering". In these courses, Matlab is used for numerically and symbolically solving problems for ordinary and partial DE and visualize real processes modeled with them.

I was very impressed by the work of Dr. Ts. Hristov in the direction of the application of mathematics in solving applied problems. He was the scientific supervisor of a master's thesis on "Convection-diffusion equations for 3D volcanic ash modeling", successfully defended in 2013. He was the leader of two projects and a participant in a third project to improve the quality

and effectiveness of education at FMI. In these projects, software for mathematical modeling and visualization of processes modeled by the equation of convection and diffusion, such as the pollution of the atmosphere by volcanic ash upon eruption of a volcano, has been developed. He also developed two chapters "Linear differential equations and systems. Mechanical oscillations" and "Wave processes and wave equations. Correct and incorrect problems in mathematical physics'' from a textbook used in a number of courses in which differential equations are studied. In these chapters, the focus is specifically on the use of differential equations to model real processes and computer simulate them using software for scientific calculations and visualization. Dr. Tsvetan Hristov also has written an article on this subject ([25] in the list of all publications) published in the Proceedings of the 48th Spring Conference of the Union of mathematicians in Bulgaria, 2019. It discusses the increase in efficiency in the education of Equations of Mathematical Physics through visualization of real processes. Impressed by the diverse apparatus with which the candidate skillfully uses in his teaching activity.

5. Substantive analysis of the scientific and applied scientific achievements of the applicant, presented in the materials for participation in the competition

I accept for review all 13 publications submitted by Dr. Tsvetan Hristov for participation in the competition. Twelve of the publications are in the field of partial differential equations and are devoted to the study of Protter problems for degenerating hyperbolic equations. One article considered the integrating electronic assessment into differential equation education.

One conditional division of scientific papers with regard to the scientific results obtained in them is the following:

Group 1. Existence, uniqueness and behavior of generalized solutions to the Protter problem for weakly hyperbolic Tricomi-type equations [3, 4, 5];

Group 2. Formulation of multidimensional Protter-Morawetz problems for weakly hyperbolic Keldysh-type equations and results for the existence, uniqueness and asymptotic behavior of their generalized solutions [1, 2, 4, 6-12];

Group 3. Integration of online identification tools developed by TeSLA system into differential equation education [13].

I will consider each of these groups separately:

Group 1. The transonic processes in gas dynamics are modeled with boundary value problems for mixed elliptic-hyperbolic type equations. For example, the two-dimensional Guderley - Morawetz problem describes the flow around airplane wing from a stream at transonic speed. This problem has been studied in a number of works by K. Morawetz, P. Lax, R. Phillips. Its multidimensional analogue, as well as the multidimensional analogs of the classical plane Darboux and Cauchy-Goursat problems, were formulated by M. Protter in the middle of the last century. The higher dimension and the complex geometry of the boundary of the domain in which the equations are considered makes these problems extremely difficult to study. Multidimensional problems are unusual and different phenomena occur. For example, in the case of the wave equation (Tong Kwnag-Chang, 1957) and the degenerating hyperbolic equations of the Tricomi type without lower order terms (N. Popivanov, M. Schneider, 1993 and Khe Kan Cher, 1998), corresponding Protter problems are not well-posed because they have infinite-

dimensional co-kernels. This means that the considered problems are not Fredholm in the frame of classical solvability. In many cases for smooth right sides, there are solutions with very strong singularities isolated at one point from the boundary of the domain. For this reason, different classes of generalized solutions in special weighted functional spaces are introduced where solutions are sought.

This is also the approach in the paper [5], which introduces special quasi-regular solutions to the Protter problems for degenerating hyperbolic Tricomi-type equations with lower order terms ($m \in \mathbf{R}, m > 0$):

$$(1) \quad t^m \left[u_{x_1 x_1} + u_{x_2 x_2} \right] - u_{tt} + b_1 u_{x_1} + b_2 u_{x_2} + b u_t + c u = f(x_1, x_2, t).$$

The equation (1) is considered in the three-dimensional domain, introduced by Protter. There are studied the first P_1^m and the second P_2^m Protter problems, and some problems connected with their adjoint problems. Results for the uniqueness of quasi-regular solutions under certain conditions on the lower order terms are formulated. One of these conditions is a three-dimensional analogue of the two-dimensional Protter condition, and it requires the vanishing of the lower order coefficients in front of the spatial derivatives on the parabolic boundary for power of degeneration $m \geq 2$. In the paper are given interesting examples for lower order terms in (1), such that each one of the adjoint problems to the P_1^m and P_2^m can has no more than one quasi-regular solution.

In [4] is obtained results for the uniqueness of another kind of generalized solutions with possible singularity to the problem P_1^m for the equation (1) without first order terms.

In [3], there are studied the third Protter problem P_α^m for weakly hyperbolic Tricomi-type equations (1). The article introduces a special generalized solution with a possible singularity at one boundary point - the origin O. The results for the existence and uniqueness of such solution are formulated under conditions for lower order terms, similar to the two-dimensional Protter condition. Smooth right-hand sides f, and additional conditions on the coefficients in (1) are given, in which the corresponding generalization solutions have a strong power-type singularity, isolated at the point O.

Group 2. In [4] is formulated new Protter-Morawetz problem PK for three-dimensional weakly hyperbolic equations of Keldysh type ($m \in \mathbf{R}, 0 < m < 2$):

$$(2) \quad u_{x_1 x_1} + u_{x_2, x_2} - (t^m u_t)_t + r u = f(x_1, x_2, t) .$$

The equation (2) is considered in a domain Ω_m, analogous to the Proter domain in Tricomi case. The degeneration here is in front of the second time derivative, not in front of the spatial second order derivatives, as in the case of Tricomi. The more complex geometry of the domain in the Keldysh case, due to the different behavior of the characteristics, leads to a non-standard setting of the Protter problems and significantly complicates its study. In the considered problem PK the boundary condition $u = 0$ is given only on one of the characteristic surfaces, but parabolic part of the boundary is free of data. In [4] infinitely many classical solutions to the homogeneous adjoint problem PK^* in the case $r = 0$ are found. This result shows that, like the case of Tricomi, the problem PK is ill-posed in the frame of classical solvability. For this reason, classes of new quasi-regular solutions are introduced in the article. A beautiful technique based

on the exact Hardy-Sobolev inequality was developed to prove the uniqueness theorem in these classes. These results are generalized in [6] for Keldysh-type equations with lower order terms

(3) $u_{x_1x_1} + u_{x_2x_2} - (t^m u_t)_t + b_1 u_{x_1} + b_2 u_{x_2} + bu_t + cu = f(x_1, x_2, t)$,

and in the more delicate case of the Protter-Morawetz problem for elliptic-hyperbolic equations involving the Gellerstedt operator. For semi-linear equations the non-existence of non-trivial solutions was established in critical and supercritical cases.

In order to avoid the infinite number of conditions for classical solvability of the problem PK in [7 - 9, 12], new generalized solutions of (3) are introduced. These solutions can have singularity at one boundary point - the origin O, but the derivative u_t can have singularity up to certain order on the parabolic boundary. Results for the existence and uniqueness of such solutions are obtained without imposing conditions for vanishing of the lower order coefficients on parabolic boundary for $m \in (0,1)$. This is natural, since the Protter condition in the case of Tricomi imposes similar conditions on the lower order coefficients only for high power of degeneration. Generalized solutions that have at least a power-type singularity, isolated at point O are found.

The papers [1, 2, 10, 11] are dedicated to the Protter-Morawetz problems PK and PK^* for four-dimensional Keldysh-typeэa equations of the form

(4) $u_{x_1x_1} + u_{x_2x_2} + u_{x_3x_3} - (t^m u_t)_t = f(x_1, x_2, x_3, t)$.

In [1, 10] generalized solutions have been introduced in an appropriate weighted functional space for the growth of the singularities. With the help of special functions – the Appel and the Horn series the Riemann-Hadamard function for problem PK is constructed. The construction of this function is not a trivial result and enables the integral representation of the generalized solution to the problem PK to be found. As a consequence, the uniqueness theorem is proved, and when the right-hand side function is a generalized harmonic polynomial of three-dimensional spherical functions the existence theorem is proved also. Finding an integral representation of the solution by special functions allows precision results for the behavior of the solution around the singular point to be obtained. In [2] and [11] infinitely many classic solutions to the four-dimensional homogeneous adjoint problem PK^* have been found. With their help, after extremely skilful calculations, conditions for the right-hand side f in equation (4) were found, which influence the growth of the singularity of the solution. Asymptotic behavior of the generalized solution of the problem PK around the singular point when the right-hand side function is a generalized harmonic polynomial of order l is found. The solution has power-type singularity $|x|^{-p-1}$, where p takes values $0,1,...,l$, depending on the right-hand side f of which solutions to the homogeneous problem PK^* is orthogonal in $L_2(\Omega_m)$.

Group 3. The paper [13] describes an experiment to use the “Face recognition” tool, developed by TeSLA system, in the assessment in the "Differential equations" course, compulsory for the bachelor program “Software Engineering” at FMI of Sofia University with the lecturer Dr.

Tsvetan Hristov. The assessment model presented combines electronic assessment and traditional summative assessment, which together with the direct relationship between differential equations and real process modeling make the learning model attractive to students. This leads to increasing student engagement in the learning process and increases their success in the course.

The candidate demonstrates a thorough knowledge of the theory of the Partial Differential Equations. He has researched important problems and obtained useful results, which have been cited many times. Impressive is the citation of his papers in articles by reputable mathematicians such as Academician of RAS E. Moiseev, Academician of NASRK T. Kalmenov, Corresponding member of RAS T. Moiseev, Corresponding member of NASRK M. Sadybekov, J. Mauersberger, K. Zhang, J. Song, Y. Cao. Ten of the citations are in journals with Impact Factor – 2 citations in Journal of Mathematical Analysis and Applications (IF2018 = 1.188, Q1-WoS), 2 citations in Integral Transforms and Special Functions (IF2017= 0.828, Q2-WoS), 3 citations in Advances in Mathematical Physics (IF2018= 0.936, Q3-WoS), 2 citations in Applied Mathematics-A Journal of Chinese Universities (IF2018= 0,806, Q3-WoS) and citation in Differential Equations (IF2016= 0.371, Q4-WoS). Fifty citations are in journals with Impact rank (SJR). Eight citations are in scientific peer-reviewed journals (outside WoS and Scopus) and 10 citations are in PhD theses, among which the two citations that impress are in the dissertation of PhD student supervised by Academician E. Moiseev, MSU "M. V. Lomonosov ” in 2017.

6. Critical notes and recommendations

I have no significant critical comments on the materials in the competition and in particular on the scientific and pedagogical works of Dr. Tsvetan Dimitrov Hristov.

7. Personal impressions of the applicant

I know Dr. Ts. Hristov from his participation in the international conference "Application of Mathematics in Engineering and Economics" (in 2017, 2018 and 2019), organized by FPMI-TU and FMI-SU. His talks have always aroused interest amongst the listeners, including among some respected foreign scientists attending the conferences. I have no personal impressions of the candidate's pedagogical activity, but I trust the attached recommendation from Prof. Nedyu Popivanov and the certificate from Assoc. Prof. Geno Dachev, according to which he has a rich and successful pedagogical activity and actively participates in the academic life at FMI and Sofia University.

8. Conclusion on the application

Having become acquainted with the materials and scientific works presented in the competition and on the basis of the analysis of their significance and the scientific and applied scientific contributions contained therein, I confirm that the academic achievements of the candidate Chief Assistant Professor Dr. Tsvetan Dimitrov Hristov meet the requirements of the ADAPRB, the Regulations for its implementation and the corresponding Regulations of SU for the occupation of the academic position of “Associate Professor” in the professional field 4.5. Mathematics (Differential equations). In particular, the applicant meets the minimum national

requirements in the professional field and no plagiarism has been detected in his scientific papers submitted at the competition.

I give a positive assessment of the application of Chief Assistant Professor Dr. Tsvetan Dimitrov Hristov.

II. GENERAL CONCLUSION

Based on the above, I **strongly recommend** the scientific jury to vote on a proposal to the Council of the Faculty of Mathematics and Informatics of Sofia University St. Kliment Ohridski to select Chief Assistant Professor Dr. Tsvetan Dimitrov Hristov for the academic position of "Associate Professor" in the professional field 4.5. Mathematics (Differential equations).

09.12.2019 Reviewer: .....

(Prof. PhD Andrey Zahariev)